PAUL CUFFEE SCHOOL

A Maritime Charter School for Providence Youth

9th Grade Mathematics Curriculum

Captain Thomas R. Beall, U. S. Navy (Ret.)

Volume I

First Trimester

Algebra I Syllabus

I. **Course Description:**

Algebra I will introduce you to basic skills in numbers, operations with numbers, single variable equations and inequalities, data analysis, modeling with linear equations, and basic polynomial and exponential functions. You will learn these skills through a combination of traditional problem-solving practice and projects linked to problems in the real world. You will be assessed on your mastery of these skills through frequent homework assignments, in – class work, tests, and projects. The objective of this course is not just skills mastery. You will also develop the ability to solve complex, multi-step tasks by a fixed deadline.

II. **Course Objectives (Academic Standards):** Upon completion of this course, you will:

A. *First Trimester:*

1. Assemble raw data into computer – based analysis tools such as Microsoft EXCEL.
2. Develop and describe linear models to explain data sets and predict future outcomes.
3. Solve equations of one variable using multiple techniques.
4. Describe observed occurrences in the real world using single variable equations.
5. Express problems presented to you in words as expressions or equations and develop solutions to those expressions or equations.

B. *Second Trimester:*

1. Solve systems of two equations of two variables using conventional and computer – based techniques.
2. Solve systems of two equations of two variables using Matrix Algebra.
3. Manipulate and simplify expressions using exponents.
4. Graph and interpret exponential functions.
5. Graph and interpret second degree polynomial functions.
6. Factor second degree polynomial functions using various techniques.
7. Use polynomial functions to solve problems.

C. *Third Trimester:*

1. Develop solutions to complex, multi – step tasks and submit results on time as established by the instructor.

III. **Classroom Expectations:** This classroom is a professional work environment. We can have fun as we learn together but to do that we must show each other respect. I respect you and will assume you can act respectfully to me and to your colleagues (classmates). You can demonstrate that respect by following a simple set of expectations which are posted in the classroom:

A. When bell rings, be in seat with homework out starting warm-up.

B. Do warm-up in silence. Remain quiet until everyone is finished.

C. When working together as a class, one person speaks at a time. Everyone else listens quietly and respectfully.

D. Raise hand if you want to be called on. Wait to be called on before speaking.

E. During pair or group work, speak quietly – don't distract other groups.

F. Stay focused – finish class work before end of class.

In addition, profane language, to include use of the word "cuff" is not to be used in the classroom.

IV. **Preparation for Class:** Each day, I will assess and grade your readiness to learn. To receive a satisfactory grade:

A. Arrive on time, in seat, quietly starting warm – up when bell rings.

B. Have your 3 – ring binder, paper, writing instrument, and agenda with you. Your 3 – ring binder should have 4 tabs: (1) Vocabulary, (2) Notes, (3) Homework, (4) Project hand-outs and project work.

C. Complete the warm – up. You need not get the right answer but you do need to make an attempt to solve the problem. ***Simply copying the problem down on paper will not earn you credit.***

V. **Homework:** To receive credit for homework, you must have it completed and ready to show me at the beginning of the class period on which it is due. *If you leave it in your locker or at home accidentally, you will not receive credit for it.*

VI. **Grading:** Grades at Paul Cuffee School are standards based. In other words, when you achieve proficiency on a particular standard, you receive a grade of "3" (proficient) or higher. Until you achieve proficiency, you will receive a grade of "2" or lower on that standard. The grades we use at Paul Cuffee School are:

- **4: Honors:** This grade will be assigned if you achieve proficiency and you demonstrate performance above what I expect of a student at the 9th grade level.
- **3: Proficient:** This grade will be assigned if you meet my expectations for proficiency at the 9th grade level.
- **2: Partially Proficient:** This grade will be assigned if you demonstrate proficiency on some of the material covered by the standard but not all of it. For example, to achieve proficiency in the standard, "Solve systems of two equations of two variables using conventional and computer – based techniques" you must demonstrate proficiency on all the techniques. If you are proficient on some but not all, you will receive a "2".
- **1: Not Proficient:** This grade will be assigned if you demonstrate no proficiency on the standard.

A. **Academic Standards Grades.** The academic standards you will be graded on throughout the year are listed under paragraph II, Course Objectives, above. Projects and assessments will be linked to these academic standards so your performance on assessments and successful completion of projects will determine your academic standards grades.

B. **Habits of a Learner Grades.** You will also be graded on your habits of a learner, using the same "1" through "4" scale. Among the things I consider in these grades are your preparation for class, your homework completion, the effort you put into projects and tests and your behavior in class. If you follow the guidelines set forth in this syllabus, you will receive a grade of at least "3" in the "habits of a learner" categories.

C. **Overall Grade.** At the end of each trimester, you will also receive an overall grade that reflects my assessment of how well you have met the academic standards and the "habits of a learner" standards. This grade will appear on your report card as:

- High Honors.

- Honors – equivalent to a “4”.
- Good Standing – equivalent to a “3”.
- Partially Satisfactory – equivalent to a “2”.
- Unsatisfactory – equivalent to a “1”. **This is a failing grade.**

VII. **Topics and Assessments:** This course is organized into units which cover the academic standards in paragraph II above. The units are:

A. **“We Choose to Go to the Moon.”** We will explore data analysis and linear modeling using computer software in the context of American space exploration since 1960. Integrating the history of America’s adventure in space with mathematics, we will explore how mathematical modeling can be used to achieve goals such as a voyage to the Moon and predict future events. Assessments and projects include:

1. Space shuttle launch data task.
2. Rocket design and construction project.
3. Space debris data analysis project.
4. Final unit assessment.

B. **Equations and Inequalities.** We will explore the use of single variable equations to model various real – life situations. This will include continuation of our analysis of the flight of spacecraft, gathering data and modeling spacecraft flight as single variable equations. Assessments and projects include:

1. Rocket altitude analysis project.
2. Final unit assessment.

C. **“A Voyage to the Panama Canal.”** We will voyage from San Diego, California to the Panama Canal, learning how to use equations to navigate and then transit that engineering marvel that was constructed one hundred years ago. Assessments and projects include:

1. Panama Canal lock scaled drawing project.
2. Panama Canal transit planning project.
3. Final unit assessment.

D. **Systems of Equations.** We will learn to solve systems of two equations and two variables using the graphing, substitution, and elimination methods. We will also use Microsoft EXCEL to solve systems and learn the Matrix Algebra technique that is used in this program. Assessments and projects include:

1. Systems of equations project.
2. Final unit assessment.

E. **Exponents and Exponential Functions.** We will learn how to manipulate expressions involving exponents and then explore how exponential functions are used to model growth and decay using computer software in a number of real – world applications. Assessments and projects include:

   1. Computer lab exercises.
   2. Final unit assessment.

F. **Polynomials.** We will learn how to graph polynomial functions using computer software and analyze their structure. We will learn multiple methods of factoring second degree polynomials into products of linear functions. Assessments and projects include:

   1. Computer lab exercises.
   2. Final unit assessment.

G. **Final Project: Mathematics and History – The Naval Battle of Midway, June 1942.** The final project will be in multiple parts and will be completed over the final trimester. It will test your ability to complete complex tasks on pre-established deadlines. The project will fall under a historical scenario (the World War II naval Battle of Midway) that will unfold as each part is given to you.

   1. Format. Each part will include instructions, some of the required materials, and all the information you will need to complete that part. It will be necessary to complete each part to move on to the next part because the answers to each part will be required when working on the next part.

   2. Grading. This project will constitute your third trimester grade. If you fail to complete any assigned task, you will fail the third trimester.

   3. Tasks. Project tasks include:

      a. Decoding task.
      b. Ship scaled drawing.
      c. Construction of three dimensional ship model.
      d. Systems of equations task.

e. Data analysis and linear modeling task.
f. War game board design.
g. War game participation.

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Geometry Syllabus (9th Grade)

I. **Course Description:**

Geometry will focus on application of geometric concepts to real – world challenges. We will explore topics such as naval architecture, cargo shipment, and ocean navigation; examining how you can use geometry to solve problems in these fields. You will also develop the ability to apply logic to proving the validity of geometric concepts. Most importantly, you will develop the ability to solve complex, multi – step problems by a fixed deadline.

II. **Course Objectives (Academic Standards):** Upon completion of this course, you will:

A. *First Trimester:*

1. Apply a general formula for surface area and volume of three – dimensional shapes to determine the surface area and volume of prisms and cylinders.
2. Construct three – dimensional scaled models from two – dimensional plans.
3. Determine the most efficient way to fill the volume of a three – dimensional object.
4. Develop the ability to apply principles of logic to geometric proof.
5. Express problems presented to you in words as expressions or equations and develop solutions to those expressions or equations.

B. *Second Trimester:*

1. Routinely convert units of measure into other units as part of problem solving.
2. Apply principles of Geometry to the practical application of celestial navigation.
3. Apply the principles of congruence and similarity when scaling two and three – dimensional shapes.
4. Develop a detailed understanding of circles and their fundamental place in representing real – world objects and concepts.
5. Apply the techniques of trigonometry to real – world applications.

C. *Third Trimester:*

1. Develop solutions to complex, multi – step tasks and submit results on time as established by the instructor.

III. **Classroom Expectations:** This classroom is a professional work environment. We can have fun as we learn together but to do that we must show each other respect. I respect you and will assume you can act respectfully to me and to your colleagues (classmates). You can demonstrate that respect by following a simple set of expectations which are posted in the classroom:

A. When bell rings, be in seat with homework out starting warm-up.

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D. Raise hand if you want to be called on. Wait to be called on before speaking.

E. During pair or group work, speak quietly – don't distract other groups.

F. Stay focused – finish class work before end of class.

In addition, profane language, to include use of the word "cuff" is not to be used in the classroom.

IV. **Preparation for Class:** Each day, I will assess and grade your readiness to learn. To receive a satisfactory grade:

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- **3: Proficient:** This grade will be assigned if you meet my expectations for proficiency at the 9th grade level.
- **2: Partially Proficient:** This grade will be assigned if you demonstrate proficiency on some of the material covered by the standard but not all of it. For example, to achieve proficiency in the standard, “Apply the principles of congruence and similarity when scaling two and three – dimensional shapes,” you must demonstrate proficiency with both congruence and similarity. If you have not achieved proficiency with both, you will receive a “2”.
- **1: Not Proficient:** This grade will be assigned if you demonstrate no proficiency on the standard.

A. **Academic Standards Grades.** The academic standards you will be graded on throughout the year are listed under paragraph II, Course Objectives, above. Projects and assessments will be linked to these academic standards so your performance on assessments and successful completion of projects will determine your academic standards grades.

B. **Habits of a Learner Grades.** You will also be graded on your habits of a learner, using the same “1” through “4” scale. Among the things I consider in these grades are your preparation for class, your homework completion, the effort you put into projects and tests and your behavior in class. If you follow the guidelines set forth in this syllabus, you will receive a grade of at least “3” in the “habits of a learner” categories.

C. **Overall Grade.** At the end of each trimester, you will also receive an overall grade that reflects my assessment of how well you have met the academic standards and the “habits of a learner” standards. This grade will appear on your report card as:

- High Honors.

- Honors – equivalent to a “4”.
- Good Standing – equivalent to a “3”.
- Partially Satisfactory – equivalent to a “2”.
- Unsatisfactory – equivalent to a “1”. **This is a failing grade.**

VII. **Topics and Assessments:** This course is organized into units which cover the academic standards in paragraph II above. The units are:

H. **“The Geometry of the Ship.”** Ships are what make world – wide commerce possible. Every day, thousands of ships are loading, moving, or offloading cargo in the great ports of the world. That cargo is packed into shipping containers which are nothing more than large rectangular prisms, three – dimensional geometric shapes. The fuel ships carry is stored into tanks which are also three – dimensional geometric shapes. In fact, ships are really just aggregations of geometric shapes. In this unit, we will explore how those shapes come together to make up a working ship. We will also explore how to measure the surface area and volume of these shapes to ensure the ship is stowed with enough fuel and cargo to make her voyage profitable. Assessments and projects will include:

1. Building a scale model of a container box project.
2. Surface area and volume of rectangular prisms assessment.
3. Building a three dimensional scaled ship model from two – dimensional drawings.
4. Surface area and volume of cylinders and other shapes assessment.
5. Ship stability lab.

I. **Logic and Proof.** We will learn concepts of logic and apply them to proving that geometric postulates are true. This unit will include many activities and conclude with an assessment.

J. **“A Voyage to the Panama Canal.”** We will voyage from San Diego, California to the Panama Canal, learning how to use equations to navigate and then transit that engineering marvel that was constructed one hundred years ago. Assessments and projects include:

4. Panama Canal lock scaled drawing project.
5. Panama Canal transit planning project.
6. Final unit assessment.

K. **Congruence and Similarity.** We will continue to use proofs to demonstrate that certain shapes are congruent, similar, or neither. These concepts are important when

creating real objects such as buildings or ships from scaled drawings or models. Assessments and projects include:

1. Solar System scale model project.
2. Final unit assessment.

L. **Circles.** Circles and lines are used to depict concepts in many fields of science and engineering including spacecraft orbits and trajectories and shipboard and submarine ballast. We will explore the characteristics of circles and lines as used in these and other applications. Assessments and projects include:

   1. Apollo lunar mission trajectory project.
   2. Final unit assessment.

M. **Trigonometry.** Acoustics, architecture, astronomy , cartography, civil engineering, geophysics, crystallography, electrical engineering, electronics, land surveying and geodesy, many physical sciences, mechanical engineering, machining, medical imaging , number theory, oceanography, optics, pharmacology, probability theory, seismology, statistics, and visual perception are all fields in which trigonometry and trigonometric functions are used. We will explore some of these applications as well as the basic relationship among the unit circle, triangle, sine wave. Assessments and projects include:

   1. Moons of Jupiter orbits project.
   2. Final unit assessment.

N. **Final Project: Mathematics and History – The Naval Battle of Midway, June 1942.** The final project will be in multiple parts and will be completed over the final trimester. It will test your ability to complete complex tasks on pre-established deadlines. The project will fall under a historical scenario (the World War II naval Battle of Midway) that will unfold as each part is given to you.

   1. Format. Each part will include instructions, some of the required materials, and all the information you will need to complete that part. It will be necessary to complete each part to move on to the next part because the answers to each part will be required when working on the next part.

   2. Grading. This project will constitute your third trimester grade. If you fail to complete any assigned task, you will fail the third trimester.

3. Tasks. Project tasks include:

    a. Decoding task.
    b. Ship scaled drawing.
    c. Vector analysis task.
    d. Systems of equations task.
    e. Data analysis and linear modeling task.
    f. War game board design.
    g. War game participation.

How likely am I to realize my dreams of becoming a professional athlete?

The odds of winning a NCAA sports scholarship are miniscule. Only about 2 percent of high school athletes win sports scholarships every year at NCAA colleges and universities. Yes, the odds are that dismal. For those who do snag one, the average scholarship is less than $11,000.[1]

1.2% of college players play professionally, 0.03% of high school players do.[2]

So the probability that you will win an NCAA scholarship and go on to play professional basketball is:[3]

NCAA Research

Estimated Probability of Competing in Athletics
Beyond the High School Interscholastic Level

Student Athletes	Men's Basketball	Women's Basketball	Football	Baseball	Men's Ice Hockey	Men's Soccer
High School Student Athletes	545,844	438,933	1,108,441	471,025	36,912	398,351
High School Senior Student Athletes	155,955	125,409	316,697	134,579	10,546	113,815
NCAA Student Athletes	17,500	15,708	67,887	31,264	3,944	22,573
NCAA Freshman Roster Positions	5,000	4,488	19,396	8,933	1,127	6,449
NCAA Senior Student Athletes	3,889	3,491	15,086	6,948	876	5,016
NCAA Student Athletes Drafted	48	32	255	806	11	49
Percent High School to NCAA	**3.2%**	**3.6%**	**6.1%**	**6.6%**	**10.7%**	**5.7%**
Percent NCAA to Professional	**1.2%**	**0.9%**	**1.7%**	**11.6%**	**1.3%**	**1.0%**
Percent High School to Professional	**0.03%**	**0.03%**	**0.08%**	**0.60%**	**0.10%**	**0.04%**

[1] http://www.cbsnews.com/8301-505145_162-57516273/8-things-you-should-know-about-sports-scholarships/
[2] http://www.businessinsider.com/odds-college-athletes-become-professionals-2012-2?op=1#ixzz2dsfrL0Nx
[3] http://www.ncaa.org/wps/wcm/connect/public/ncaa/pdfs/2011/2011+probability+of+going+pro

Was the United States justified in using atomic bombs on Japan?

Approximately 350,000 Japanese were killed or wounded by the two atomic bombs dropped on Japanese cities in August of 1945.[4]

American leaders expected that an invasion of Kyushu would be at least as costly as the battles to take Luzon in the Philippines and Okinawa. Fleet Admiral King, Commander-in-Chief of the U. S. Fleet, expected that casualty rates for the preliminary invasion of Kyushu (Operation OLYMPIC) would fall somewhere between those of Luzon (which had a casualty ratio of 5 Japanese for every American) and Okinawa (which had a casualty ratio of 2 Japanese for every American). If American planners had accepted King's estimate and assumed the casualty ratio would be 3.5 Japanese for every American (1:3.5) and then conducted a Lanchester force-on-force analysis, they might have estimated that American casualties would fall between 66,000 and 67,000. Japanese casualties of course would have comprised their entire estimated force of 350,000 since it is logical to assume that all Japanese would be killed, wounded, or captured. Therefore, American military planners might have estimated that total military casualties would be on the order of 416,000 to 417,000.

In 1914, British engineer F. W. Lanchester developed simple differential equations to model interaction of opposing forces in a land battle:

$$\frac{d(U.S.)}{dt} = -\beta(Japan)$$
$$\frac{d(Japan)}{dt} = -\alpha(U.S.)$$

If we use the 1:3.5 casualty ratio where $\beta = 0.01$ is Japanese combat effectiveness and $\alpha = 0.035$ is United States combat effectiveness then the result is that U. S. forces defeat the Japanese, incurring 66,642 casualties. Japanese casualties number its entire force of 350,000 (killed, wounded, or captured). Therefore, it was quite possible that the invasion would have cost the same number of Japanese lives and several tens of thousands of American lives as well.

[4] http://www.atomcentral.com/hiroshima-nagasaki.aspx

How much will it cost to go to college?

Annual in-state tuition at the University of Rhode Island is $ 14,086.00.

As of October 2012, the average amount of student loan debt for the Class of 2011 was $26,600, a 5 percent increase from approximately $25,350 in 2010. (Source: The Project on Student Debt).[5]

The life of a loan is typically 10 years with an Variable interest rates range from 3.17% APR – 9.37% APR.

Assuming a 10 year, $26,000 loan with a 6.2% APR:

$$A = P\left(1 + \left(\frac{r}{t}\right)\right)^{nt}$$

A = total cost of loan

P = amount of money loaned

r = interest rate

t = number of payments per year

n = life of the loan in years

$$A = \$26{,}000\left(1 + \frac{0.062}{12}\right)^{120}$$

Total URI tuition:	$4 \times \$14{,}086 = \$56{,}344$
Amount paid for by you:	$\$56{,}344 - \$26{,}000 = \$30344$
Amount paid for by loan:	$\$26{,}000$
Total cost of loan:	$\$65739.65$
Total cost of education:	$\$65{,}739.65 - \$96{,}083.65$
Total cost with typical athletic scholarship:	$\$96{,}083.65 - \$11{,}000 = \$85{,}083.65$

[5] http://www.asa.org/policy/resources/stats/

Name: ______________________
Date: ________________________
Advisor: _____________________

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“We Choose to Go to the Moon”

Algebra I Unit Plan

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Guiding Idea

There is no strife, no prejudice, no national conflict in outer space as yet. Its hazards are hostile to us all. Its conquest deserves the best of all mankind, and its opportunity for peaceful cooperation many never come again. But why, some say, the moon? Why choose this as our goal? And they may well ask "why climb the highest mountain?" Why, 35 years ago, fly the Atlantic? Why does Rice play Texas?

We choose to go to the moon. We choose to go to the moon in this decade and do the other things, not because they are easy, but because they are hard, because that goal will serve to organize and measure the best of our energies and skills, because that challenge is one that we are willing to accept, one we are unwilling to postpone, and one which we intend to win, and the others, too.

President John F. Kennedy, September 12, 1962, at Rice University, Houston, Texas

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President Kennedy's words inspired a generation of scientists, technicians, and pilots to create new ideas, develop new technologies, and make a dream a reality in just eight years. These men and women brought mathematics, science, and engineering to bear to achieve this dream. My hope, in developing this unit plan is to inspire a future mathematician, scientist, or engineer to embrace new frontiers in the exploration of space.

P C H S

CURRICULUM MAPPING: Middle and High School Algebra – We Choose to Go to the Moon

CENTRAL QUESTION(S)/THEMES

1. How do we use Algebra to predict the future?
2. What is the best model to use with a particular set of data?

TEXT(S) AND RESOURCES

1. National Aeronautics and Space Administration: Space Shuttle Launch Data.
2. National Aeronautics and Space Administration: High Power Paper Rockets Activity.
3. National Aeronautics and Space Administration: Space Debris Data.

STUDENTS WILL KNOW AND BE ABLE TO . . .

Content, Skills, & Standards to be assessed by rubric(s) in this unit (Common Core, GSE, HOL and PCHS Expectations)

1. **M(F&A)-10-1:** Identifies, extends, and generalizes a variety of patterns (linear and nonlinear) represented by models, tables, sequences, or graphs to solve problems.
2. **M(DSP)–10–1:** Interprets a given representation (e.g., box-and-whisker plots, scatter plots, bar graphs, line graphs, circle graphs, histograms, frequency charts) to make observations, to answer questions, to analyze the data to formulate or justify conclusions, critique conclusions, make predictions, or to solve problems within mathematics or across disciplines or contexts (e.g. media, workplace, social and environmental situations).
3. **M(DSP)-10-2:** Analyzes patterns, trends, or distributions in data in a variety of contexts by determining or using measures of central tendency (mean, median, or mode), dispersion (range or variation), outliers, quartile values, or estimated line of best fit to analyze situations, or to solve problems; and evaluates the sample from which the statistics were developed (bias, random, or non-random).
4. **M(DSP)-10-6:** Analyzes data consistent with concepts and skills in M(DSP)–10–2.

PROJECT/PRODUCT & PUBLIC DEMONSTRATION

1. Rocket construction, experimentation, and data analysis project.
2. Space Debris Project.

OTHER EVIDENCE OF STUDENT LEARNING

1. Practice worksheets.
2. Pencil and Paper Post-assessments.

SCAFFOLDED TEACHING AND LEARNING ACTIVITIES

1. **"Space Shuttle Launch" Activity.** Students will graph data on the launch of a space shuttle, fit a linear model, determine the characteristics of that model, and use the model to predict future outcomes.

2. **"Building and Testing a Rocket" Project.** Students will design and construct rockets out of poster board and paper, seeking to build a rocket that will fly higher than any other. Students will test fire their rockets using an air pump, measure the rocket's final altitude over at least four flights, and analyze the data gathered using linear modeling.

3. **"Space Debris" Project.** Students will enter data on the increase of man – made debris in Earth orbit over a 30-year period into Microsoft EXCEL. Students will graph the data, fit linear models to it, determine the characteristics of those models, and use those models to predict future outcomes.

Unit Goals

Students will:

1. Place learning in a real-world context by exploring the connections between basic algebra and astronautical science.

2. Design and build aerodynamic shapes.

3. Gather accurate data through experimentation.

4. Accurately enter data and develop simple, meaningful graphs to display that data.

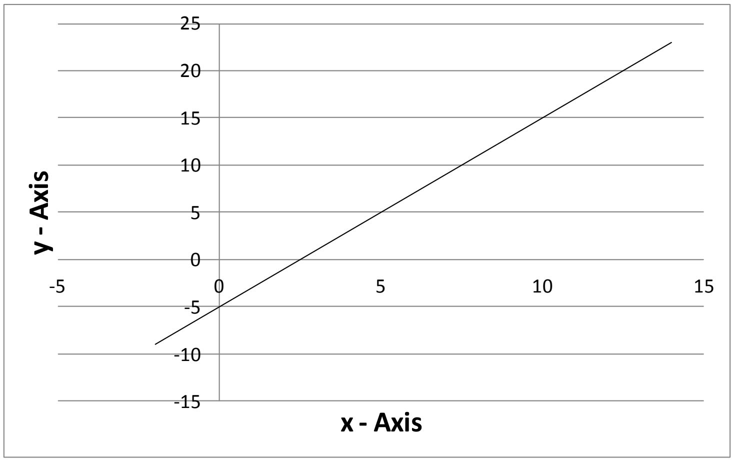
5. Develop linear models and use those models to predict future outcomes.

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:________________________________
Advisor:______________________________
Date:_________________________________

Linear Modeling Unit Pre-Assessment

Directions: For each graph below, determine the slope (m) and y-intercept (b) of the line and write the equation of the line.



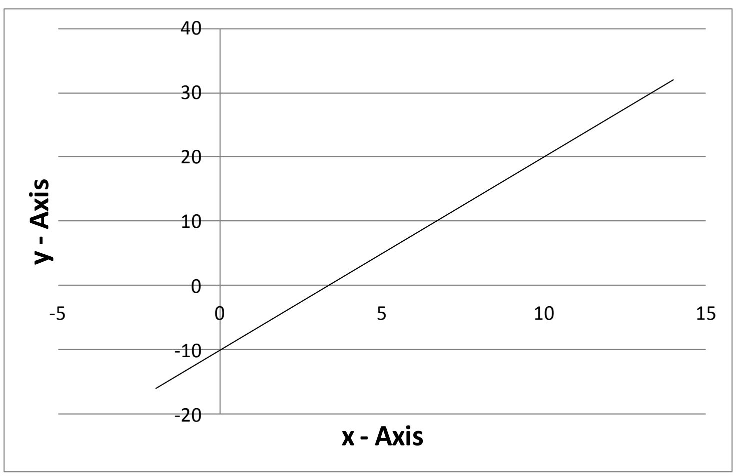
$y_2 =$ ________ $y_1 =$ ________

$x_2 =$ ________ $x_1 =$ ________

Slope: $\frac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ____________

Equation: ____________



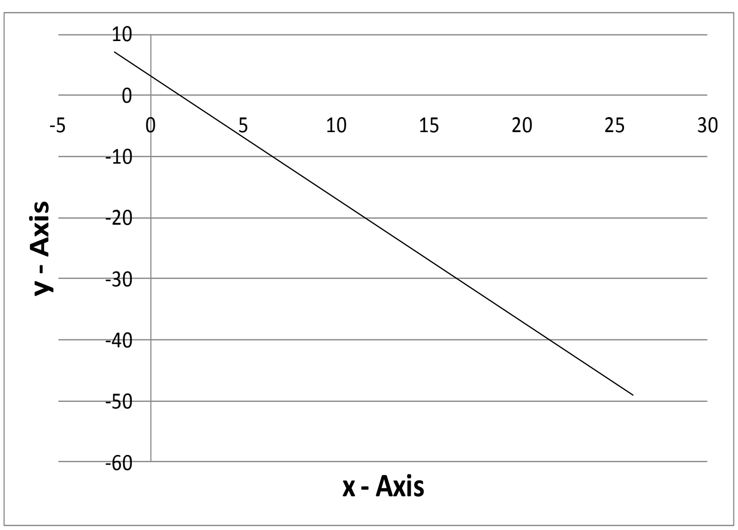
$y_2 =$ ________ $y_1 =$ ________

$x_2 =$ ________ $x_1 =$ ________

Slope: $\frac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ____________

Equation: ____________



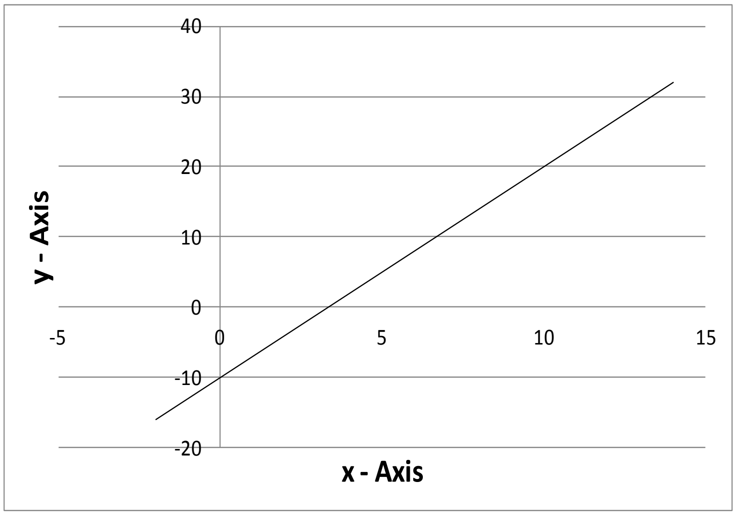
$y_2 =$ ________ $y_1 =$ ________

$x_2 =$ ________ $x_1 =$ ________

Slope: $\frac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ____________

Equation: ____________



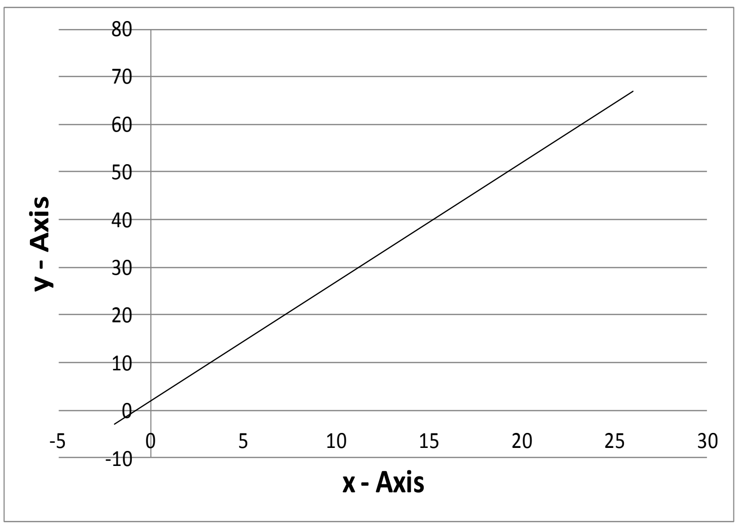
$y_2 =$ ________ $y_1 =$ ________

$x_2 =$ ________ $x_1 =$ ________

Slope: $\dfrac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ________________

Equation: ________________



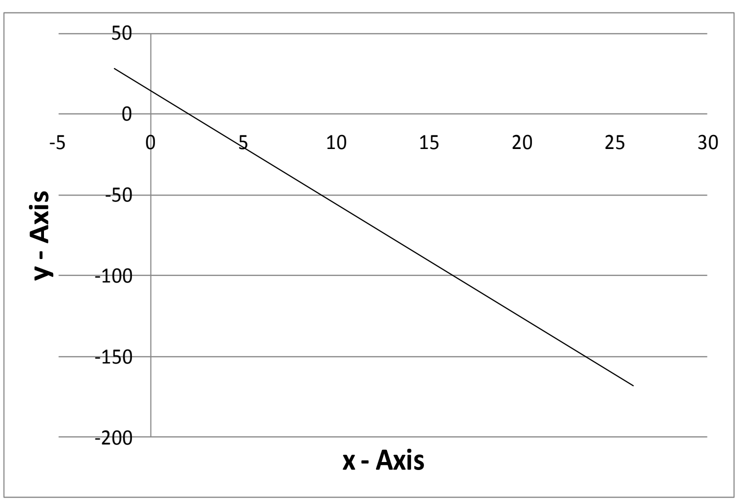
$y_2 =$ ________ $y_1 =$ ________

$x_2 =$ ________ $x_1 =$ ________

Slope: $\dfrac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ________________

Equation: ________________



$y_2 =$ ________ $y_1 =$ ________

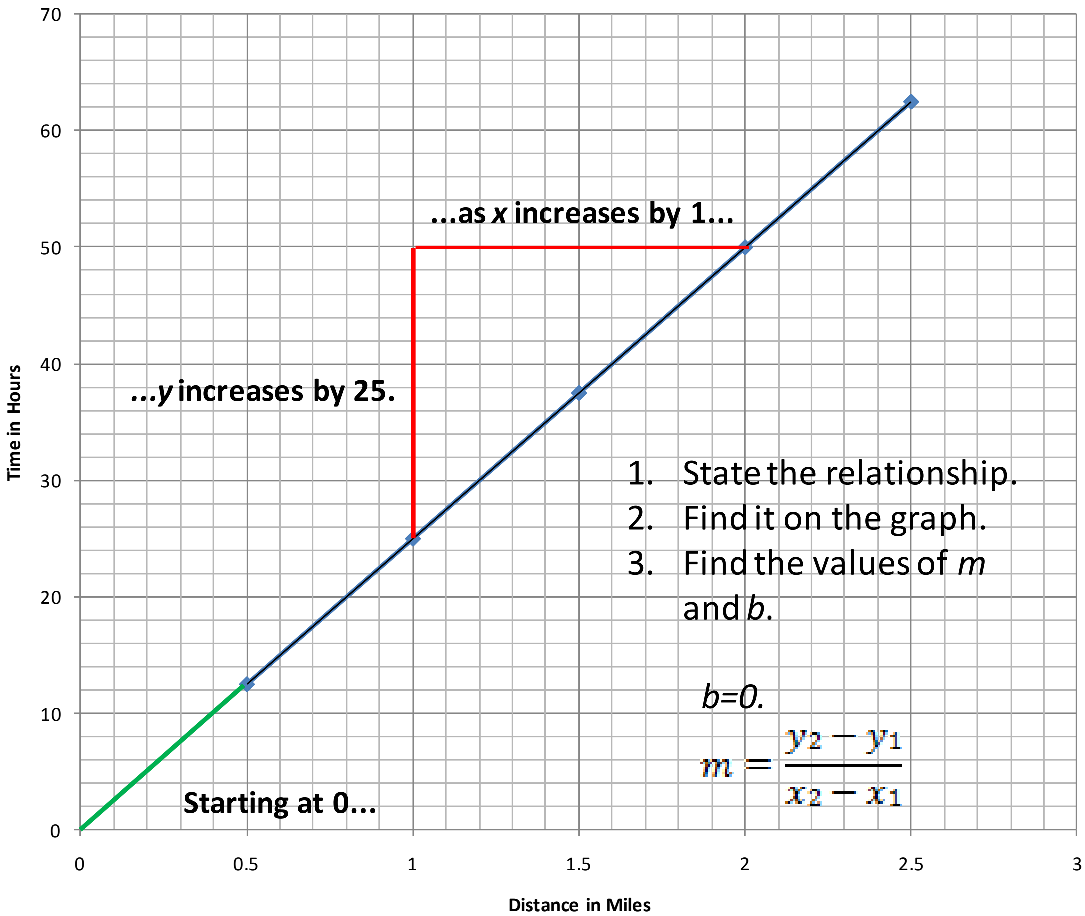
$x_2 =$ ________ $x_1 =$ ________

Slope: $\dfrac{y_2 - y_1}{x_2 - x_1} =$ ________

y-intercept: ________________

Equation: ________________

Lines Graphed on the Coordinate Plane – *Keep this forever!!!!*



Slope: A measure of the steepness of a line. Given two points with coordinates (x_1, y_1) and (x_2, y_2) on a line, the slope, *m*, of the line is given by:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept: the *y*-coordinate of a point where a line crosses the *y*-axis.

<table>
<tr><td>Grade / Content Area</td><td>9th Grade: Algebra I</td></tr>
<tr><td>Lesson Title</td><td>Introduction to Algebra / Linear Relationships</td></tr>
<tr><td>Guiding Question</td><td>What is a relationship in Algebra?</td></tr>
<tr><td rowspan="2">Content Standards</td><td>State Content Standards:

I. M(F&A)-10-1
II. M(DSP)-10-1
III. M(DSP)-10-6</td></tr>
<tr><td>Common Core Standards:

I. Graph linear functions expressed symbolically and show key features of the graph.
II. Represent data with plots.</td></tr>
<tr><td>Preparation</td><td>I. Classroom Organization: Students will work in pairs.

II. Differentiation. Group work will provide opportunity for students to leverage talents of their peers.

III. Materials:

A. For each group of students:

1. Relationships Worksheet.</td></tr>
<tr><td>Instruction and Engagement</td><td>I. Warm-up (10 minutes). Go over classroom expectations, homework expectations.</td></tr>
</table>

	Classroom Expectations 1. ***When bell rings, be in seat with homework out starting warm-up.*** 2. ***Do warm-up in silence. Remain quiet until everyone is finished.*** 3. ***When working together as a class, one person speaks at a time. Everyone else listens quietly and respectfully.*** 4. ***Raise hand if you want to be called on. Wait to be called on before speaking.*** 5. ***During pair or group work, speak quietly – don't distract other groups.*** 6. ***Stay focused – finish class work before end of class.*** II. *Launch* (**15 minutes**). A. I will ask students, *"How can we use mathematics to predict the future?"* I will then ask them to recall some of the questions they asked. These may include: a. *"Did it happen before?"* b. *"How often has it happened?"* B. *"These questions lead us to the basic rationale for the study of algebra. Algebra is about the relationships between numbers. Who can tell me the definition of the word "relationship"?* **Vocabulary:** ***Relationship*** – is the state of belonging or working together (The scientist studied the relationship between the variables).

C. *If we can identify the relationship between numbers or sets of numbers, such as time of flight and altitude of the space shuttle, we can predict the future altitude of the shuttle at any future time of flight. Let's look at another example."*

D. *"The speed of light is approximately 186.000 miles per second. The speed of sound is approximately 1116 feet per second. Why do you see lightning before you hear thunder?"* I will ask students to copy the following table which represents the approximate distance from a lightning strike to an observer based on the time it takes her to here the thunder:[7]

Seconds	2	4	6	?	?
Miles	0.4	0.8	1.2	1.6	2.4

E. I will ask students to predict how many seconds it will take for us to hear the thunder at 1.6 miles and 2.4 miles.

F. *"You could express the relationship between seconds and miles as an algebraic equation (Anyone want to try?) but we will take that up later. For now, let's just focus on finding the relationships between numbers."*

$$seconds = 5 \times miles$$

$$s = 5m$$

III. *Engagement* (**50 minutes**). Working in pairs, students will work to solve the following problems:

1. **Relationships between numbers in sequence.** Find the next three terms in the following sequences:

   a. 87, 78, 69, 60, 51, ...

To solve this problem, take the differences:

$$78 - 87 = -9$$

[7] Schultz, James E., et al. *Algebra I*. NY: Holt, 2001, p. 4.

$$69 - 78 = -9$$

$$60 - 69 = -9$$

$$51 - 60 = -9$$

$$51 - 9 = 42$$

$$42 - 9 = 33$$

$$33 - 9 = 24$$

b. 1, 4, 9, 16, 25, ...
c. 10, 17, 26, 37, 50, ...
d. 3, 13, 27, 45, 67, ...
e. 18, 32, 46, 60, 74, ...
f. 100, 94, 88, 82, 76, ...
g. 20, 21, 26, 35, 48, ...

To solve this problem, you need to look at the second differences:

$$21 - 20 = 1$$
$$26 - 21 = 5, 5 - 1 = 4$$
$$35 - 26 = 9, 9 - 5 = 4$$
$$48 - 35 = 13, 13 - 9 = 4$$
$$35 - (13 + 4) = 18$$
$$18 - (17 + 4) = -3$$
$$-3 - (21 + 4) = -28$$

h. 1, 8, 18, 31, 47, ...
i. 1, 2.5, 4, 5.5, 7.0, ...

2. Relationships between numbers related to other numbers.

a. Complete each table.

x	0	1	2	3	4	5
y	17	23	29	35	41	47

x	0	1	2	3	4	5
y	25	31	37	43	49	55

x	0	1	2	3	4	5
y	60	180	300			

x	0	1	2	3	4	5
y	500	455				

x	0	1	2	3	4	5
y	5	26	47			

x	0	1	2	3	4	5
y	201	181				81

b. A passenger ferry from Seattle, Washington to Victoria, British Columbia.

i. The table below shows how many miles the ferry travels after so many hours. Complete the table.

Time in hours	0.5	1.0	1.5	2.0	2.5
Distance in miles	26.5	53.0	79.5	?	?

ii. How many miles per hour does the ferry travel?

At this point, I may ask the students to focus on me as I model one or two of the examples below.

c. Exercise physiologists suggest that a reasonable estimate for the maximum heart rate during exercise is no more than 220 beats per minute minus the person's age.

i. The table depicts ages (the letter *a*) and their corresponding maximum exercise heart rates (the letter *r*). Complete the table.

a	10	20	25	37	49	60	65
r	210	200					

ii. Write an equation for the maximum exercise heart rates in

	terms of *a* and *r*. *One way to solve this equation would be to use our problem solving strategy:* 1. *What do I need to find out? An equation that has an equals sign and* a *and* r *in it? "a" should be on one side, "r" on the other of the equals sign.* 2. *What do I know already? I know the corresponding value of* r *for every* a *I am given. I also know that the maximum heart rate should be no more than 220 beats per minute minus the person's age. Maybe I could try writing out the data using that fact:* 220 – 10 = 210 220 – 20 = 200 220 – 25 = 195 etc. 3. *So maybe I could write the equation as:* 220 – a = r IV. *Closing* (**10 minutes**). I will say, *"Today you have taken your first step into the world of Algebra by exploring the relationships between numbers. Tomorrow we will look more closely at how we can represent those numbers as equations. Your homework tonight is to finish this worksheet."*
Assessment	I. Completed worksheet.

Name: ________________________

Class:_________________________

Date:_________________________

Worksheet

1. **Relationships between numbers in sequence.** Find the next three terms in the following sequences:

   a. 87, 78, 69, 60, 51, ...

   b. 1, 4, 9, 16, 25, ...

   c. 10, 17, 26, 37, 50, ...

   d. 3, 13, 27, 45, 67, ...

   e. 18, 32, 46, 60, 74, ...

   88, 102, 116

   f. 100, 94, 88, 82, 76, ...

   70, 64, 58

   g. 20, 21, 26, 35, 48, ...

   65, 86, 111

2. **Relationships between numbers related to other numbers.**

a. Complete each table.

x	0	1	2	3	4	5
y	17	23	29	35	41	47

x	0	1	2	3	4	5
y	25	31	37	43	49	55

x	0	1	2	3	4	5
y	60	180	300	420	540	660

x	0	1	2	3	4	5
y	500	455	410	365	320	275

x	0	1	2	3	4	5
y	5	26	47	68	89	110

x	0	1	2	3	4	5
y	201	181	161	141	121	101

b. A passenger ferry from Seattle, Washington to Victoria, British Columbia.

i. The table below shows how many miles the ferry travels after so many hours. Complete the table.

Time in hours	0.5	1.0	1.5	2.0	2.5
Distance in miles	26.5	53.0	79.5	106	132.5

ii. How many miles per hour does the ferry travel?

53.0 mph

c. Exercise physiologists suggest that a reasonable estimate for the maximum heart rate during exercise is no more than 220 beats per minute minus the person's age.

i. The table depicts ages (the letter a) and their corresponding maximum exercise heart rates (the letter r). Complete the table.

a	10	20	25	37	49	60	65
r	210	200	195	184	171	160	155

ii. Write an equation for the maximum exercise heart rates in terms of a and r.

$$\boldsymbol{r = 220 - a}$$

Name: ___________________________

Class:____________________________

Date:____________________________

Worksheet

1. **Relationships between numbers in sequence.** Find the next three terms in the following sequences:

    a. 87, 78, 69, 60, 51, ...

    b. 1, 4, 9, 16, 25, ...

    c. 10, 17, 26, 37, 50, ...

    d. 3, 13, 27, 45, 67, ...

    e. 18, 32, 46, 60, 74, ...

    f. 100, 94, 88, 82, 76, ...

    g. 20, 21, 26, 35, 48, ...

2. **Relationships between numbers related to other numbers.**

a. Complete each table.

x	0	1	2	3	4	5
y	17	23	29			

x	0	1	2	3	4	5
y	25	31	37			

x	0	1	2	3	4	5
y	60	180	300			

x	0	1	2	3	4	5
y	500	455	410			

x	0	1	2	3	4	5
y	5	26	47			

x	0	1	2	3	4	5
y	201	181	161			

b. A passenger ferry from Seattle, Washington to Victoria, British Columbia.

i. The table below shows how many miles the ferry travels after so many hours. Complete the table.

Time in hours	0.5	1.0	1.5	2.0	2.5
Distance in miles	26.5	53.0	79.5		

ii. How many miles per hour does the ferry travel?

c. Exercise physiologists suggest that a reasonable estimate for the maximum heart rate during exercise is no more than 220 beats per minute minus the person's age.

i. The table depicts ages (the letter *a*) and their corresponding maximum exercise heart rates (the letter *r*). Complete the table.

a	10	20	25	37	49	60	65
r	210	200					

ii. Write an equation for the maximum exercise heart rates in terms of *a* and *r*.

Grade / Content Area	**Grade 9 Algebra**
Lesson Title	**Linear Relationships – Launching a Spacecraft (2 days)**
Guiding Question	*How can we use Algebra to predict the future?*
Content Standards	*State Content Standards*: 1. **M(F&A)–8–2:** Demonstrates conceptual understanding of linear relationships ($y = kx$; $y = mx + b$) as a constant rate of change by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change. 2. **M(F&A)–8–3:** Demonstrates conceptual understanding of algebraic expressions by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when x = 4 given $y = 7\sqrt{x} + 2x$).
	NCTM Standards: In middle and high school all students should: 1. Use mathematical models to represent and understand quantitative relationships. 2. Analyze change in various contexts. Use graphs to analyze the nature of changes in quantities in linear relationships.
	Common Core Standards: I. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
Preparation	I. *Classroom Organization*: Students will work in pairs. II. *Differentiation*. Working in pairs promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with

the course material.

III. *Materials:* For each pair of students:

A. Graphing paper.
B. Pencils.
C. Handouts with questions and the following instructions:

1. On graph paper plot the following points:

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100	110	120
Shuttle Altitude (meters)	0	241	1244	2872	5377	8130	11617	15380	19872	25608	31412	38309	44726

- Time of Flight on the x-axis.
- Shuttle Altitude on the y-axis.

2. Answer the following questions:

- Is this a linear relationship?
- Does it look linear anywhere?
- Could we draw a line that comes close to any points?
- What is the equation of that line?
- Can we use it to predict the shuttle's altitude 140 seconds into the flight?

3. On a graph paper plot the following points:

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100
Shuttle Mass (kilograms)	2,051,113	1,935,155	1,799,290	1,681,120	1,567,611	1,475,282	1,376,301	1,277,921	1,177,704	1,075,683	991,872

- Time of Flight on the x-axis.
- Shuttle Mass on the y-axis.

4. Answer the following questions:

- Is this a linear relationship?
- Does it look linear anywhere?
- Could we draw a line that comes close to any points?
- What is the equation of that line?
- Can we use it to predict the shuttle's mass 120 seconds into the flight?

	IV. *Technology Integration:* If available, we will use PowerPoint and a Smart Board located in the classroom. If not, paper handouts will be sufficient. V. *Task Modification:* Plotting on a Cartesian graph is required to complete the task. Because visualizing how to construct the graph may be difficult, I will have visual representations of a correctly designed graph ready for students to examine.
Student Learning Objectives	I. Use Cartesian coordinate system to plot real-world data. II. Evaluate data to determine relationship between independent and dependent variables. III. Fit a linear model to the data provided. IV. Evaluate the validity of the linear model as a predictor for future outcomes.
Instruction and Engagement	I. *Warm-up* (**10 minutes**).

A. Complete the table below and derive the equation that expresses the relationship.

x	0	1	2	3	4	5
y	5	15	25			

$$y = 5 + 10x$$

B. Write down the definition of an expression (you may use your notes).

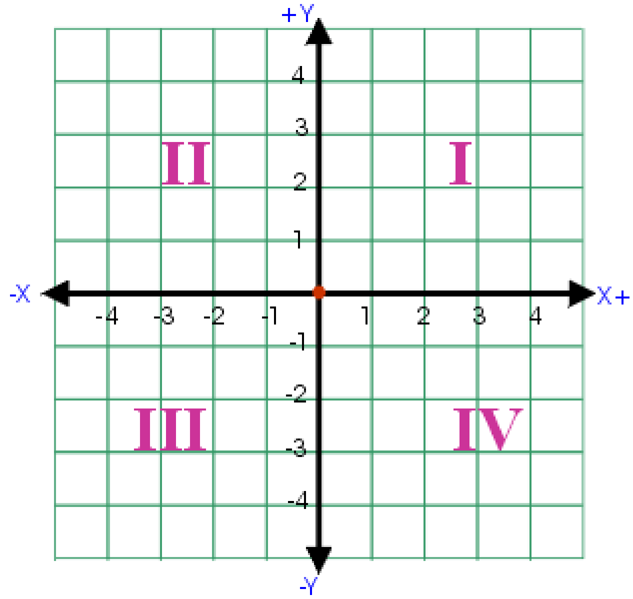
II. *Launch* (**20 minutes**). I will begin with, *"Let's go back to our earlier question, 'How can we use mathematics to predict the future?' Let's look at some data. This table depicts the total distance* (**y**) *a cyclist in the 'Tour de France' will travel after the given number of hours* (**x**)*.*

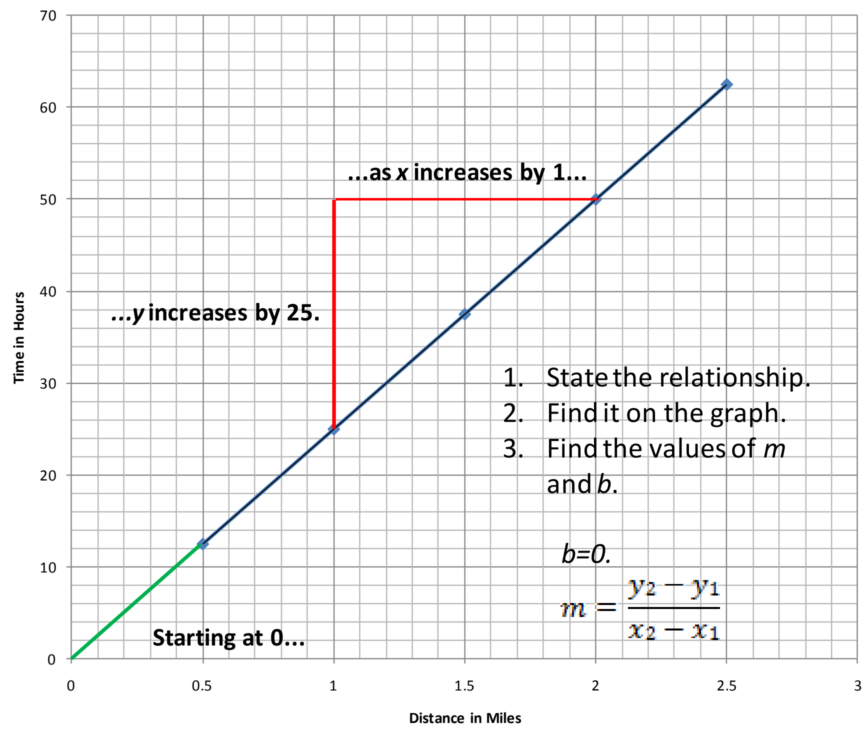
Time in hours (*x*)	0.5	1.0	1.5	2.0	2.5
Distance in miles (*y*)	12.5	25	37.5	50	67.5

A. *"What is the average speed at which the cyclist travels?"*

Answer: *"25 miles per hour".*

B. *"If the cyclist needs to cover 230 miles in a ten-hour riding day, will he make it?"*

	Answer: *"Yes, he will cover at least that much if he maintains his current speed."* C. *"You have just predicted the future. We could easily see this relation if we plotted the table on a coordinate plane.* D. First, we will discuss / review how to draw a coordinate plane. I will ask, *"Can anyone tell me what a coordinate plane looks like?"* Students will draw one based on their answers. Once complete, it should have all of the following components.  E. I will ask students to describe the components. F. *"Can anyone tell me what the equation is that describes this relationship?"* **Answer:** $y = 25x$ *"Right. Now we solved that by looking at the relationship between all the numbers – trying to find the values of* m *and* b *in the equation* $y = mx + b$. *Can anyone think of a way we can use a graph on the coordinate plane to find the values of* m *and* b*?*



Answer: *"*m *equals 'rise over run' and is called the slope,* b *is the* y *intercept."*

G. **Vocabulary:**

Slope: A measure of the steepness of a line. Given two points with coordinates (x_1, y_1) and (x_2, y_2) on a line, the slope, m, of the line is given by:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept: the y-coordinate of a point where a line crosses the y-axis.

III. *Engagement* (**50 minutes**). Students will work in groups with graph paper, rulers and pencils. We will begin with the following question:

How does a space craft like the space shuttle get into orbit? What keeps it from falling to earth?

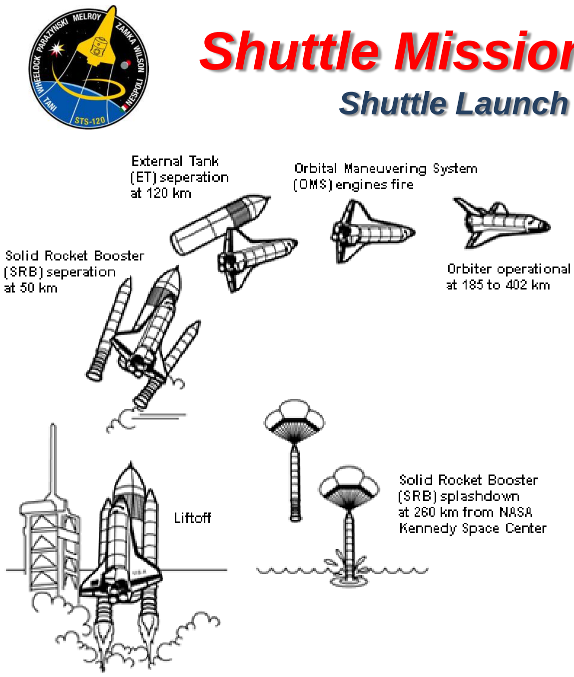
A. After we ponder this question for a few moments, I will introduce the lesson with the following background information:

Today, we are going to look at a real-world application of rates and linear relationships – the launch of a space shuttle. Let's take a look at the mission profile (I will show the following slides from a PowerPoint presentation):

This slide includes a link to a video of a shuttle launch which we will watch if there is internet connectivity.

B. Having shown the introductory slides and the video, we will now

consider a typical shuttle launch profile.



Time (s)	Mass (kg)	Altitude (m)
0	2,051,113	0
10	1,935,155	241
20	1,799,290	1,244
30	1,681,120	2,872
40	1,567,611	5,377
50	1,475,282	8,130
60	1,376,301	11,617
70	1,277,921	15,380
80	1,177,704	19,872
90	1,075,683	25,608
100	991,872	31,412
110	913,254	38,309
120	880,377	44,726

Why does the mass decrease as the altitude increases?

C. This graphic will be accompanied by a couple of questions:

We know that the thrust of the burning fuel pushes against the Earth to lift the shuttle into space. We can also see that the shuttle gets lighter as it goes into space. This helps the shuttle achieve orbit because, as it gets lighter, it needs less fuel to keep pushing it into orbit.

Why does the shuttle get lighter (shed mass)?

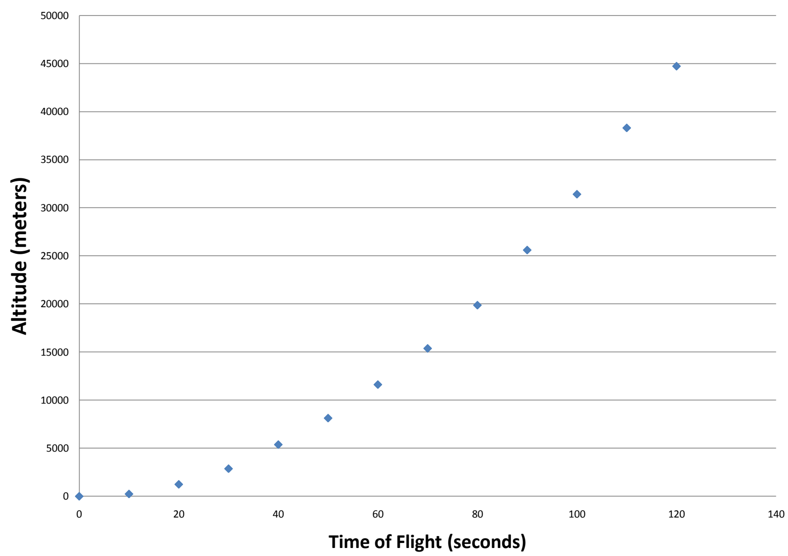
Ans: Because it expends fuel and also sheds the fuel tank and boosters.

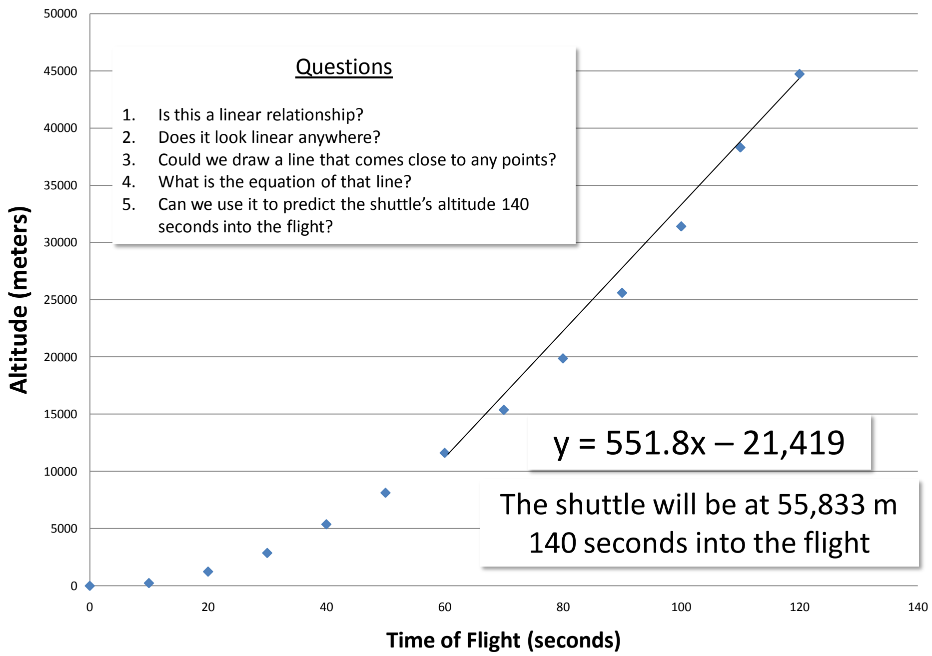
Why is it important to know the mass of the shuttle at a given altitude?

Ans: If the shuttle needs to burn less fuel as it gets lighter, knowing its mass at a given altitude will help us determine how much fuel we need to put aboard.

D. Students will now conduct the first exercise, plotting the altitude data and then fitting a line to the "linear" portion of the data. As I walk around the class, observing student work, I will be mindful that I may need to assist with the following:

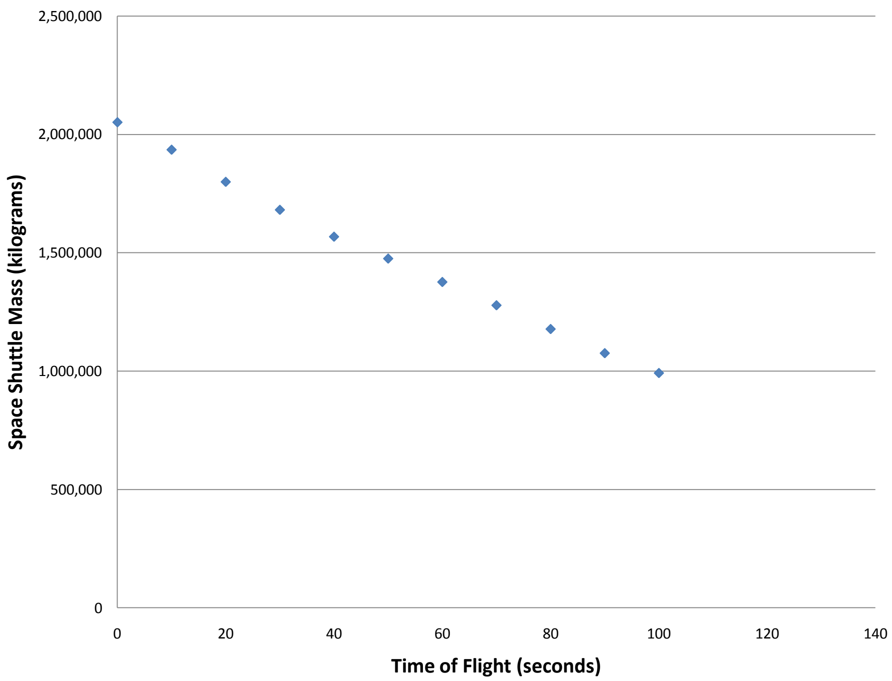
1. The data sets include large numbers. Students may have difficulty setting up a good scale on the graph paper. I will have the following

	slide, with only the scale and data points visible, to give students visual help if necessary:  2. Plotting the points may also be challenging. The numbers are very precise and the graph has a large scale. Again, the graphic above will be helpful. 3. Once the students plot the data, recognizing that there may be a linear relationship from 60 to 120 seconds may be difficult to grasp. I will ask them to think about their own bicycle or an automobile. When an auto first begins to move, it increases in speed until it reaches the cruising speed the driver wants. The same thing happens with the shuttle. Once the desired velocity (rate) is achieved, it remains essentially the same; yielding a linear relationship. 4. Deciding where exactly to place the line will also be a challenge. It will be important to emphasize that there is no one right answer. Students should be encouraged to place the line where they think it best fits. 5. Once the line is fit, students in groups should be able to determine the equation of the line. If not, I will help them by giving them hints about how to find the slope and the y-intercept. Again, the large numbers may be a distracter – something I need to keep an eye on. I will use the following graphic as we review the answers:

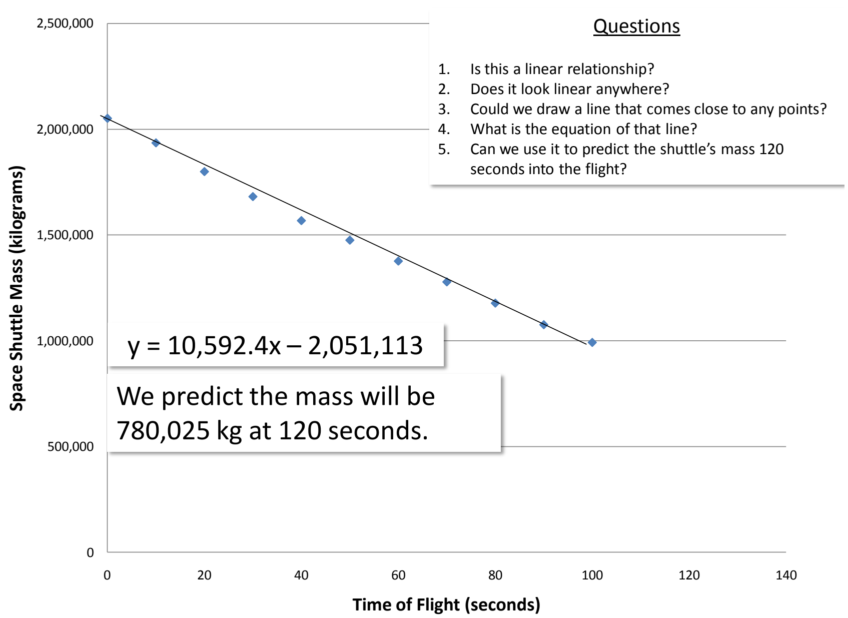


E. Upon completion of the first exercise, we will review the answers students got. They will discover that each group probably got a different answer but that the answers were not all that different if they all did essentially the same thing. I hope this will give them insight into how difficult it is to achieve precision with raw data – *one of the reasons we design computers to manipulate and plot such data.*

F. Because of time constraints, I will be ready with a handout to conduct the second exercise that already has a drawn graph with the plotted data.



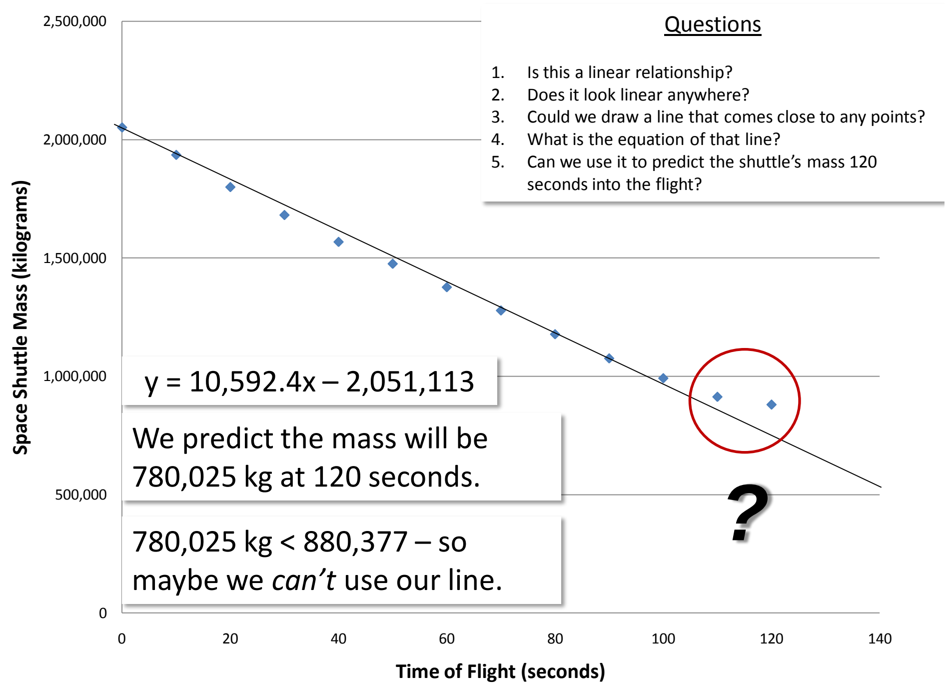
G. The plotted data looks much more like a line. Those students who successfully completed the first exercise should not find it difficult to fit a line to this data and determine the equation. From the equation, they can make a prediction as to the mass of the shuttle at 120 seconds of flight.



H. Once they do that we will once again compare answers. This will lead into the next question:

What do we do if we gather additional data that doesn't fit our model?

I. In the case of the shuttle, the rate of decrease of mass diminishes dramatically between 100 and 110 seconds of flight. Our prediction of the shuttle's mass, therefore, is too low. What can we do? The answer is outside the scope of the lesson but we will speculate a bit – setting the stage for learning later on.



Closing (**10 minutes**)*:* To conclude, we will consider the following questions:

I. Does data always plot in a straight line or in a smooth curve? Why / why not?

II. If it does not, can we still "fit" a line to the data? How?

III. If we can "fit" a line, will it always give us good predictions? How will we know if it does or does not?

Assessment

I will (informally) assess students' achievement of lesson objectives throughout the lesson and particularly as follows (these are also opportunities for self-assessment by the students):

I. I will observe how well students design and draw the Cartesian graph. Can they develop well-scaled axes without my help?

	II. I will observe how well students plot the data. Can they translate data in a table into data points on a graph? III. Can students make a reasonable estimate of where a fitted line should be plotted? IV. Can students derive the equation of the line and use it to make predictions? Do they understand the limitations of such a linear model?

Space Shuttle *Discovery*

Launch Analysis

1. On graph paper plot the following points (*time of flight* on the x-axis, *shuttle altitude* on the y-axis):

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100	110	120
Shuttle Altitude (meters)	0	241	1244	2872	5377	8130	11617	15380	19872	25608	31412	38309	44726

2. Answer the following questions:

- Is this a linear relationship?
- Does it look linear anywhere?
- Could we draw a line that comes close to any points?
- What is the equation of that line?

- Can we use it to predict the shuttle's altitude 140 seconds into the flight? If yes, what is your prediction?

Space Shuttle *Discovery*

Launch Analysis

1. On a graph paper plot the following points (*time of flight* on the x-axis, *shuttle mass* on the y-axis)::

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100
Shuttle Mass (kilograms)	2,051,113	1,935,155	1,799,290	1,681,120	1,567,611	1,475,282	1,376,301	1,277,921	1,177,704	1,075,683	991,872

2. Answer the following questions:

- Is this a linear relationship?
- Does it look linear anywhere?
- Could we draw a line that comes close to any points?
- What is the equation of that line?

- Can we use it to predict the shuttle's mass 120 seconds into the flight? If yes, what is your prediction?

Mission of the Space Shuttle DISCOVERY

Launch Operations Directorate
John F. Kennedy Space Center
Cape Canaveral, Florida

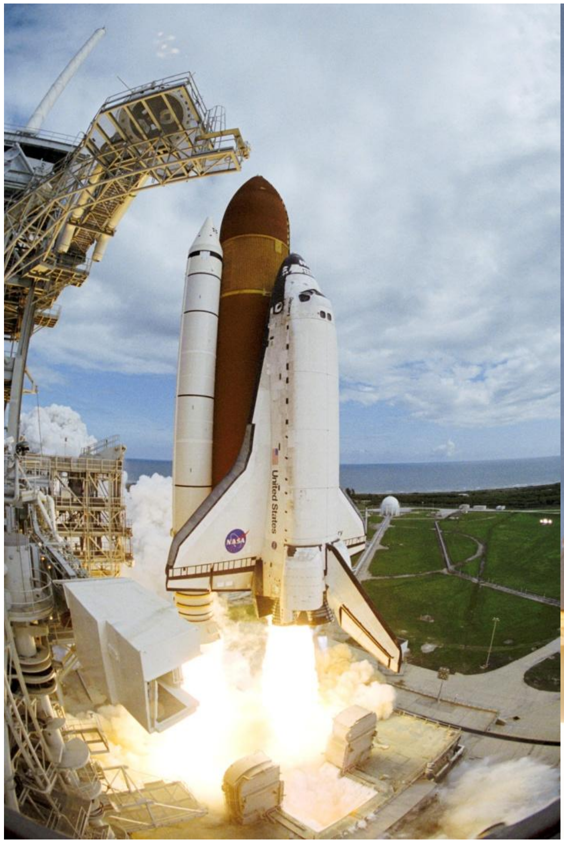
Shuttle Mission STS-120

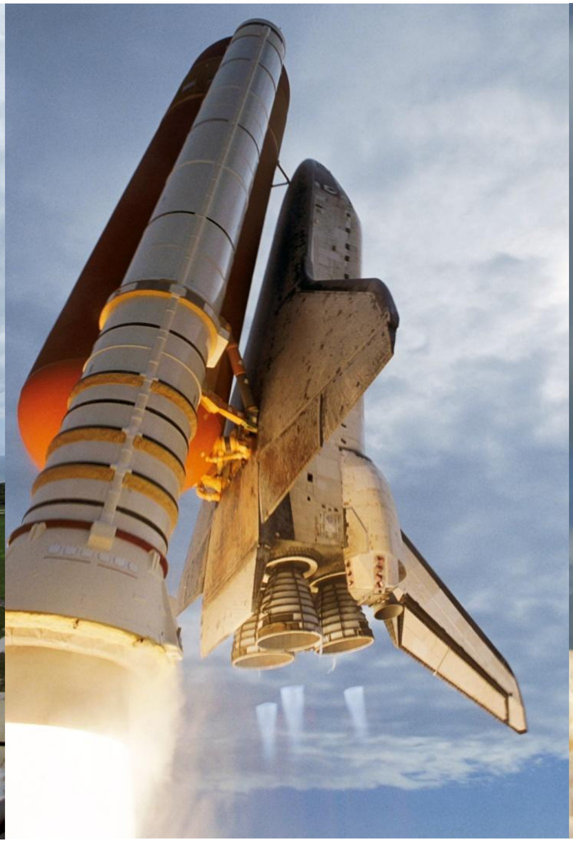
Space Shuttle DISCOVERY

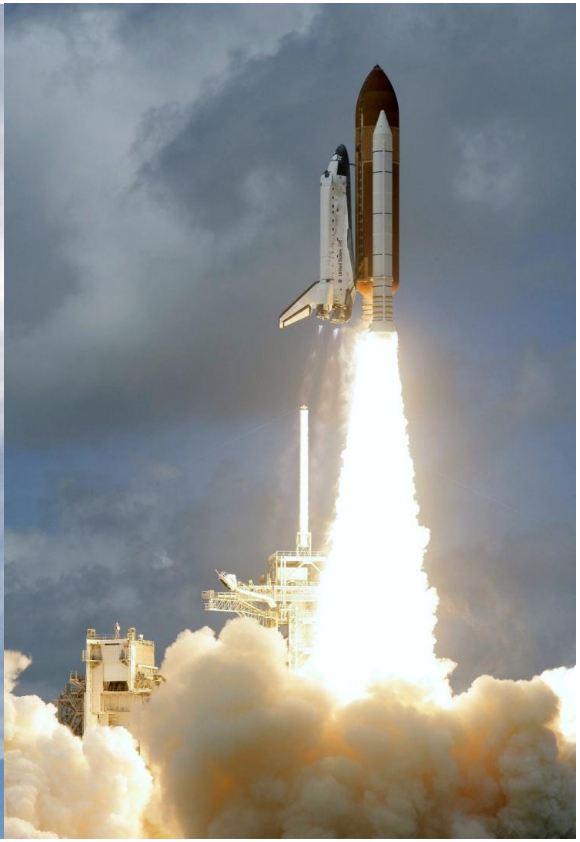
- **Mission**
  - Deliver a laboratory component to the International Space Station (ISS).
- **Commander**
  - Colonel Pamela Ann Melroy, USAF

Shuttle Mission STS-120

Space Shuttle DISCOVERY







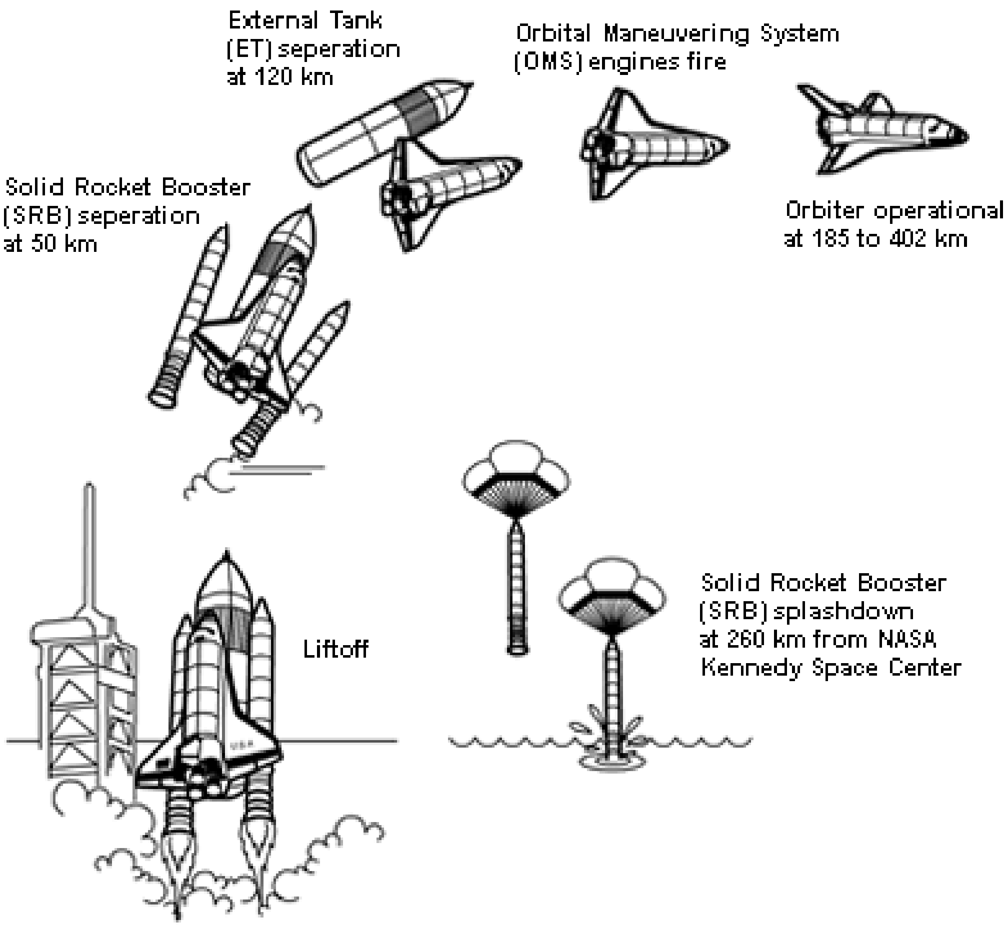
Liftoff!

Shuttle Mission STS-120

International Space Station

Shuttle Mission STS-120

Shuttle Launch Profile



Time (s)	Mass (kg)	Altitude (m)
0	2,051,113	0
10	1,935,155	241
20	1,799,290	1,244
30	1,681,120	2,872
40	1,567,611	5,377
50	1,475,282	8,130
60	1,376,301	11,617
70	1,277,921	15,380
80	1,177,704	19,872
90	1,075,683	25,608
100	991,872	31,412
110	913,254	38,309
120	880,377	44,726

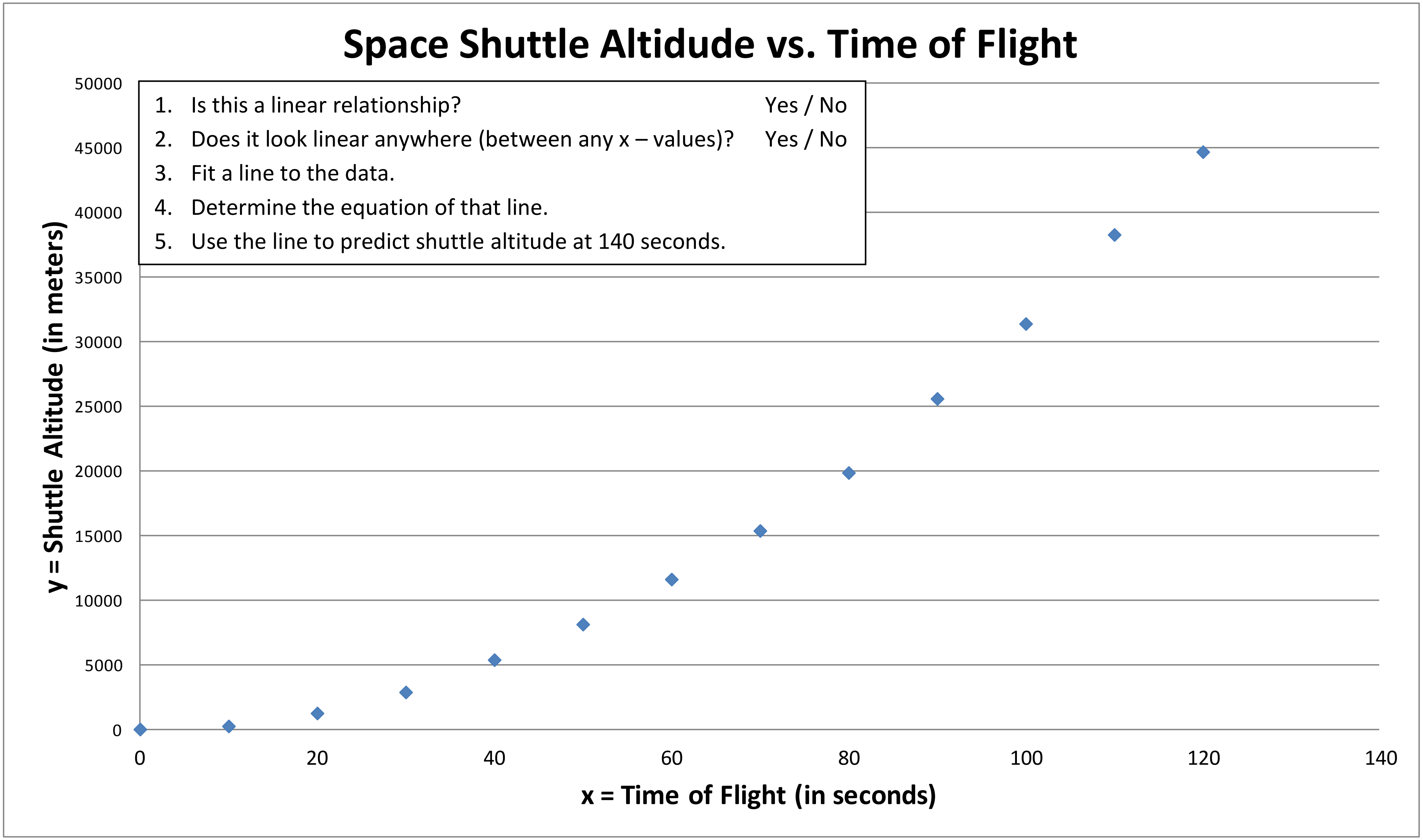
Why does the mass decrease as the altitude increases?

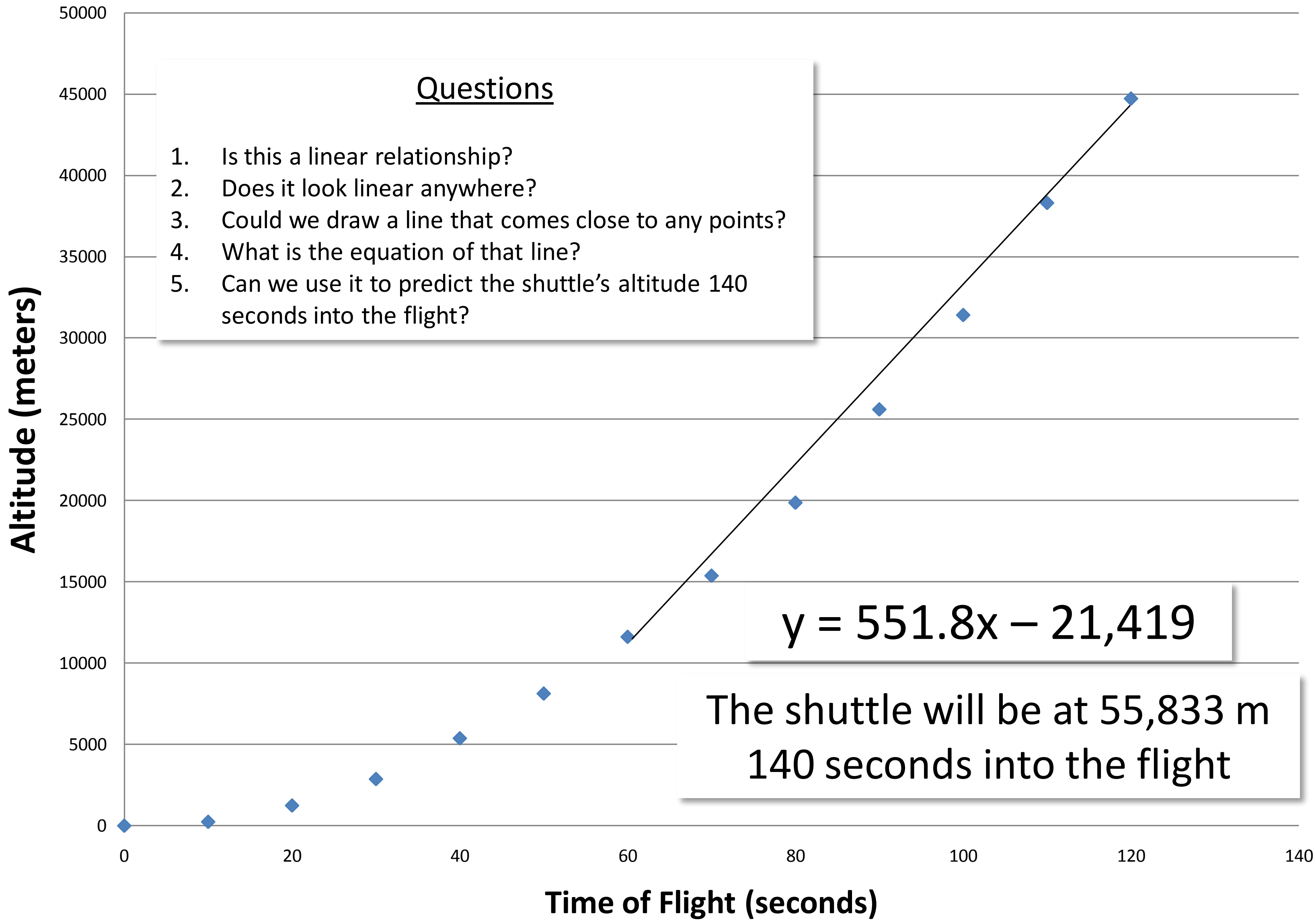
Shuttle Mission STS-120

Liftoff Data

- **On graph paper:**
  - **Plot the following points:**
    - *Time of Flight on the x-axis.*
    - *Shuttle Altitude on the y-axis.*

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100	110	120
Shuttle Altitude (meters)	0	241	1244	2872	5377	8130	11617	15380	19872	25608	31412	38309	44726


Space Shuttle Altitude vs. Time of Flight
1. Is this a linear relationship? Yes / No
2. Does it look linear anywhere (between any x – values)? Yes / No
3. Fit a line to the data.
4. Determine the equation of that line.
5. Use the line to predict shuttle altitude at 140 seconds.
y = Shuttle Altitude (in meters)
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
50000
0
20
40
60
80
100
120
140
x = Time of Flight (in seconds)

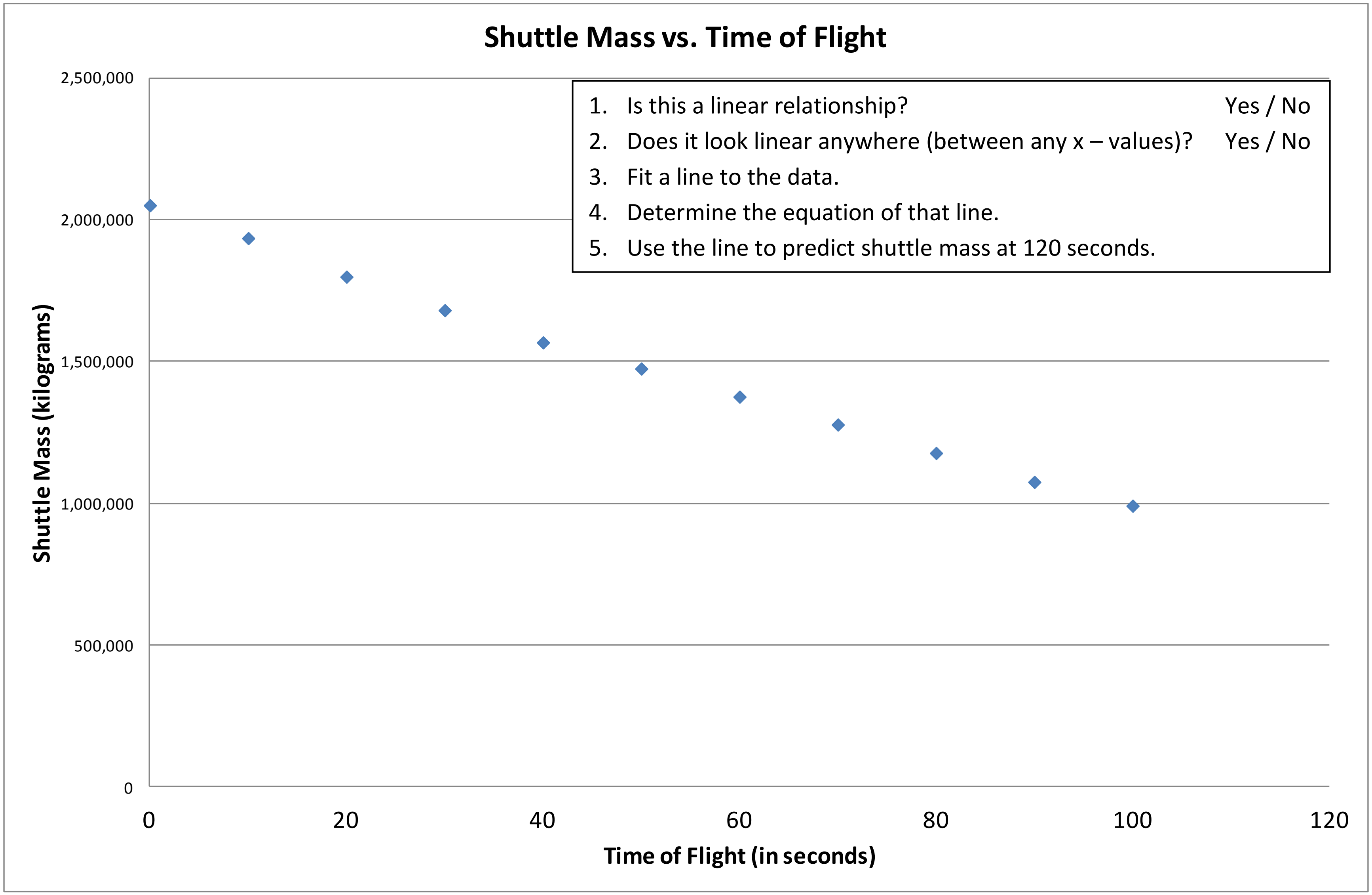

Questions
1. Is this a linear relationship?
2. Does it look linear anywhere?
3. Could we draw a line that comes close to any points?
4. What is the equation of that line?
5. Can we use it to predict the shuttle's altitude 140 seconds into the flight?
y = 551.8x – 21,419
The shuttle will be at 55,833 m 140 seconds into the flight
Altitude (meters)
Time of Flight (seconds)
50000
45000
40000
35000
30000
25000
20000
15000
10000
5000
0
0
20
40
60
80
100
120
140

Shuttle Mission STS-120

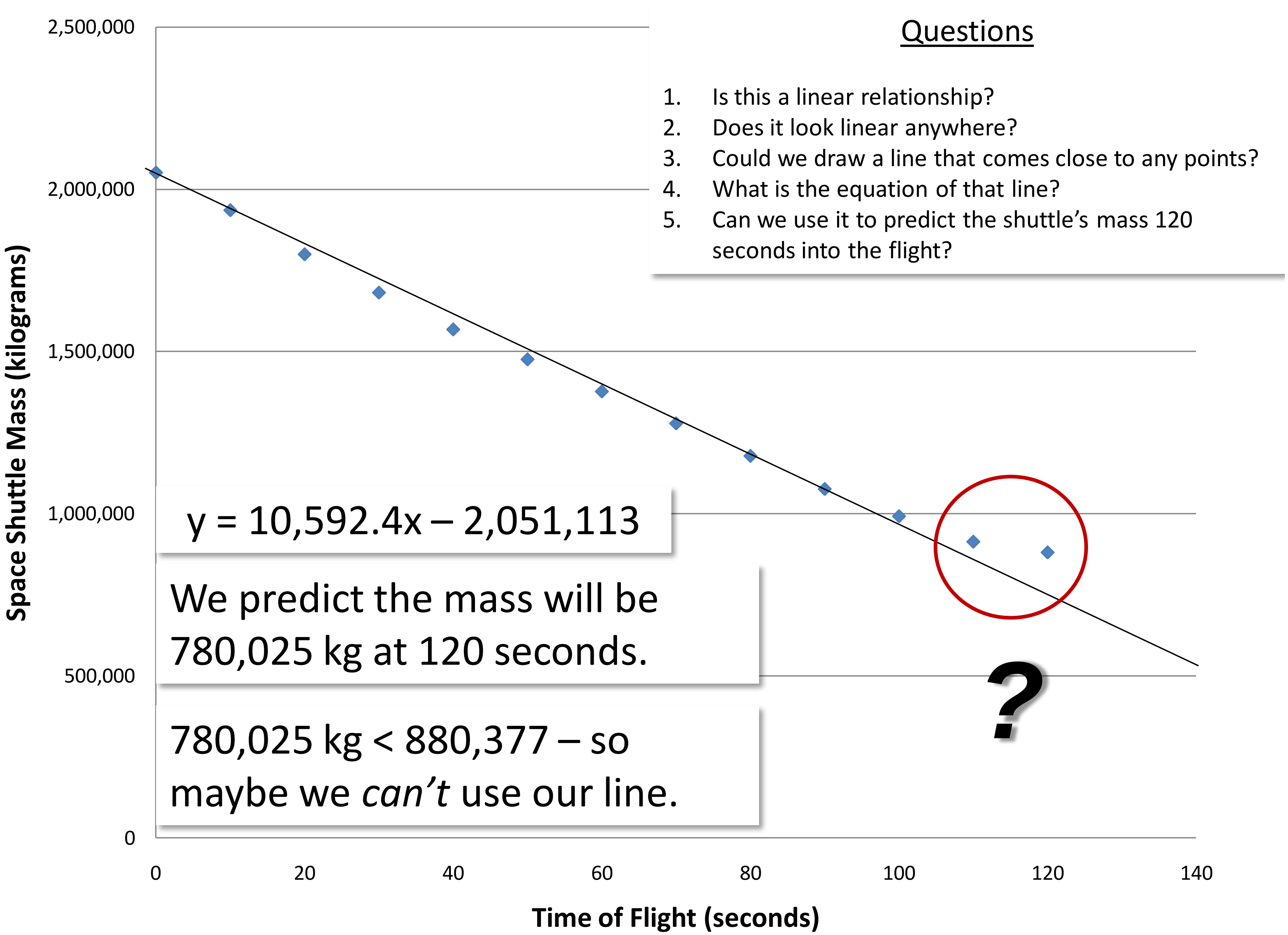
Liftoff Data

- **On graph paper:**
  - **Plot the following points:**
    - *Time of Flight on the x-axis.*
    - *Shuttle Mass on the y-axis.*

Time of Flight (seconds)	0	10	20	30	40	50	60	70	80	90	100
Shuttle Mass (kilograms)	2,051,113	1,935,155	1,799,290	1,681,120	1,567,611	1,475,282	1,376,301	1,277,921	1,177,704	1,075,683	991,872



1. Is this a linear relationship? Yes / No
2. Does it look linear anywhere (between any x – values)? Yes / No
3. Fit a line to the data.
4. Determine the equation of that line.
5. Use the line to predict shuttle mass at 120 seconds.


Questions
1. Is this a linear relationship?
2. Does it look linear anywhere?
3. Could we draw a line that comes close to any points?
4. What is the equation of that line?
5. Can we use it to predict the shuttle's mass 120 seconds into the flight?
2,500,000
2,000,000
1,500,000
1,000,000
500,000
0
Space Shuttle Mass (kilograms)
0
20
40
60
80
100
120
140
Time of Flight (seconds)
y = 10,592.4x – 2,051,113
We predict the mass will be 780,025 kg at 120 seconds.
780,025 kg < 880,377 – so maybe we *can't* use our line.
?

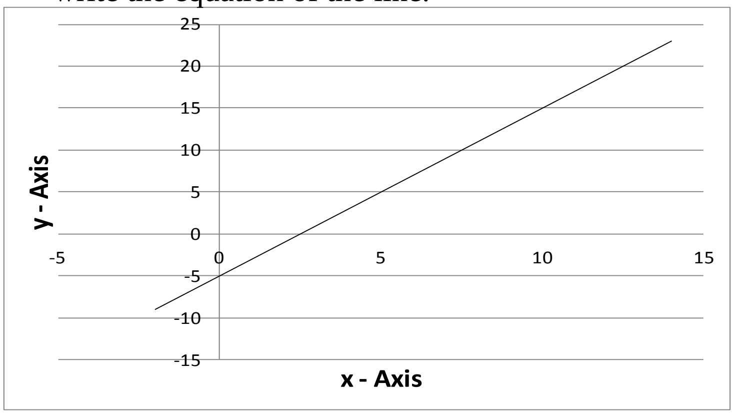
What Have We Learned?

- ***Does data always plot in a straight line or in a smooth curve? Why / why not?***
- ***If it does not, can we still "fit" a line to the data? How?***
- ***If we can "fit" a line, will it always give us good predictions? How will we know if it does or does not?***

Name: ____________________________

Linear Modeling Practice Worksheet Nr. 1

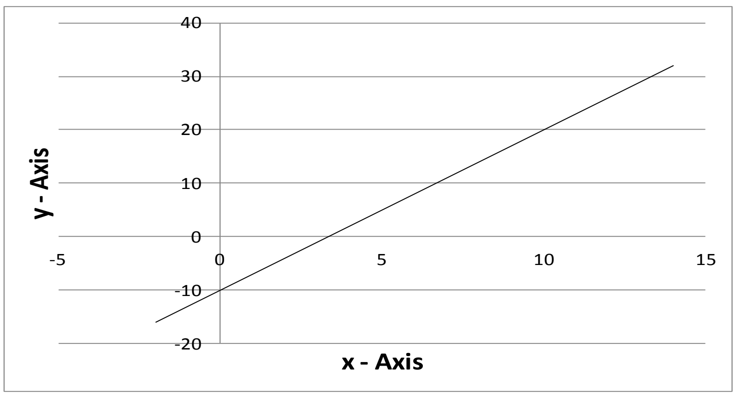
Directions: For each graph below, determine the slope (m) and y-intercept (b) of the line and write the equation of the line.



Slope: ____________________

y-intercept: ____________________

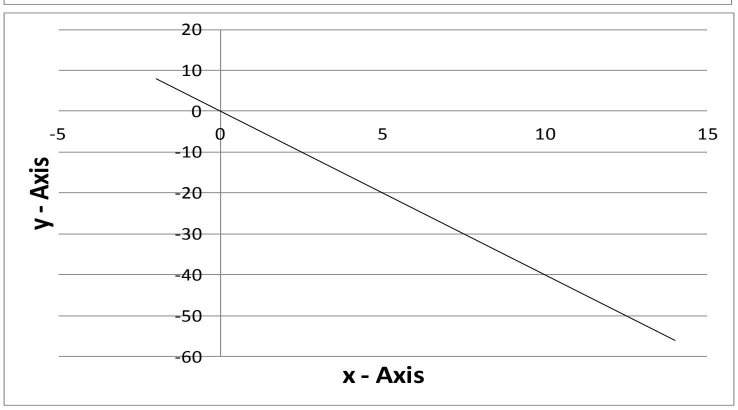
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

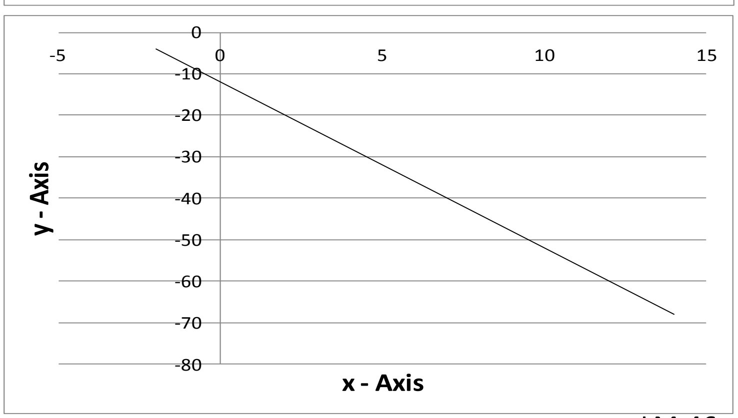
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

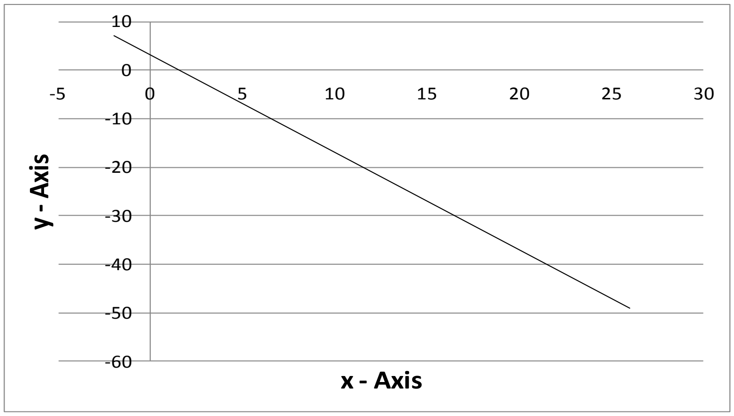
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

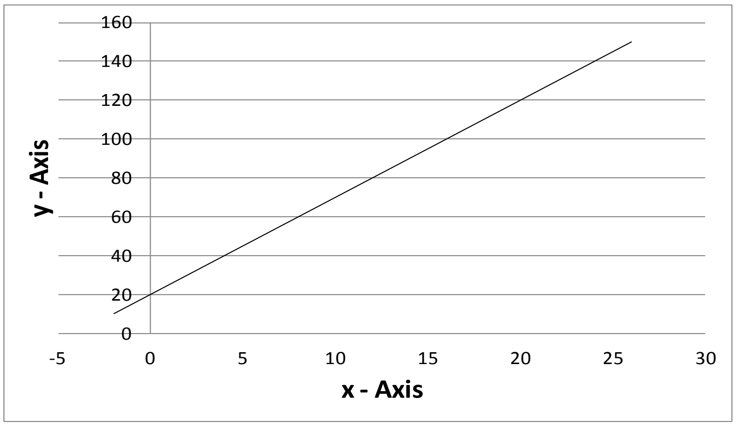
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

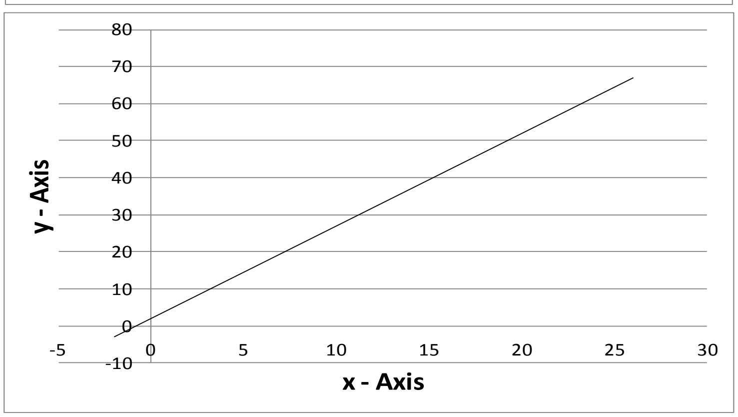
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

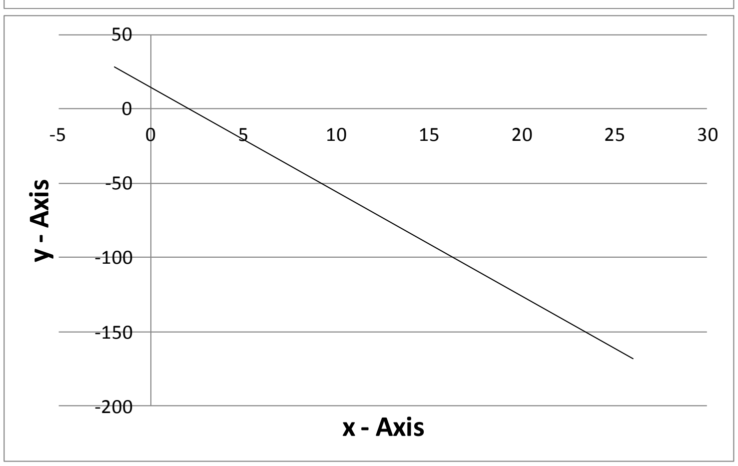
Equation: ____________________



Slope: ____________________

y-intercept: ____________________

Equation: ____________________



Slope: ____________________

y-intercept: ____________________

Equation: ____________________

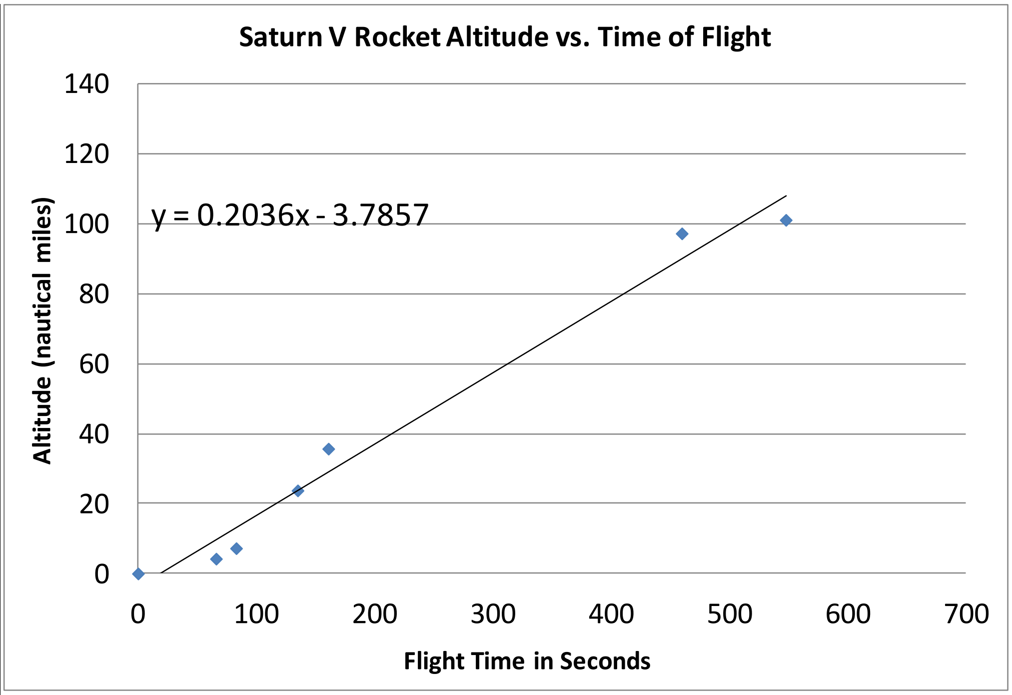
Name: ______________________________

Linear Modeling Practice Worksheet Nr. 2

Directions: Answer the questions about each of the following graphs.

1. The following data shows the altitude (height) of a Saturn V lunar rocket as a function of its time of flight. As time progresses, the rocket gets higher. The data is plotted on this graph and a line is fitted to the data as shown. The equation of the fitted line is also shown.

x = Flight Time in Seconds	y = Altitude (nautical miles)
0	0
66	4.24
83	7.24
135	23.76
161	35.70
460	97.28
548	101.14



a. What is the slope of the equation of the fitted line? ____________________

b. What is the y-intercept? ____________________

c. Predict the Saturn V rocket's altitude at 600 seconds: ____________________

d. Predict the Saturn V rocket's altitude at 700 seconds: ____________________

e. Do you think a line is a good model for this data? Why or why not?

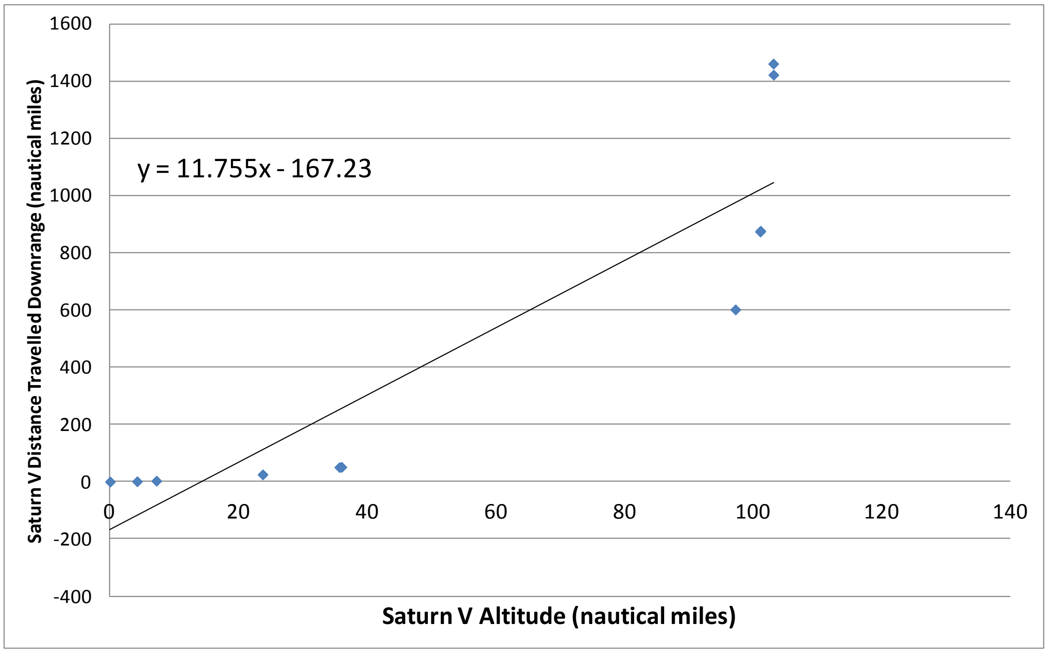
____

____

____

2. The following data shows the distance a Saturn V rocket travels across the Earth's surface (distance travelled downrange) as a function of the rocket's altitude. As the rocket gets higher, it also travels further downrange. The data is plotted on this graph and a line is fitted to the data as shown. The equation of the fitted line is also shown.

x = Altitude (nm)	y = Downrange (nm)
0.032	0
4.236	1.044
7.236	3.012
23.761	25.067
35.701	50.529
36.029	51.323
97.28	601.678
101.142	873.886
101.175	876.55
103.202	1421.959
103.176	1460.697



a. What is the slope of the equation of the fitted line? ____________________

b. What is the y-intercept? ____________________

c. Predict the Saturn V rocket's downrange distance at 120 nm altitude: ____________________

d. Predict the Saturn V rocket's downrange distance at 140 nm altitude: ____________________

e. Do you think a line is a good model for this data? Why or why not?

Grade / Content Area	**Grade 9 Algebra**
Lesson Title	**"Can We Do This?"**
Guiding Question	*Why did the United States choose to go to the Moon?*
Content Standards	I. No mathematics standards for this lesson.
Preparation	I. Students will watch the first episode of the HBO miniseries "From the Earth to the Moon," entitled *Can We Do This?* II. Desks will be arranged to permit watching the movie. We will need the TV and DVD player. III. Students will be provided with the following questions to reflect on what they have watched.
Student Learning Objectives	I. Students will answer questions about the history of the American effort to land on the Moon. II. Students will understand the connection between these events and the rocket construction and data analysis project they will begin in the next lesson.
Instruction and Engagement	I. *Warm-up* (**5 minutes**). II. *Launch* (**5 minutes**). I will handout the question sheets and instruct students to write answers during the movie on a separate sheet of paper for collection at the end of class. III. *Engagement* (**60 minutes**). We will watch the movie.

Name:_________________________________
Date:_________________________________
Class:_________________________________

Movie Study Sheet: Can We Do This?

1. Why did President John F. Kennedy establish the goal that the United States would land a man on the Moon by the end of the 1960's?

   To beat the Soviet Union to the Moon.

2. What was the name of the first American astronaut?

   Alan B. Shephard, Jr.

3. Chris Kraft, the Flight Director for NASA, talked about all of the steps necessary to prepare to land on the Moon. What were those steps?

   a. *Orbit.*
   b. *Space walk.*
   c. *Rendezvous.*
   d. *Docking.*
   e. *Long-duration space flight.*

4. Why did the "new nine" astronauts introduce themselves at the hotel as "Max Peck?"

   To keep their meeting secret from reporters.

5. What was the name of the first man to walk in space? The first American?

   Alexi Leonov – a Russian cosmonaut.
   Edward H. White, II

6. How did Astronaut Eliot See die?

   His plane crashed in St. Louis.

7. Why were the leaders of NASA concerned about See's death?

   They were afraid that Congress would slow down or cancel the space program.

8. Why was Gemini 8 brought back to Earth early?

   The spacecraft became out of control and the astronauts used the RCS to control it.

9. What was the name of the project to put Americans on the Moon.

   Project Apollo.

Homework: In the movie, we saw many different types of rockets that were used to put man into space. By far the biggest (although not shown in the movie) was the Saturn V rocket.

1. Go to the following link on your computer: http://www.nasa.gov/audience/foreducators/rocketry/home/what-was-the-saturn-v-58.html

2. Read the article about the Saturn V rocket. The article has 4 paragraphs.

3. **In your own words**, write two paragraphs describing what the Saturn V rocket was used for and how it worked.

4. Write your two paragraphs on a separate sheet of lined paper, stapled to this paper.

Grade / Content Area	**Grade 9 Algebra**
Lesson Title	**Building an X-15 Model (2 weeks)**
Guiding Question	*How Can We Build an X-15 Model and Measure Its Performance in Flight*
Content Standards	I. No mathematics standards for this lesson.
Preparation	I. Students will be provided with construction materials and instructions to construct paper model X-15.
Student Learning Objectives	I. Students will answer questions about the history of the American effort to land on the Moon. II. Students will understand the connection between these events and the X-15 model construction and data analysis project they will begin in the next lesson.
Instruction and Engagement	I. *Warm-up* (**10 minutes**). II. *Homework Review* (**10 minutes**). Collect the Saturn V paragraphs. III. *Launch* (**10 minutes**). A. Review how we have been looking at number relationships, graphing lines and looking at data – specifically the space shuttle data. 1. **Relationship.** The state of belonging or working together (The scientist studied the relationship between the variables). B. **New Vocabulary:** 1. **Variable.** A letter that is used to represent a number in algebraic expressions. 2. **Expression.** An expression is one or more algebraic terms in a phrase. It can include variables, constants, and operating symbols, such as plus and minus signs. 3. **Equation.** An equation is two equivalent expressions separated by an equals sign. 4. **Scatter Plot.** A scatter plot or scatter graph is a type of mathematical diagram using the coordinate plane to display values for two variables for a set of data.

	5. **Line of Best Fit.** A "line of best fit" is line that shows the overall trend of data in a scatter plot. 6. **Correlation.** A measure of the relationship between two or more variables. a. **Positive Correlation.** As *x* (the independent variable) moves in one direction *y* moves in the same direction. b. **Negative Correlation.** As *x* (the independent variable) moves in one direction *y* moves in the opposite direction. 7. **Linear Model.** A line of best fit to a data set that is used to predict future values of y (the dependent variable), as x (the independent variable) changes. C. I will then display the "Time of Flight vs. Shuttle Altitude" graph and indicate each new vocabulary word. IV. *Engagement.* A. **First Activity** (**10 minutes**). I will ask students to insert a line of best fit to the "Time of Flight vs. Shuttle Mass" graph, determine its slope, y-intercept and equation, and predict shuttle mass at 120 and 140 seconds. *This is how data is analyzed and used. We will now design and conduct our own experiment by building and testing air X-15 models.* B. **Second Activity** (**20 minutes**). We will begin building our X-15 models in accordance with the *X-15 model instructions*. V. *Closing* (**5 minutes**). "Ticket Out the Door"

Name: ___________________________

Class:___________________________

Date:___________________________

Homework Worksheet

1. Make a table showing the value of each expression when the value of the variable is 1, 2, 3, 4, 5.

   a. $4x$

   b. $5y$

   c. $7s + 4$

   d. $3n$

   e. $5p - 3$

   f. $2f - 1$

2. Derive equations from the following tables of relationships.

x	0	1	2	3	4	5
y	17	23	29	35	41	47

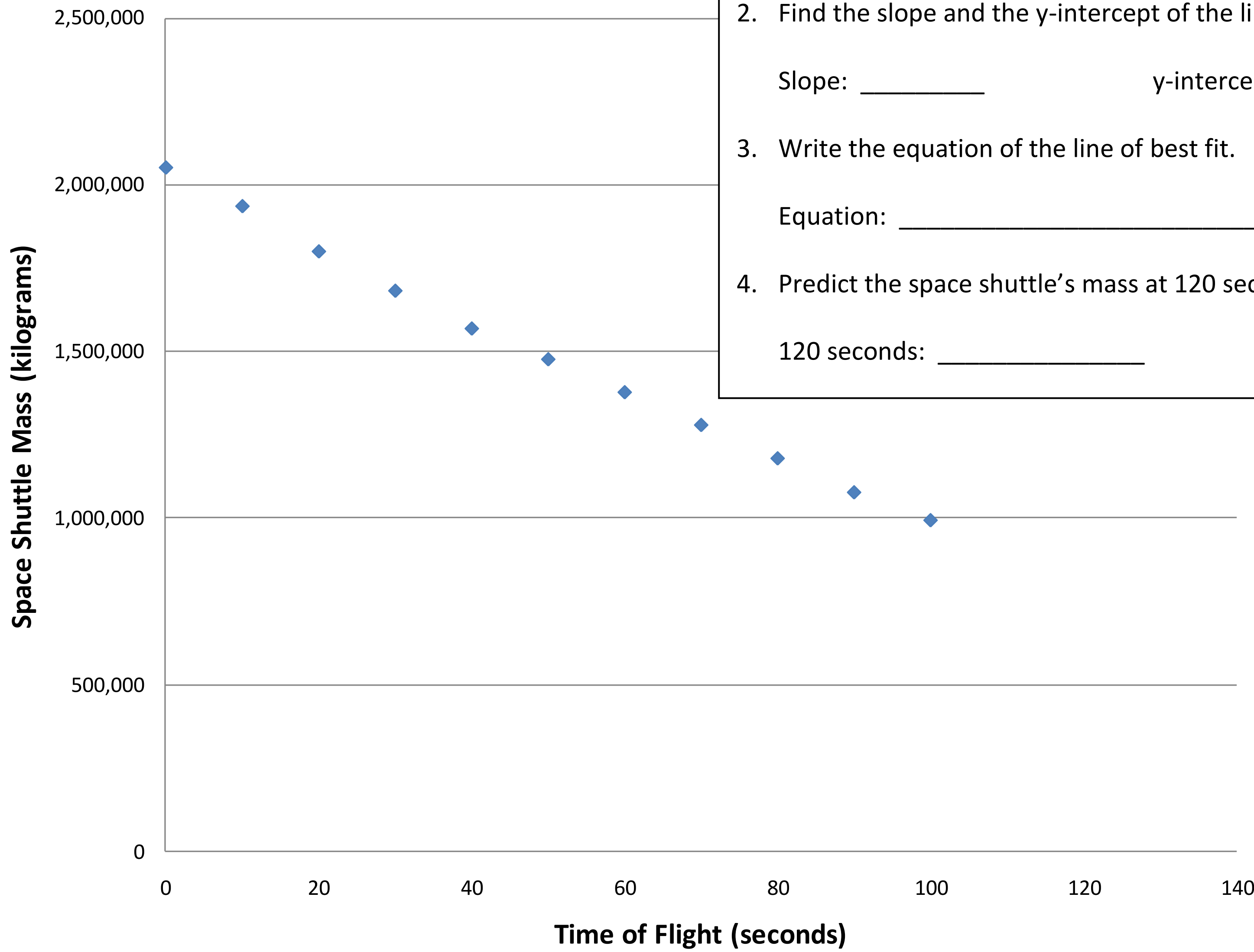
x	0	1	2	3	4	5
y	0	10	20	30	40	50

x	0	1	2	3	4	5
y	2	4	6	8	10	12

x	0	1	2	3	4	5
y	500	455	410	365	320	275

x	0	1	2	3	4	5
y	5	25	45	65	85	105

x	0	1	2	3	4	5
y	200	180	160	140	120	100



1. Draw a line of best fit.
2. Find the slope and the y-intercept of the line of best fit.

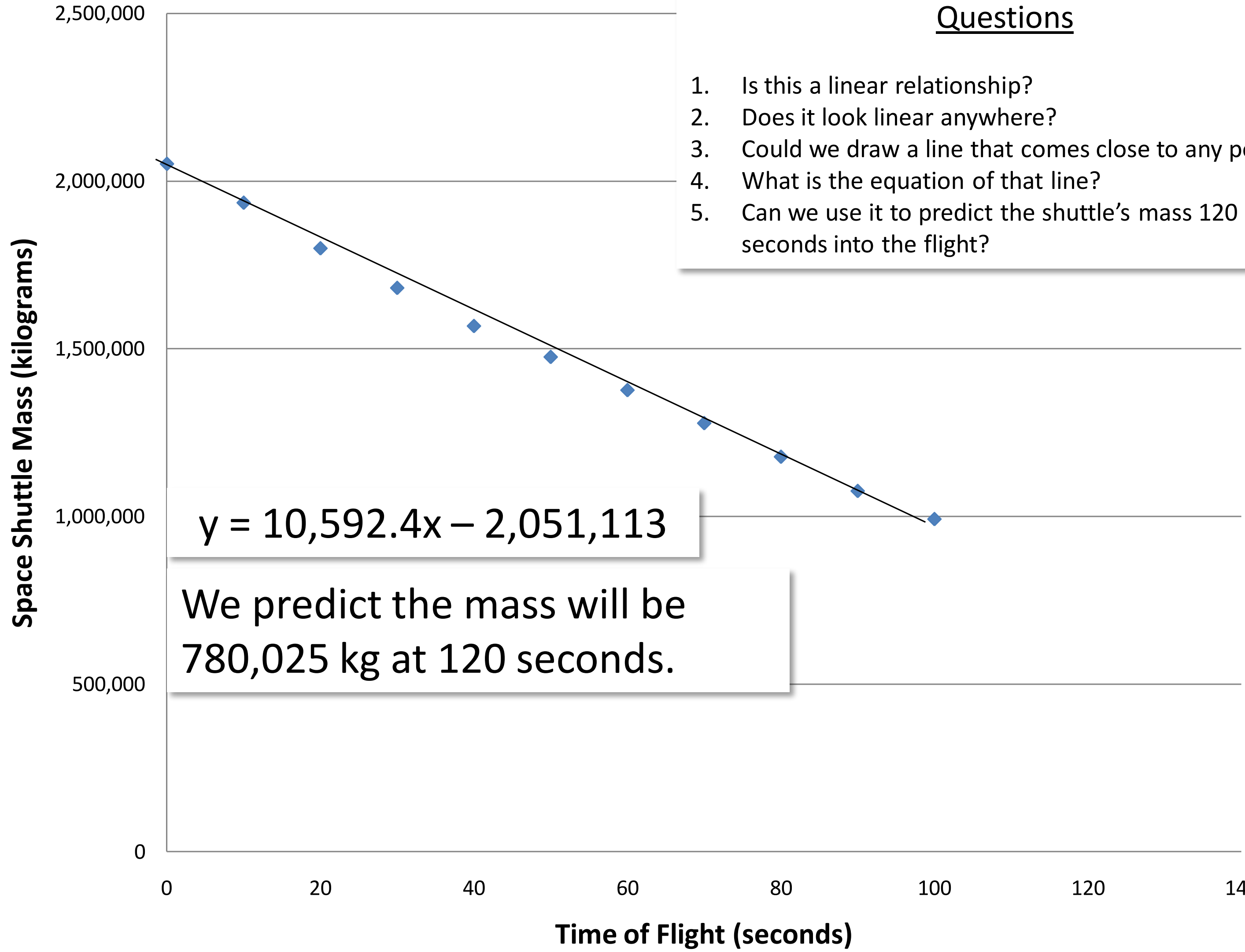
   Slope: __________ y-intercept: ____________

3. Write the equation of the line of best fit.

   Equation: ____

4. Predict the space shuttle's mass at 120 seconds.

   120 seconds: ________________



Questions

1. Is this a linear relationship?
2. Does it look linear anywhere?
3. Could we draw a line that comes close to any points?
4. What is the equation of that line?
5. Can we use it to predict the shuttle's mass 120 seconds into the flight?

$y = 10{,}592.4x – 2{,}051{,}113$

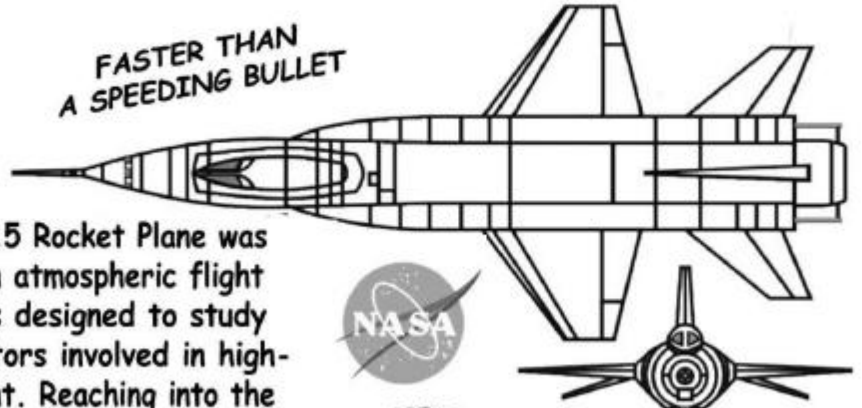
We predict the mass will be 780,025 kg at 120 seconds.

NORTH AMERICAN

X-15

MANNED ROCKET

FASTER THAN A SPEEDING BULLET

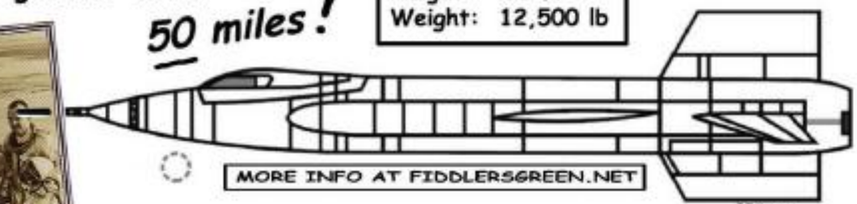


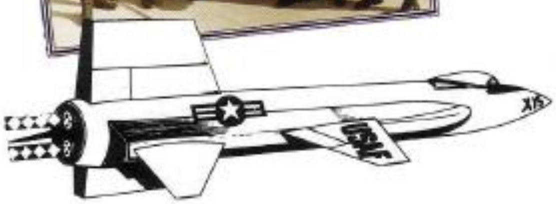
The North American X-15 Rocket Plane was a stepping stone between atmospheric flight and space flight. It was designed to study human and material factors involved in high-speed, high-altitude flight. Reaching into the fringes of space, several of the pilots of this manned missile earned "astronaut" rating, traveling to altitudes of greater than

50 miles!

SPECS..

Wingspan:	22ft 4in
Length:	50ft 7in
Height:	13 ft
Weight:	12,500 lb

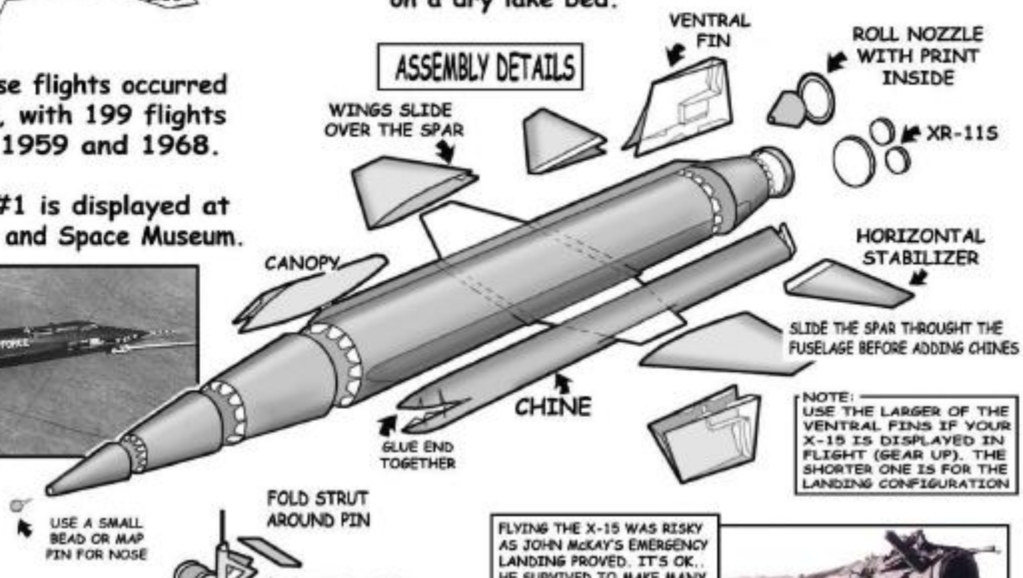




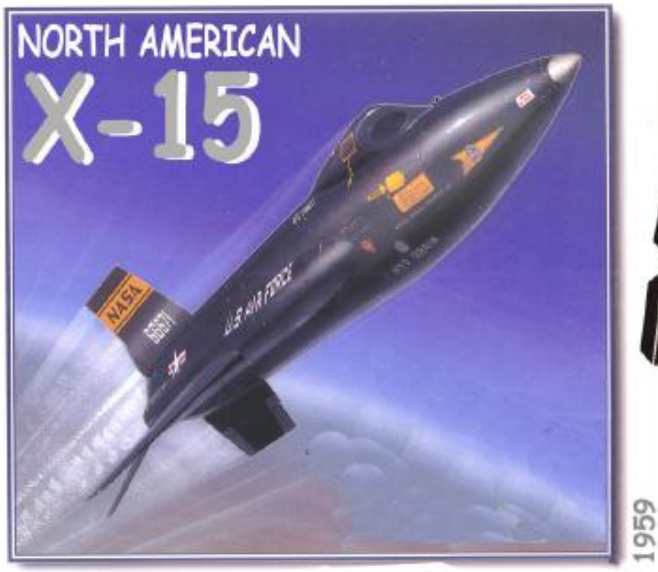
Holding only enough fuel for 80 to 120 seconds of powered flight, the X-15 was carried aloft under the wing of a B-52 for a release at 45,000 ft.and 500 mph After the brief rocket powered phase of the flight, the X-15 would continue unpowered for an additional 10 to 11 minutes ending with a 200 mph dead-stick landing on a dry lake bed.

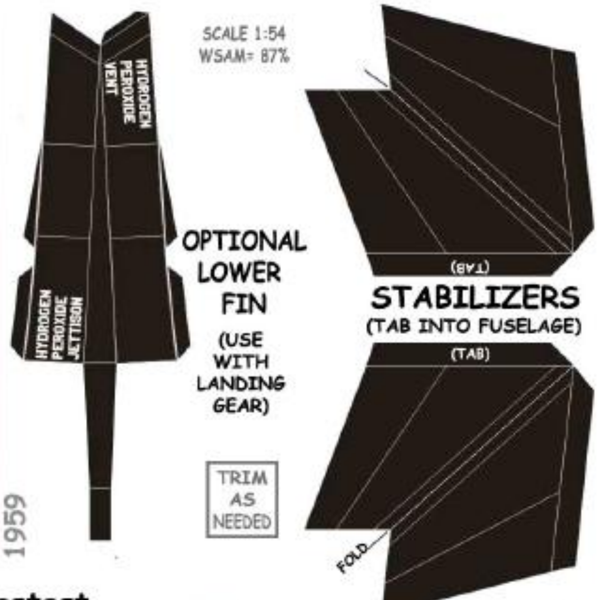
The first of these flights occurred Sept. 17, 1959, with 199 flights made between 1959 and 1968.

Historic X-15 #1 is displayed at the National Air and Space Museum.



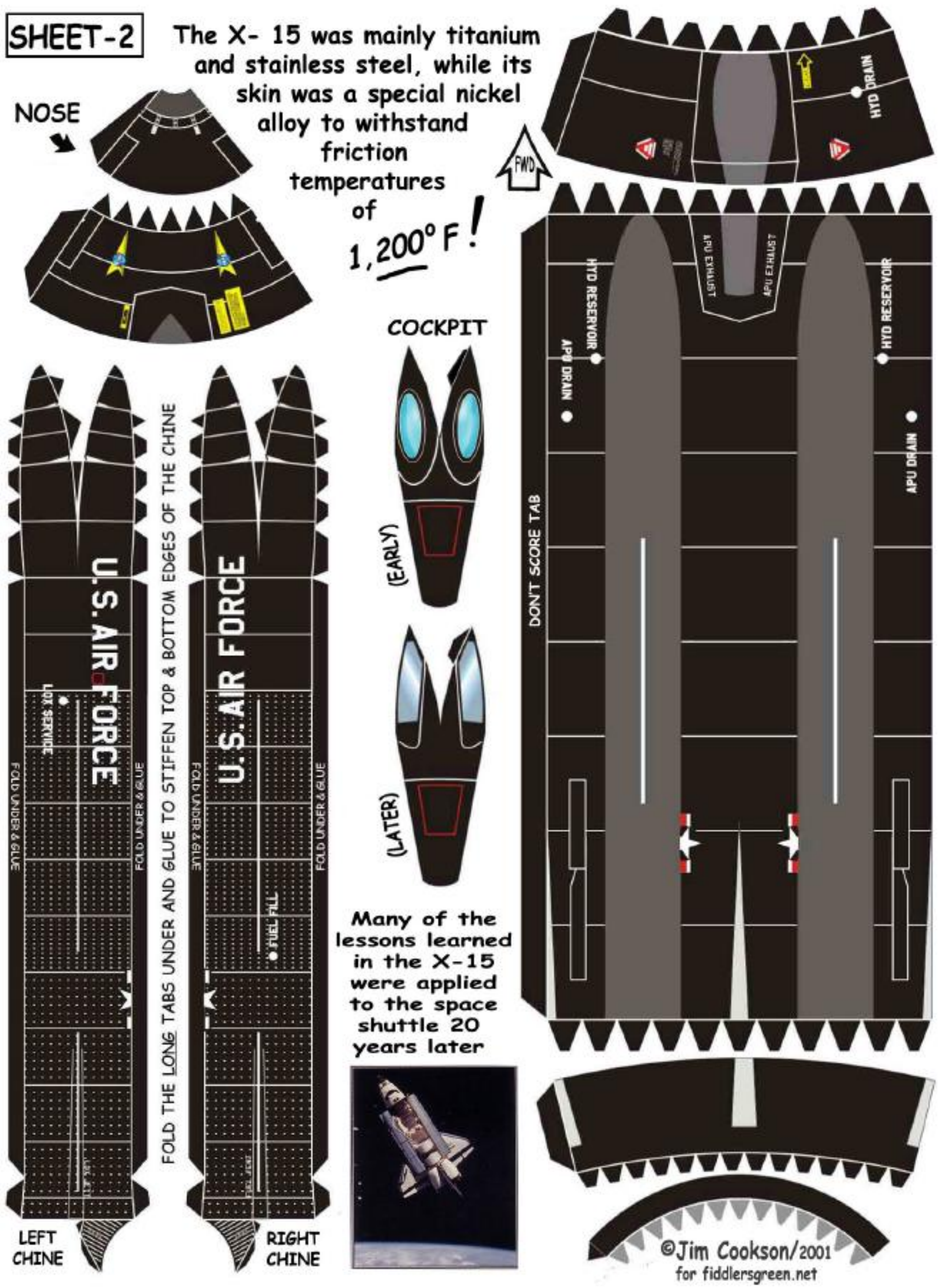
Thanks to Dr.Jim Cookson for this great X-15 Design

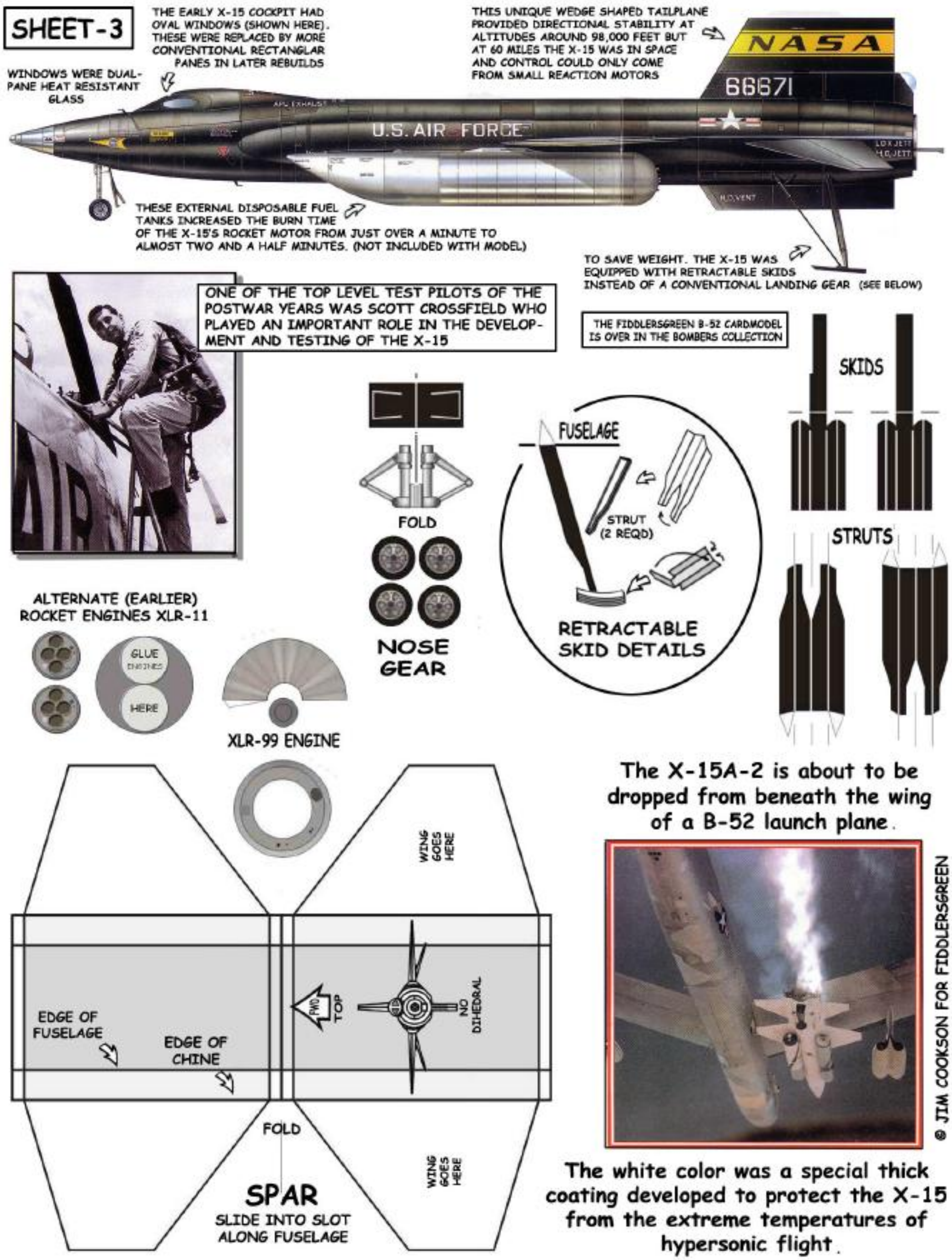




The North American X-15 is still the fastest, highest-flying aerodynamically supported machine man has yet produced, even though it has not flown for more than a quarter of a century. First dropped from its B-52 mother ship on June 8, 1959, the sinister-looking blue-black bullet pushed the boundaries of aviation to the edge of space and beyond.

Although overshadowed by the Mercury, Gemini and Apollo programs, the X-15 contributed its share to the exploration of space, and even more to the understanding of practical hypersonic flight. The knowledge gleaned by North American's manned bullet was to be a vital factor in the success of the SPACE SHUTTLE!


SHEET-2
The X- 15 was mainly titanium and stainless steel, while its skin was a special nickel alloy to withstand friction temperatures of 1,200° F!
NOSE
FWD
HYD DRAIN
HYD RESERVOIR
APU DRAIN
APU EXHAUST
COCKPIT
(EARLY)
(LATER)
DON'T SCORE TAB
FOLD THE LONG TABS UNDER AND GLUE TO STIFFEN TOP & BOTTOM EDGES OF THE CHINE
U.S. AIR FORCE
LOX SERVICE
FUEL FILL
FOLD UNDER & GLUE
Many of the lessons learned in the X-15 were applied to the space shuttle 20 years later
LEFT CHINE
RIGHT CHINE
©Jim Cookson/2001
for fiddlersgreen.net


SHEET-3
THE EARLY X-15 COCKPIT HAD OVAL WINDOWS (SHOWN HERE). THESE WERE REPLACED BY MORE CONVENTIONAL RECTANGLAR PANES IN LATER REBUILDS
WINDOWS WERE DUAL-PANE HEAT RESISTANT GLASS
THIS UNIQUE WEDGE SHAPED TAILPLANE PROVIDED DIRECTIONAL STABILITY AT ALTITUDES AROUND 98,000 FEET BUT AT 60 MILES THE X-15 WAS IN SPACE AND CONTROL COULD ONLY COME FROM SMALL REACTION MOTORS
NASA
66671
U.S. AIR FORCE
THESE EXTERNAL DISPOSABLE FUEL TANKS INCREASED THE BURN TIME OF THE X-15'S ROCKET MOTOR FROM JUST OVER A MINUTE TO ALMOST TWO AND A HALF MINUTES. (NOT INCLUDED WITH MODEL)
TO SAVE WEIGHT, THE X-15 WAS EQUIPPED WITH RETRACTABLE SKIDS INSTEAD OF A CONVENTIONAL LANDING GEAR (SEE BELOW)
ONE OF THE TOP LEVEL TEST PILOTS OF THE POSTWAR YEARS WAS SCOTT CROSSFIELD WHO PLAYED AN IMPORTANT ROLE IN THE DEVELOPMENT AND TESTING OF THE X-15
THE FIDDLERSGREEN B-52 CARDMODEL IS OVER IN THE BOMBERS COLLECTION
SKIDS
FUSELAGE
STRUT (2 REQD)
STRUTS
FOLD
NOSE GEAR
RETRACTABLE SKID DETAILS
ALTERNATE (EARLIER) ROCKET ENGINES XLR-11
GLUE ENGINES HERE
XLR-99 ENGINE
WING GOES HERE
FWD
TOP
NO DIHEDRAL
EDGE OF FUSELAGE
EDGE OF CHINE
FOLD
WING GOES HERE
SPAR
SLIDE INTO SLOT ALONG FUSELAGE
The X-15A-2 is about to be dropped from beneath the wing of a B-52 launch plane.
The white color was a special thick coating developed to protect the X-15 from the extreme temperatures of hypersonic flight.
© JIM COOKSON FOR FIDDLERSGREEN

Name: ___________________________

Date:_____________________________

Advisor:___________________________

X-15 Distance Analysis Project

I. **Step 1:** Place the X-15 model in the wind tunnel and record the maximum velocity it “flies” without spinning out of control.

____

II. **Step 2:**

a. Place the X-15 model on the launcher and pump to **20 pounds per square inch (PSI)**.
b. Launch the X-15 model and record the distance flown.
c. Repeat a. and b.

III. **Step 3:**

a. Place the X-15 model on the launcher and pump to **30 PSI**.
b. Launch the X-15 model and record the distance flown.
c. Repeat a. and b.

IV. **Step 4:** Repeat step 2 at **40 PSI**.

	20 PSI	**30 PSI**	**40 PSI**
Launch 1 Distance (feet)			
Launch 2 Distance (feet)			

V. **Step 4 (on graph paper or in the computer lab using Microsoft EXCEL):**

a. Make a table of your launch distances using the data you recorded above.
b. Graph the distance data. Your x – axis will be PSI. Your y – axis will be launch distance. **Print this table and graph and attach them to this sheet.**
c. Fit a trend line to your data. Find the equation of the trend line:

Slope: ______________________________

y-intercept: ______________________________

Equation: ______________________________

d. Predict how far the X-15 model will travel at:

10 PSI: ________________________ 50: PSI: ________________________

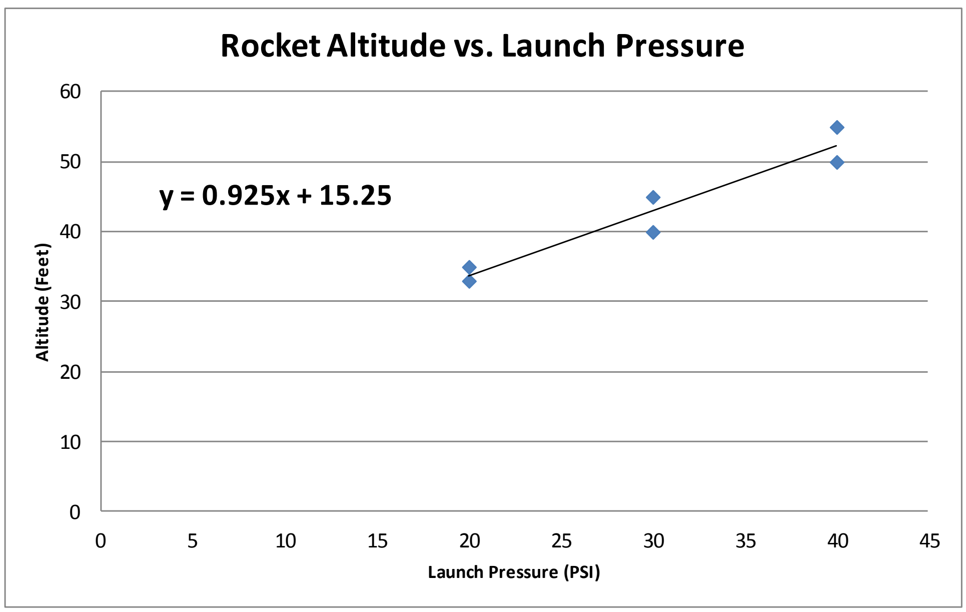
Example:

1. If my data table looks like this:

	20 PSI	**30 PSI**	**40 PSI**
Launch 1 Distance (feet)	35	40	55
Launch 2 Distance (feet)	33	45	50

2. My table and graph in EXCEL should look like this:

PSI	Distance
20	35
20	33
30	40
30	45
40	55
40	50



3. And my answers to the questions should look like this:

Slope: 0.925

y-intercept: 15.25

Equation: $y = 0.925x + 15.25$

10 PSI: 24.5 ft. 50: PSI: 61.5 ft.

Name: ________________________

Date:_________________________

Advisor:_________________________

Linear Modeling Review Nr. 1

1. **Tour de France cyclist.** This table depicts the total distance (**y**) a cyclist in the 'Tour de France' will travel after the given number of hours (**x**).

Time in hours (*x*)	0	0.5	1.0	1.5	2.0	2.5
Distance in miles (*y*)	0	12.5	25	37.5	50	62.5

A. What is the average speed at which the cyclist travels?

B. If the cyclist needs to cover 230 miles in a ten-hour riding day, will he make it? Why?

C. What is the slope, the y – intercept, and the equation of the line represented in this table?

Slope: ________________________

y – intercept: ________________________

Equation: ________________________

2. In each graph, fit a line to the data as we did in class. Find the slope, the y – intercept, and the equation of the fitted line. Predict a value for *y* for the given value of *x*.

A. (1, -3), (2, -6), (3, -9)

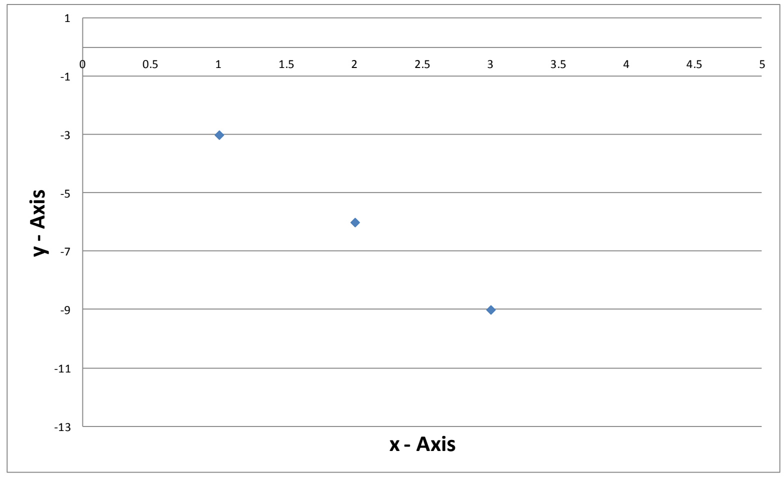
Slope: ______________________

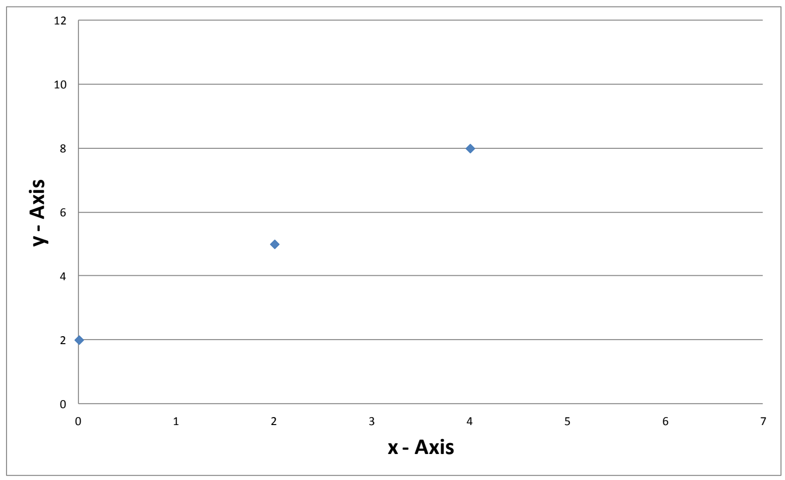
y – intercept: ________________

Equation: ___________________

Predict a value for *y* if *x* = 4

____





B. (0, 2), (2, 5), (4, 8)

Slope: ______________________

y – intercept: ________________

Equation: ___________________

Predict a value for *y* if *x* = 6

____

C. (0, 15), (4, 12), (8, 6), (12, 3)
Predict a value for *y* if *x* = 16

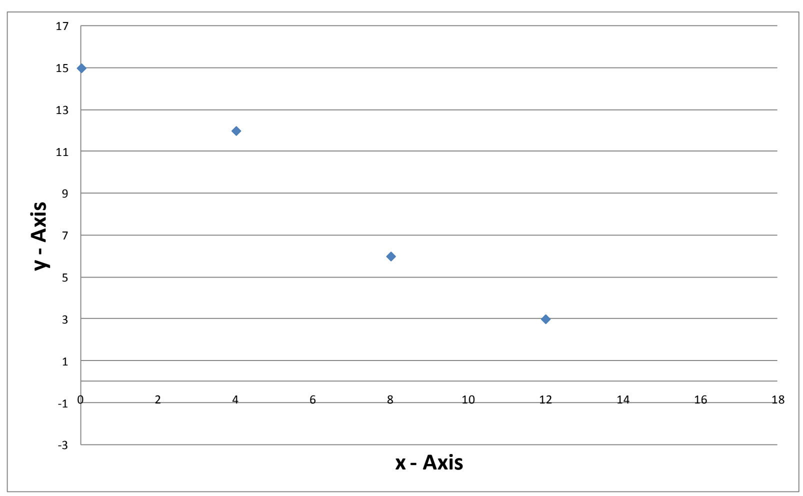
Slope: ______________________

y – intercept: ________________

Equation: ___________________

Predict a value for *y* if *x* = 16

____



Name: ___________________________

Class:____________________________

Date:____________________________

Linear Modeling – Review Nr. 2

1. Find the slope (*m*) and *y*-intercept of the linear equation and state the linear equation.

x	0	1	2	3	4	5
y	0	10	20	30	40	50

a. Slope: ________________________

b. *y*-intercept: ________________________

c. *y* = ________________________

x	0	3	6	9	12	15
y	17	23	29	35	41	47

a. Slope: ________________________

b. *y*-intercept: ________________________

c. *y* = ________________________

x	6	8	10	12	14	16
y	325	300	275	250	225	200

a. Slope: ________________________

b. *y*-intercept: ________________________

c. *y* = ________________________

1. Show all work on a separate sheet of paper stapled to this one.

Name: ___________________________

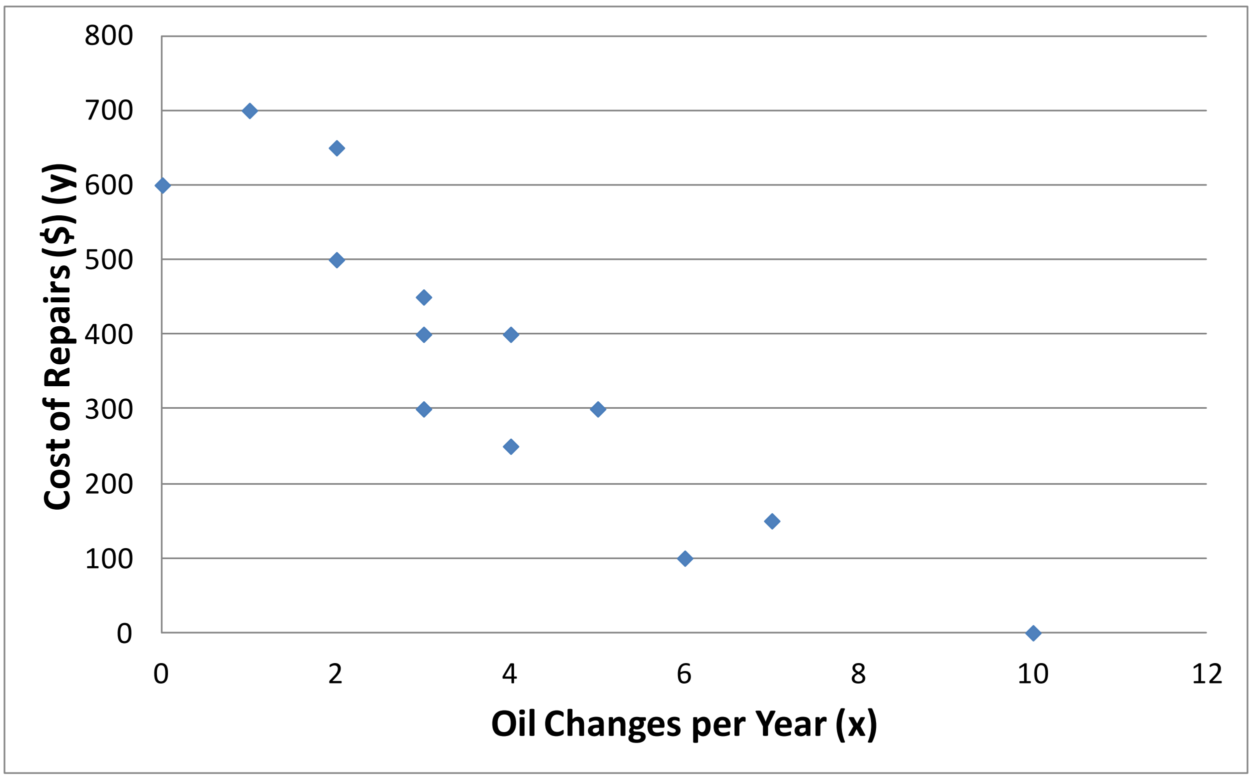
Class: ___________________________

Date: ___________________________

Linear Modeling Review Nr. 3

1. The table and grap below display data that relate to the number of oil changes per year and the cost of engine repairs.

Oil Changes Per Year (x)	3	5	2	3	1	4	6	4	3	2	0	10	7
Cost of Repairs ($) (y)	300	300	500	400	700	400	100	250	450	650	600	0	150



a. Is this a positive or a negative correlation?

b. Draw a line of best fit to the data. Determine the slope, y – intercept, and equation of your line.

Slope: ___________ y – intercept: ___________ Equation: ___________

2. For the tables below, provide the requested information**.**

a.

x	0	2	4
y	-4	-8	-12

Is this a positive or a negative correlation? ___________________________________

Determine the: Slope: __________ y – intercept: __________ Equation: __________

Predict a value for *y* if *x* = 8: ____________________

b. Suppose that a certain wildlife reserve has an increase in its buffalo population. The data is shown below:

Year	2	4	6	8	10
Population	50	65	80	95	110

Is this a positive or a negative correlation? ___________________________________

Determine the: Slope: __________ y – intercept: __________ Equation: __________

Predict a cost for year 14: ____________________

c. The table below shows the price of a half-gallon of whole milk from 1989 to 1995:

Year	1989	1990	1991	1992	1993	1994	1995
Cost ($)	1.37	1.39	1.41	1.43	1.45	1.47	1.49

Is this a positive or a negative correlation? ___________________________________

Determine the: Slope: __________ y – intercept: __________ Equation: __________

Predict the cost for 1999: ____________________

Algebra I – Review 3 Answers

1. **Problem – solving strategy.** What are the three questions we ask in our problem – solving strategy?

    a. What do I need to find out?

    b. What do I know already?

    c. How will I solve it?

2. **Definitions.** Write the definitions of the following algebra terms.

    a. **Relationship.** the state of belonging or working together (The scientist studied the relationship between the variables).

    b. **Variable.** A letter that is used to represent a number in algebraic expressions

    c. **Expression.** An expression is one or more algebraic terms in a phrase. It can include variables, constants, and operating symbols, such as plus and minus signs.

    d. **Equation.** An equation is two equivalent expressions separated by an equals sign.

3. Find the next three terms in the sequence.

    a. 100, 92, 84, 76, 68, ... **60, 52, 44**

    b. 1, 3, 7, 13, 21, ... **31, 43, 57**

    c. 17, 25, 35, 47, 61, ... **77, 95, 115**

4. Complete each table.

x	0	1	2	3	4	5
y	25	36	47	**58**	**69**	**80**

x	0	1	2	3	4	5
y	150	135	120	**105**	**90**	**75**

x	0	1	2	3	4	5
y	-12	-7	-2	**3**	**8**	**13**

5. Make a table showing the value of each expression when the value of the variable is 1, 2, 3, 4, 5.

   a. 6x

1	2	3	4	5
6	12	18	24	30

   b. 4x + 6

1	2	3	4	5
10	14	18	22	26

6. Derive an equation for the following relationship.

x	0	1	2	3	4	5
y	6	9	12	15	18	21

$$y = 3x + 6$$

Name: ___________________________

Date:____________________________

Advisor:_________________________

Linear Modeling Assessment Nr. 1

Show all work on separate sheets of paper attached to this one.
Place all answers on this sheet.

1. For each linear relationship, find the following:

x	0	1	2	3	4	5
y	15	20	25	30	35	40

a. Slope (*m*): ______________________

b. *y*-intercept (*b*):______________________

c. Equation: *y* = ______________________

d. Predict the value of **y** when **x** = 8: ______________________

x	12	14	16	18	20	22
y	18	25	32	39	46	53

e. Slope (*m*): ______________________

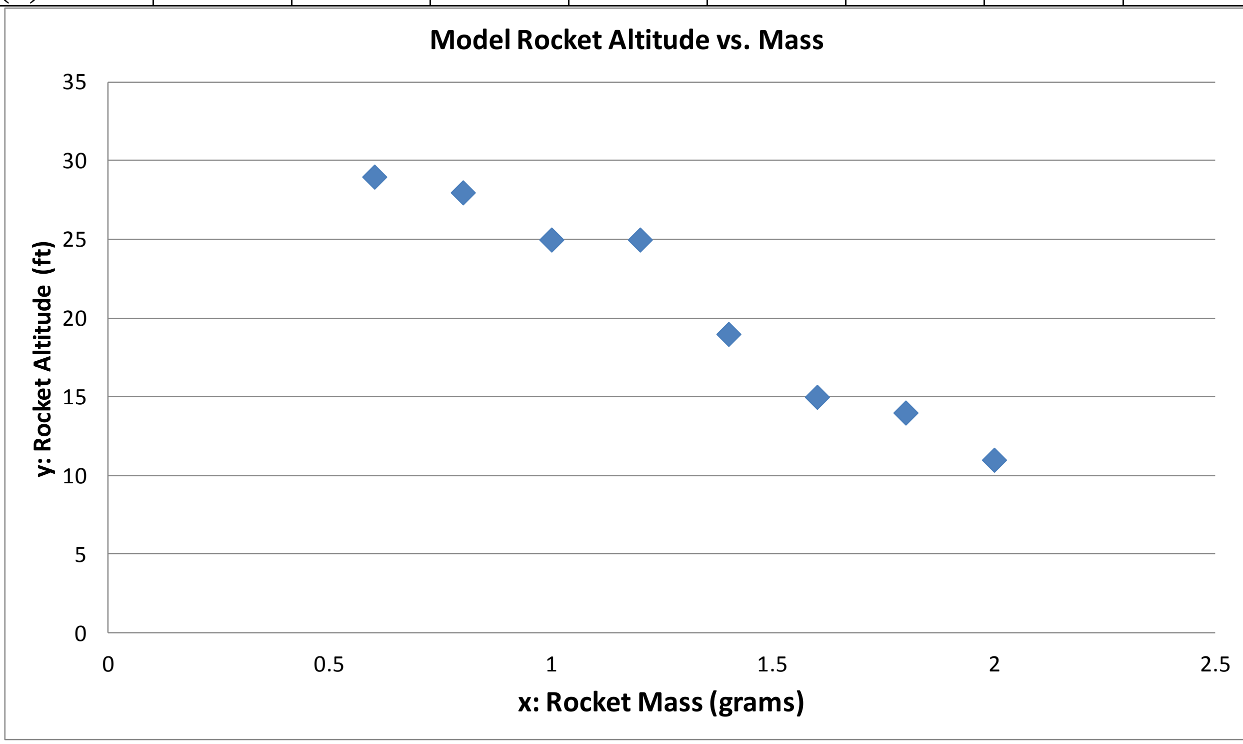
f. *y*-intercept (*b*):______________________

g. Equation: *y* = ______________________

h. Predict the value of **y** when **x** = 26: ______________________

2. A student is trying to determine the best weight (mass) for a model rocket with a given engine. The student's goal is to have the rocket go as high as possible. She has conducted several launches of model rockets with different mass and gathered the following data:

x: Rocket Mass (g)	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
y: Rocket Altitude (ft)	29	28	25	25	19	15	14	11



a. Is this a positive or a negative correlation? ______________________

b. Insert a line of best fit.

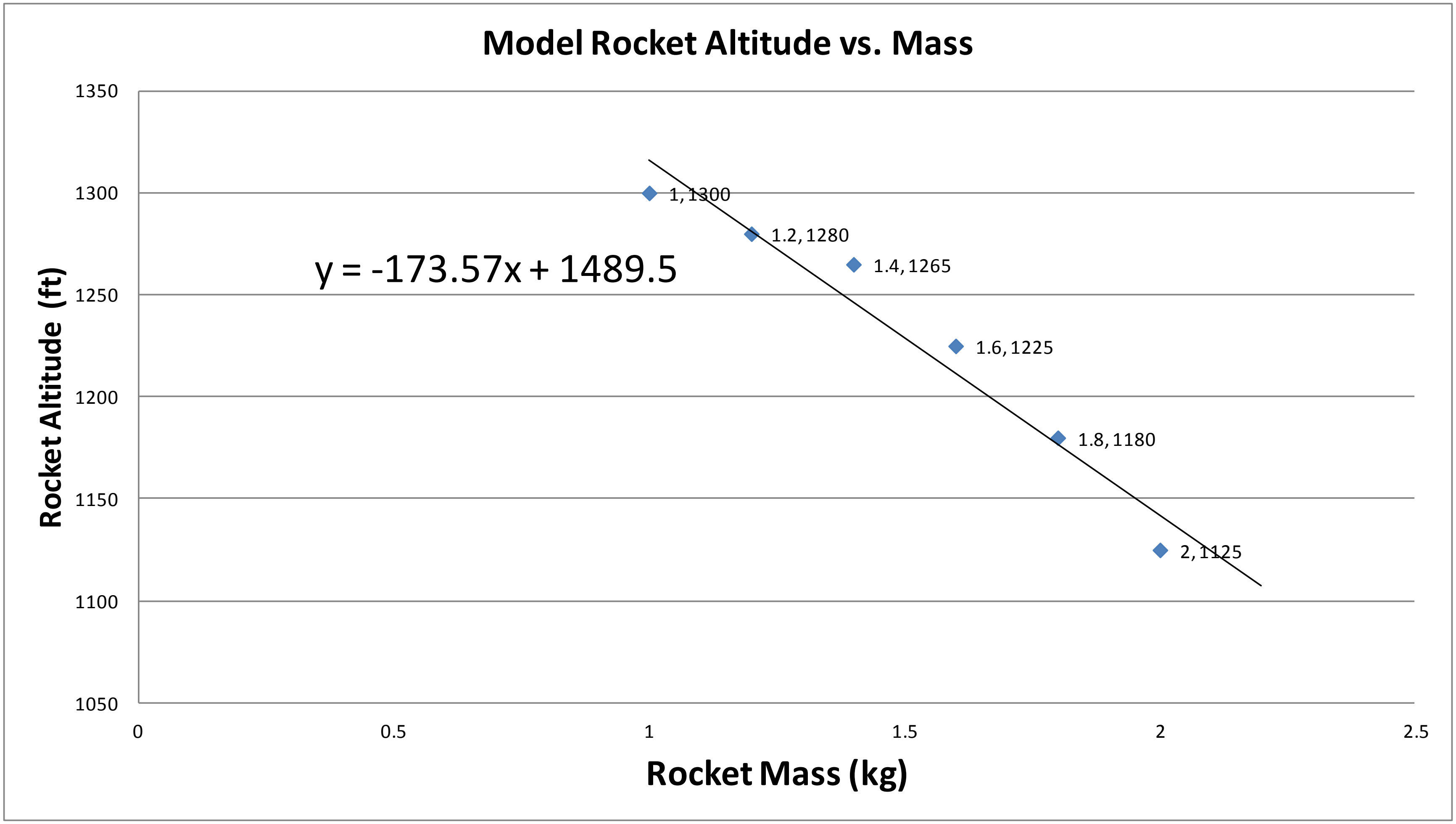
c. Find the following from the line of best fit:

i. Slope: ________________________________

ii. y-intercept: ____________________________

iii. y = ____________________________________

d. Predict the rocket's altitude, **y**, if the mass, **x** = 2.5 grams. ______________________


Model Rocket Altitude vs. Mass
1350
1300
1250
1200
1150
1100
1050
0
0.5
1
1.5
2
2.5
Rocket Altitude (ft)
Rocket Mass (kg)
y = -173.57x + 1489.5
1, 1300
1.2, 1280
1.4, 1265
1.6, 1225
1.8, 1180
2, 1125

Grade / Content Area	**Grade 9 Algebra**
Lesson Title	**“Apollo 1”**
Guiding Question	*What caused the fire that destroyed the Apollo 1 spacecraft?*
Content Standards	II. No mathematics standards for this lesson.
Preparation	IV. Students will watch the first episode of the HBO miniseries “From the Earth to the Moon,” entitled *Apollo 1*. V. Desks will be arranged to permit watching the movie. We will need the TV and DVD player. VI. Students will be provided with the following questions to reflect on what they have watched.
Student Learning Objectives	III. Students will answer questions about the history of the American effort to land on the Moon. IV. Students will understand the connection between these events and the rocket construction and data analysis project they will begin in the next lesson.
Instruction and Engagement	IV. *Warm-up* (**5 minutes**). V. *Launch* (**5 minutes**). I will handout the question sheets and instruct students to write answers during the movie on a separate sheet of paper for collection at the end of class. VI. *Engagement* (**60 minutes**). We will watch the movie.

Name:________________________________
Date:_________________________________
Class:_________________________________

Movie Study Sheet: "Apollo 1"

1. Who were the three astronauts who died in the Apollo 1 fire?

   Gus Grissom, Ed White, and Roger Chaffee.

2. What was the name of NASA's Director of Flight Crew Operations?

   Deke Slayton.

3. What did the fire investigation team find in the Apollo 1 spacecraft that should not have been there?

   A socket wrench.

4. Why couldn't the astronauts open the hatch and escape the burning spacecraft?

   The hatch opened inward and was held closed by the pressure caused by the fire.

5. What did the North American corporation, which built the Apollo spacecraft, believe was the cause of the Apollo 1 fire?

   Use of pure oxygen at high pressure in the spacecraft during the "plugs out test".

6. What was the name of the U.S. Senator who wanted to end the Apollo program after the fire?

   Walter Mondale.

7. Why did Joe Shea, the Apollo Program Manager, say he wished he could have been in the spacecraft when the fire started?

   He believed he would have been in a position to put out the fire before it spread.

8. What was the name of the astronaut who helped investigate the cause of the fire and testified before the Senate committee?

   Colonel Frank Borman.

Homework: In the movie, we saw that the temperature in the spacecraft became very high as the fired burned. We also saw that the astronauts were unable to open the hatch to get out.

The following table shows the increase in pressure (in a space whose size does not change) as the temperature in the space increases.

Temp (ºF)	90	95	100	105	110	115	120
Pressure (PSI)	165	180	200	215	230	250	270

a. Graph the data on the coordinate plane.

b. Insert a line of best fit.

c. Find the following from the line of best fit:

   i. Slope: ____________________________________

   ii. y-intercept: ________________________________

   iii. y = _______________________________________

d. What is the rate of change of pressure as the temperature increases?

e. Predict the pressure at 130 ºF.

Name: ___________________________

Class:____________________________

Date:____________________________

Linear Modeling – Review 5

1. **Problem – solving strategy.** What are the three questions we ask in our problem – solving strategy? (**3 points each**)

   a. ____

   b. ____

   c. ____

2. **Definitions.** Write the definitions of the following algebra terms.

   a. **Relationship.** ____

   ____

   b. **Variable.** ____

   ____

   c. **Expression.** ____

   ____

   d. **Equation.** ____

   ____

3. Find the next three terms in the sequence.

   a. 100, 92, 84, 76, 68, ...

   b. 1, 3, 7, 13, 21, ...

   c. 17, 25, 35, 47, 61, ...

4. Complete each table.

x	0	1	2	3	4	5
y	25	36	47			

x	0	1	2	3	4	5
y	150	135	120			

x	0	1	2	3	4	5
y	-12	-7	-2			

5. Make a table showing the value of each expression when the value of the variable is 1, 2, 3, 4, 5.

   a. 6x

   b. 4x + 6

6. Derive an equation for the following relationship.

x	0	1	2	3	4	5
y	6	9	12	15	18	21

Name: ________________________

Class:________________________

Date:________________________

Linear Modeling – Review 6

1. Graph the following data in the coordinate plane (***hint:*** *there will be two lines*):

The dosage chart below was prepared by a drug company for doctors who prescribed Tobramycin, a drug that combats serious bacterial infections such as those in the central nervous system, for life-threatening situations.

Weight (pounds)	Usual Dosage (mg)	Maximum Dosage (mg)
88	40	66
99	45	75
110	50	83
121	55	91
132	60	100
143	65	108
154	70	116
165	75	125
176	80	133
187	85	141
198	90	150
209	95	158

2. What is the *x*?

3. What are the two *y's*?

4. Insert lines of best fit to each data set and find the following:

   a. Slopes of the two lines: ____________________ ____________________

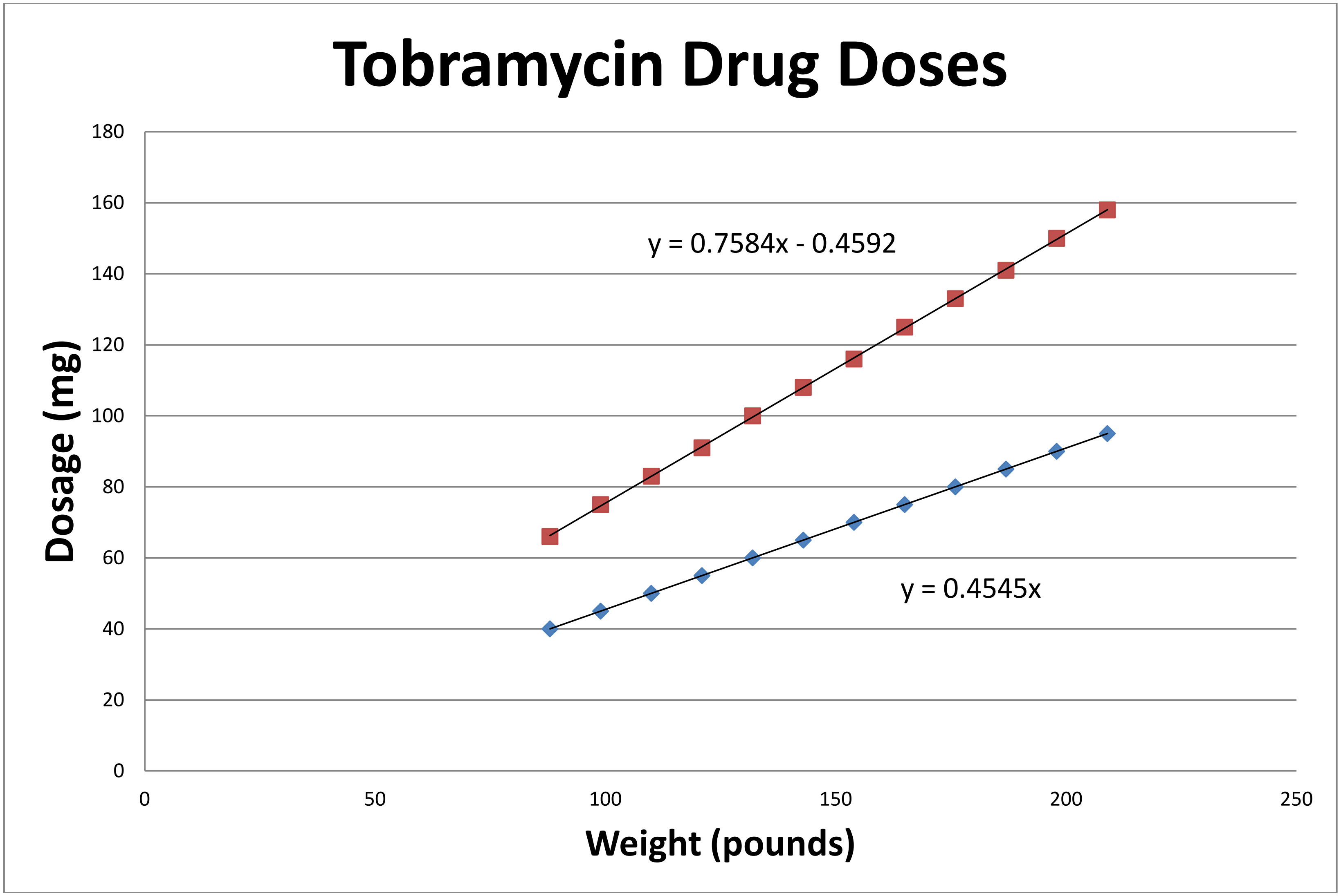
   b. y-intercepts of the two lines: ____________________ ____________________

   c. Equations of the two lines: ____________________ ____________________

5. Predict the usual dosage and maximum dosage for someone weighing 220 lbs.

Usual dosage: ________________________

Maximum dosage: ________________________


Tobramycin Drug Doses
Dosage (mg)
180
160
140
120
100
80
60
40
20
0
0
50
100
150
200
250
Weight (pounds)
y = 0.7584x - 0.4592
y = 0.4545x

1. Show all work on a separate sheet of paper stapled to this one.

Name: ______________________

Class:______________________

Date:______________________

Algebra I – Linear Modeling Practice Assessment

1. **Definitions.** Write the definitions of the following algebra terms.

   a. **Slope.** ____

   ____

   b. **y-intercept.** ____

   ____

   c. **Scatter Plot.** ____

   ____

   d. **Line of Best Fit.** ____

   ____

   e. **Correlation.** ____

   ____

2. For the following table:

x	6	9	12	15	18	21
y	25	30	35	40	45	50

a. Graph the data in the coordinate plane.

b. Find the slope (m) and y-intercept (b) of the linear equation and state the linear equation.

i. Slope: ______________________

ii. y-intercept: ______________________

iii. y = ______________________

3. The table below shows flight data for the Saturn V rocket.

x = Time of Flight in Seconds	0	66	83	135	161	460	548
y= Altitude in Nautical Miles	0	4.24	7.24	23.76	35.70	97.28	101.14

a. Graph the data in the coordinate plane.

b. Insert a line of best fit to the data.

c. Find the slope (m) and y-intercept (b) of the linear equation and state the linear equation.

i. Slope: ______________________

ii. y-intercept: ______________________

iii. y = ______________________

d. Predict the altitude of the Saturn V at 700 seconds into the flight.

1. **Show all work on a separate sheet of paper stapled to this one.**
2. **Attach all printed graphs to this sheet.**

Name: ___________________________

Date:____________________________

Advisor:__________________________

Algebra I – Linear Modeling Assessment

1. **Definitions.** Match the terms with the following definition by writing the letter of each term next to the correct definition.

a. Slope	________The *y*-coordinate of a point where a line crosses the *y*-axis.
b. y-intercept	________A line that shows the overall trend of data in a scatter plot.
c. Scatter Plot	________A measure of the relationship between two or more variables.
d. Line of Best Fit	________A type of mathematical diagram using the coordinate plane to display values for two variables for a set of data.
e. Correlation	________A measure of the steepness of a line. Given two points with coordinates (x_1, y_1) and (x_2, y_2) on a line, the slope, *m*, of the line is given by: $$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

1. For the following table:

x	0	1	2	3	4	5
y	12	17	22	27	32	37

a. Graph the data in the coordinate plane, insert a line of best fit, and display that line's equation on the graph.

b. Find the slope (*m*) and *y*-intercept (*b*) of the linear equation and state the linear equation.

i. Slope: ______________________

ii. *y*-intercept: ______________________

iii. *y* = ______________________

2. Maximum normal temperature in the month of January varies with latitude above the equator. Answer the questions below the table.

x = Latitude (in degrees)	25	30	40	45	50
y= Temperature (°F)	75	50	43	36	31

a. Graph the data in the coordinate plane.

b. Insert a line of best fit to the data and display that line's equation on the graph.

c. Is this a positive or a negative correlation?

d. Predict the maximum normal temperature at 50 degrees of latitude.

Algebra I – Linear Modeling Assessment

1. **Definitions.** Write the definitions of the following algebra terms.

   a. **Slope.** A measure of the steepness of a line. Given two points with coordinates (x_1, y_1) and (x_2, y_2) on a line, the slope, m, of the line is given by:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

   b. **y-intercept.** The y-coordinate of a point where a line crosses the y-axis.

   c. **Scatter Plot.** A type of mathematical diagram using the coordinate plane to display values for two variables for a set of data.

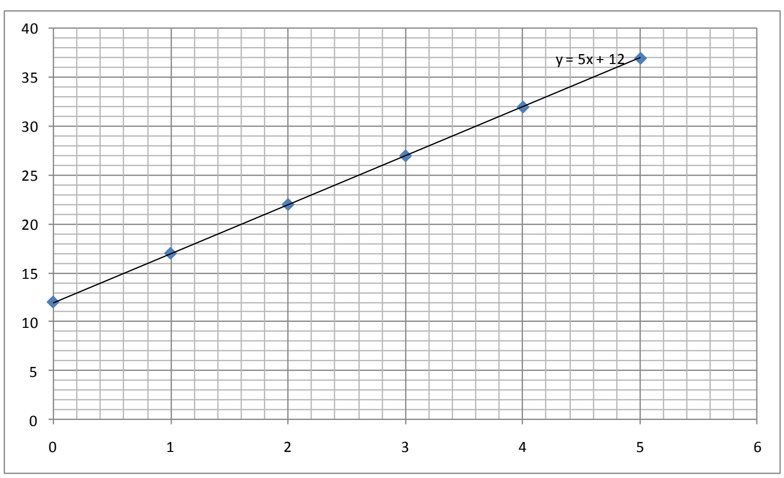
   d. **Line of Best Fit.** A line that shows the overall trend of data in a scatter plot.

   e. **Correlation.** A measure of the relationship between two or more variables.

2. For the following table:

x	0	1	2	3	4	5
y	12	17	22	27	32	37

   a. Graph the data in the coordinate plane.



b. Find the slope (m) and y-intercept (b) of the linear equation and state the linear equation.

i. Slope: 5

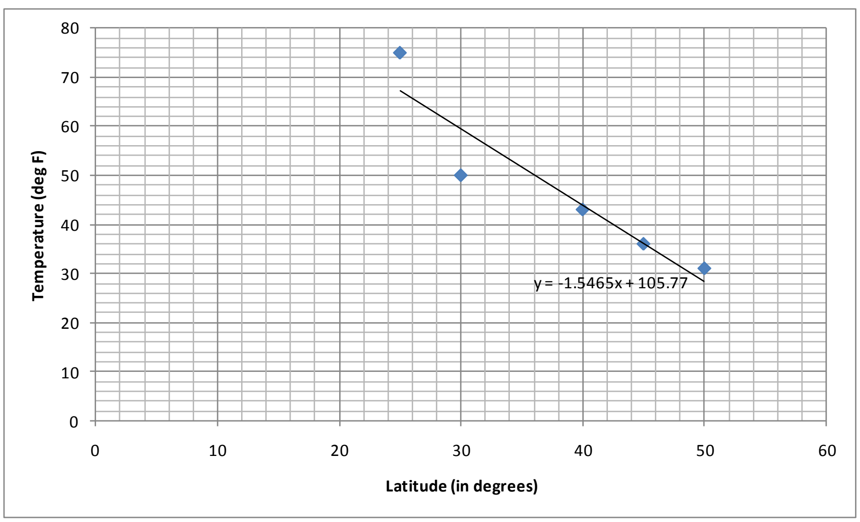
ii. y-intercept: 12

iii. $y =$ $5x + 12$

3. Maximum normal temperature in the month of January varies with latitude above the equator. Answer the questions below the table.

x = Latitude (in degrees)	25	30	40	45	50
y= Temperature (°F)	75	50	43	36	31

a. Graph the data in the coordinate plane.



b. Insert a line of best fit to the data.

c. Is this a positive or a negative correlation?

Negative

d. Predict the maximum normal temperature at 60 degrees of latitude.

1. Show all work on a separate sheet of paper stapled to this one.

Name: ________________________

Date:_________________________

Advisor:_______________________

Algebra I – Linear Modeling Assessment (if computers not available)

1. **Definitions.** Match the terms with the following definition by writing the letter of each term next to the correct definition.

a. Slope	_________The *y*-coordinate of a point where a line crosses the *y*-axis.
b. y-intercept	_________A line that shows the overall trend of data in a scatter plot.
c. Scatter Plot	_________A measure of the relationship between two or more variables.
d. Line of Best Fit	_________A type of mathematical diagram using the coordinate plane to display values for two variables for a set of data.
e. Correlation	_________A measure of the steepness of a line. Given two points with coordinates (x_1, y_1) and (x_2, y_2) on a line, the slope, *m*, of the line is given by: $$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

2. For the following table:

x	0	1	2	3	4	5
y	18	27	36	45	54	63

a. Find the slope (m) and y-intercept (b) of the linear equation and state the linear equation.

iv. Slope: ______________________

v. y-intercept: ______________________

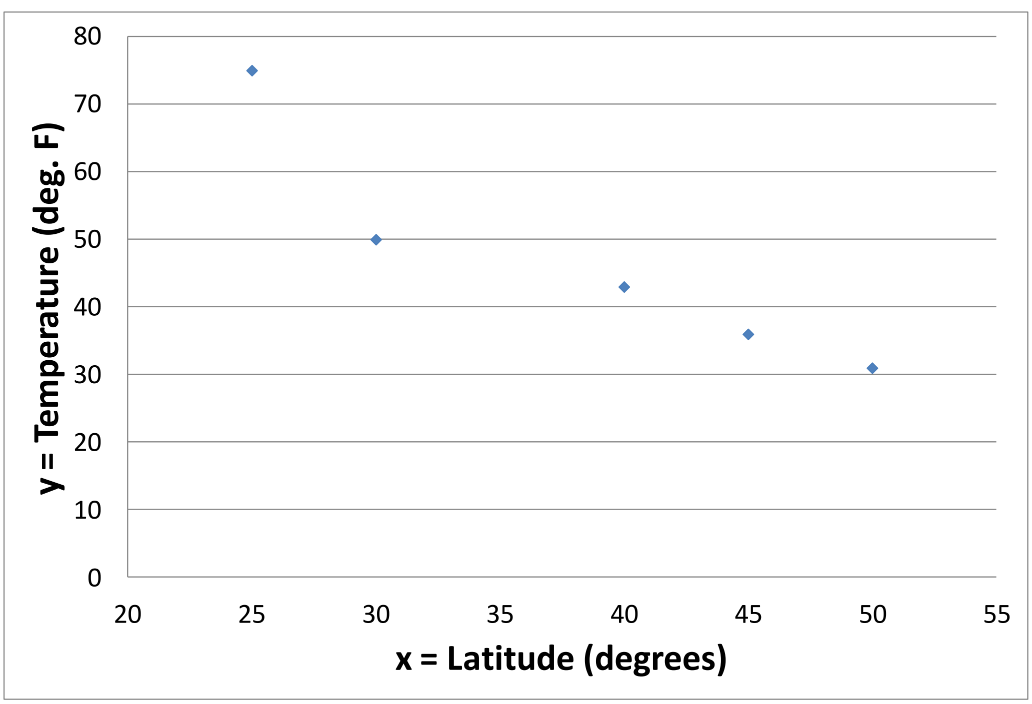
vi. y = ______________________

3. Maximum normal temperature in the month of January varies with latitude above the equator. Answer the questions below the table and graph.

x = Latitude (in degrees)	25	30	40	45	50
y= Temperature (°F)	75	50	43	36	31

a. Insert a line of best fit to the data.

b. Is this a positive or a negative correlation? ___________________________

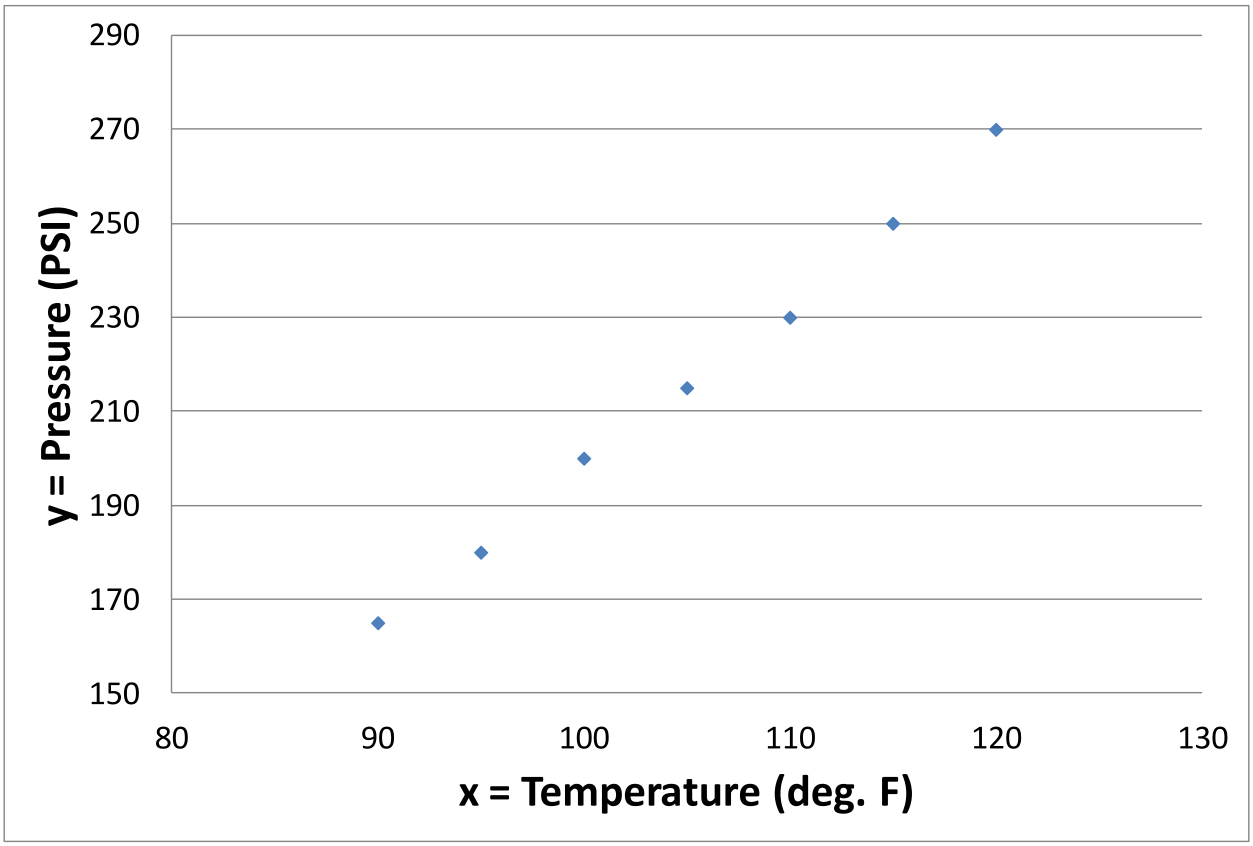


c. Predict the maximum normal temperature at 50 degrees of latitude.

____

4. The following table shows the increase in pressure (in a space whose size does not change) as the temperature in the space increases.

Temp (°F)	90	95	100	105	110	115	120
Pressure (PSI)	165	180	200	215	230	250	270



a. Insert a line of best fit.

b. Find the following from the line of best fit:

i. Slope: ____________________________

ii. y-intercept: ______________________________

iii. y = ______________________________

c. What is the rate of change of pressure as the temperature increases?

____

d. Predict the pressure at 130 ºF.

____

Grade / Content Area	**Grade 9 Algebra**
Lesson Title	**“1968”**
Guiding Question	*Why was the Apollo 8 mission an important step on mankind’s journey to the Moon?*
Content Standards	I. No mathematics standards for this lesson.
Preparation	I. Students will watch the first episode of the HBO miniseries “From the Earth to the Moon,” entitled *1968*. II. Desks will be arranged to permit watching the movie. We will need the TV and DVD player. III. Students will be provided with the following questions to reflect on what they have watched.
Student Learning Objectives	I. Students will answer questions about the history of the American effort to land on the Moon. II. Students will understand the connection between these events and the rocket construction and data analysis project they will begin in the next lesson.
Instruction and Engagement	I. *Warm-up* (**5 minutes**). II. *Launch* (**5 minutes**). I will handout the question sheets and instruct students to write answers during the movie on a separate sheet of paper for collection at the end of class. III. *Engagement* (**60 minutes**). We will watch the movie.

Name: ___________________________

Class:____________________________

Date:____________________________

Movie Questions: 1968

1. What is the war that Americans are fighting in as shown at the start of the movie?

2. Who is the President of the United States who is heard to say, "All I want is for the killing to stop?"

3. What is the solution to opening the spacecraft hatch faster than 20 seconds?

4. Who are the three astronauts who are the crew of Apollo 8?

5. What is the name of the wife of the commander of Apollo 8?

6. In what city is the Democratic National Convention being held where a lot of rioting is taking place?

7. From what book did the crew of Apollo 8 read to Earth on Christmas Eve, 1968?

8. Why did one of the astronauts say, "Houston, be advised, there is a Santa Claus?"

9. What is the name of the woman who sent the telegram to the astronauts in space saying, "You saved 1968"?

End of Unit Reflection

We have completed our first unit in Algebra I on linear modeling. Many of the problems and examples we studied involved the American space program. In this unit, we studied:

a. How to create a coordinate plane with correct scales on both axes by and with the computer.
b. How to plot data on the coordinate plane by hand and with the computer.
c. How to insert a line of best fit to that data by hand and with the computer.
d. How to determine the slope, y-intercept, and equation of the line of best fit by hand and with the computer.
e. How to make predictions using the line of best fit and its equation by hand and with the computer.
f. How to build a paper model rocket and use it to gather and study data on how far it flies.
g. The history of man's effort to travel to the Moon.

Directions: Answer the questions below in at least two complete sentences:

1. Of the seven things we studied (listed above), which do you feel you have mastered and which do you feel you still need practice on?

____

____

____

____

____

2. What did you find most interesting about this unit?

____

____

____

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____

3. What would you change in the unit to help students better learn the seven things we studied?

Name: ___________________________

Date:_____________________________

Advisor:__________________________

How Do We Use Math To Predict the Future?"

Project: A Model Problem

Directions

1. This is a project in which you will plot and analyze real-world data concerning the growing amount of man-made debris in orbit around the Earth.

2. Read all questions carefully and show all work. The following is an example of what I mean by **<u>show all work</u>**:

   Example Problem: Given the following ordered pairs: (4, 2) and (5, 4)

   1) Find the slope (m) and the y-intercept (b) of the line containing these points.

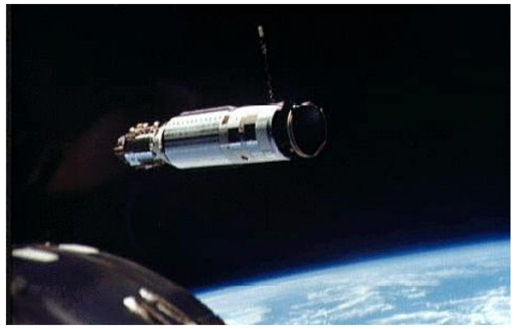
   2) Write the equation of the line in slope-intercept form.

Step 1: Solve for m.	Step 2: Solve for b.	Step 3: Check Step 2.
1) $m = \frac{y_2-y_1}{x_2-x_1} = \frac{4-2}{5-4} = \frac{2}{1} = 2$	$y = mx + b$	$2 = 2(4) - 6$
	$4 = 2(5) + b$	$2 = 8 - 6$
	$4 = 10 + (-6)$	$b = -6$
	$4 = 10 - 6$	
	$b = -6$	

2) $y = 2x - 6$

3. This project will be completed using Microsoft EXCEL or Open Office. **You will** a**ttach to your completed worksheets printouts of all data tables and all graphs.**

A Model Problem[8]



One problem with which National Aeronautics and Space Administration (NASA) scientists and space scientists from other countries must deal is the accumulation of space debris in orbit around Earth. Such debris includes satellites that are no longer operating; spent stages of rockets (like the Saturn V), assorted parts and lost tools; debris from the breakup of larger objects or from collisions between objects; and countless small pieces, such as flakes of paint and even smaller objects. Because bodies in Earth orbit travel at approximately 17,500 miles per hour, a collision with even a tiny object can have catastrophic effects. At the end of 1989, scientists estimated that a total of 4 million pounds of debris was in Earth orbit.

I. Since 1990, data on the amount of debris in orbit has been gathered each year. The following is data for the years 1990 to 1999:

A. Make a data table and plot this data on the coordinate plane. Ensure you give this coordinate plane a title and also labels for the X-axis and Y-axis.

Year	**Amount of Debris in Orbit (millions of pounds)**
1 (1990)	5.80
2 (1991)	7.69
3 (1992)	9.67
4 (1993)	11.74
5 (1994)	13.90
6 (1995)	16.15
7 (1996)	18.49
8 (1997)	20.92
9 (1998)	23.44
10 (1999)	26.05

Debris in Orbit 1990 – 1999

[8] This project is derived from the lesson plan "Modeling Orbital Debris Problems", copyright 2010 The National Council of Teachers of Mathematics, downloaded 27 July 2010 from http://illuminations.nctm.org/LessonDetail.aspx?id=L376.

B. Does the data appear to be linear? Why or why not?

C. Insert a line of best fit to the data.

1. Find the slope and y-intercept of this line and write the equation for this line in slope-intercept form ($y = mx + b$).

   a. Slope: ____________________________________

   b. y-intercept: ____________________________________

   c. y = ____________________________________

2. Using your line of best fit, what is the rate at which debris is increasing in orbit annually?

3. Use your line of best fit to predict the amount of debris that will be in orbit by 2019. What is your prediction?

II. The following is data for the years 1990 – 2009 (what you plotted before plus the years 2000 – 2009).

Year	**Amount of Debris in Orbit (millions of pounds)**
1 (1990)	5.80
2 (1991)	7.69
3 (1992)	9.67
4 (1993)	11.74
5 (1994)	13.90
6 (1995)	16.15
7 (1996)	18.49
8 (1997)	20.92
9 (1998)	23.44
10 (1999)	26.05
11 (2000)	28.75
12 (2001)	31.54
13 (2002)	34.42
14 (2003)	37.39
15 (2004)	40.45
16 (2005)	43.60
17 (2006)	46.84
18 (2007)	50.17
19 (2008)	53.59
20 (2009)	57.10

Debris in Orbit 1990 – 2009

A. Make a data table and plot this data on the coordinate plane. **It should be a separate table and graph from the one you created for the first 10 years**.

B. Insert a line of best fit to the data for the entire 20 years.

1. Find the slope and y-intercept of this line and write the equation for this line in slope-intercept form ($y = mx + b$).

   a. Slope: ______________________

   b. *y*-intercept: ______________________

   c. *y* = ______________________

2. Using your line of best fit, what is the rate at which debris is increasing in orbit annually?

3. Use your line of best fit to predict the amount of debris that will be in orbit by 2019. What is your prediction? **Is your prediction the same as before? Why or why not?**

C. Is this line of best fit as good a predictor for future amounts of debris in orbit as your first line of best fit appeared to be? Why or why not?

III. The following is data for the years 1990 – 2020 (2010 – 2020 are projections).

Projected Debris in Orbit 2010 – 2019

Year	Amount of Debris in Orbit (millions of pounds)
1 (1990)	5.80
2 (1991)	7.69
3 (1992)	9.67
4 (1993)	11.74
5 (1994)	13.90
6 (1995)	16.15
7 (1996)	18.49
8 (1997)	20.92
9 (1998)	23.44
10 (1999)	26.05
11 (2000)	28.75
12 (2001)	31.54
13 (2002)	34.42
14 (2003)	37.39
15 (2004)	40.45
16 (2005)	43.60
17 (2006)	46.84
18 (2007)	50.17
19 (2008)	53.59
20 (2009)	57.10
21 (2010)	60.70
22 (2011)	64.39
23 (2012)	68.17
24 (2013)	72.04
25 (2014)	76.00
26 (2015)	80.05
27 (2016)	84.19
28 (2017)	88.42
29 (2018)	92.74
30 (2019)	97.15

A. Make a data table and plot this data on the coordinate plane. **It should be a separate table and graph from the those you have already created**.

B. Insert a line of best fit to the data for the entire 30 years.

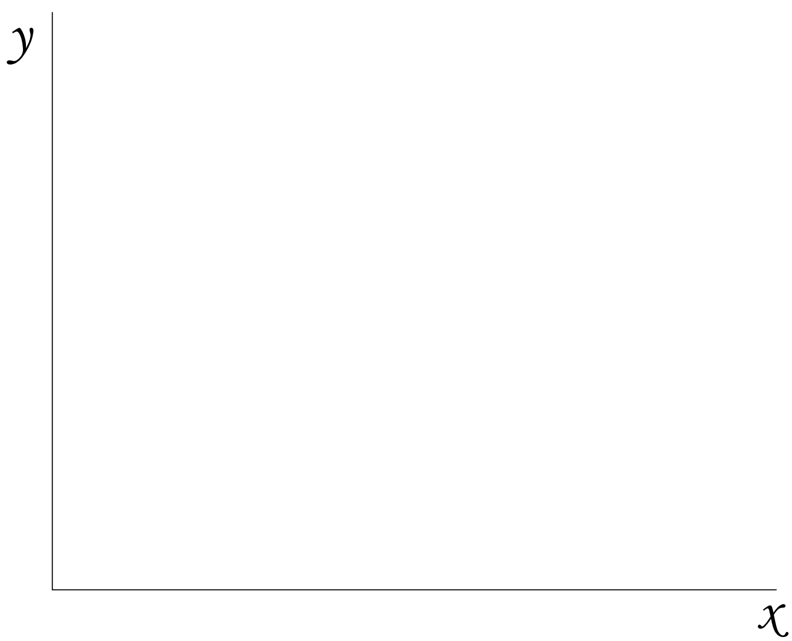
1. Find the slope and y-intercept of this line and write the equation for this line in slope-intercept form.

   a. Slope: ____________________

   b. *y*-intercept: ____________________

   c. *y* = ____________________

C. If you were to connect all of the data points for all 30 years together, what would it look like? *Sketch it in the Cartesian coordinate plane below.*



D. If all lines of best fit are of the form $y = mx + b$, what do you think the form of this curve might be?

Performance Task Rubric

I will use this rubric to assign points to each of problems I, II, and III.

CATEGORY	**4 – Highly Proficient**	**3 – Proficient**	**2 – Partially Proficient**	**1 – Below Proficient**	**0 – Not Proficient**
Coordinate Plane Title	Title is creative and clearly relates to the data being graphed and the linear model of the data.	Title clearly relates to the data being graphed and the linear model of the data.	Title somewhat relates to the data being graphed and the linear model of the data.	Title exists but it does not relate to the data being graphed or the linear model of the data.	There is no title.
Labeling of X - Axis		The X - axis has a label and describes the units used for the independent variable.	The X - axis does not have a label but does describe the units used for the independent variable.	The X - axis has a label but does not describe the units used for the independent variable.	There is no label and the independent variable's units are not described.
Labeling of Y - Axis		The Y - axis has a label and describes the units used for the dependent variable.	The Y - axis does not have a label but does describe the units used for the dependent variable.	The Y - axis has a label but does not describe the units used for the dependent variable.	There is no label and the dependent variable's units are not described.
Line of Best Fit		Line of best fit has been included and linear equation is shown.	Line of best fit has been included.		Line of best fit has not been included.

CATEGORY	4 – Highly Proficient	3 – Proficient	2 – Partially Proficient	1 – Below Proficient	0 – Not Proficient
Determination of Slope, Y – Intercept, and Equation of the Line of Best Fit		Slope, y-intercept, and equation of line of best fit are correctly identified.	One of these is incorrect or missing.	Two of these are incorrect or missing.	Slope, y-intercept, and equation of line of best fit are not correctly identified.
Analysis		Student understands that the slope of the line of best fit models the rate of change in the data over time and correctly identifies the modeled rate of change from the slope of the line of best fit.	Student understands that the slope of the line of best fit models the rate of change in the data over time but does not derive a logical rate of change from the slope of the line of best fit.		Student does not make the connection between slope of the line of best fit and the modeled rate of change; unable to derive a logical rate of change from the slope of the line of best fit.
Prediction Using Linear Model		Makes a defensible prediction of a future outcome using the line of best fit as a linear model; prediction is defended by graphical representation of the line of best fit and the	Makes a prediction of a future outcome which is reasonable but which is not defensible by use of a linear model.		Makes no prediction.

CATEGORY	4 – Highly Proficient	3 – Proficient	2 – Partially Proficient	1 – Below Proficient	0 – Not Proficient
		data point it intersects at the year of prediction.			
Mathematical Accuracy	All arithmetic operations are performed correctly with each step clearly shown.	All steps are shown but one or two arithmetic operations are performed incorrectly.	All arithmetic operations are performed correctly but some steps are not shown or more than two arithmetic operations are performed inaccurately.	More that two arithmetic operations are performed incorrectly and some steps are not shown.	Problem is not attempted or answers are given with no work shown.
Reasoning **Extra Credit**	Identifies a non-linear model that better explains the trend in the data; justifies answer with a logical, clearly-stated argument that reflects ability to reason logically and mathematically.		Identifies a non-linear model that better explains the trend in the data. Justification is not logical, not clearly stated or reasoning is unclear.		Does not identify a non-linear model that better explains the trend in the data.
Visual Presentation		Tables and graphs are attached to project worksheets, neatly and completely presented.	Tables and graphs are attached but are not neatly presented.		Tables and graphs are not attached.

Name: ________________________

Class: __________________________

Date: __________________________

Alternate Final Project: Orbital Data and the International Space Station

Directions: The purpose of this project is to link the unit on linear modeling we just finished to our current unit on equation writing. It is also designed to give you the opportunity to improve your grade on the linear modeling unit.

1. Read all directions very carefully.

2. You must answer questions 1 and 2 to get credit for this project under the equations unit.

3. To raise your grade on the linear modeling unit to a “3”, you must answer questions 2.a. – 2.d completely and correctly ***on your own with no help from anyone.***

4. To raise your grade on the linear modeling unit to a “4”, you must answer all parts of questions 2 – 5 completely and correctly *on your own with no help from anyone.*

5. You must complete and print out all graphs, attaching them to this document.

6. To get any credit at all, you must submit this document to Captain Beall no later than 051430L OCT 11.

Orbital Data of the International Space Station

1. The International Space Station is currently in orbit 340 kilometers (km) from the surface of the Earth. Each day, it loses about 90 meters in altitude.

Write an equation to express its altitude after:

a. One week (7 days).

b. One month (30 days).

c. One year (365 days).

Source: National Aeronautics and Space Administration: http://spacemath.gsfc.nasa.gov/algebra1.html

2. **Twice, NASA has pushed the station back into a higher orbit** (once in 2002 and once in 2008). The table below shows ISS altitude since 1998.

Year	Altitude (in km)
1998	400
1999	380
2000	360
2001	380
2002	390
2003	380
2004	360
2005	350
2006	340
2007	330
2008	340
2009	360
2010	340
2011	340

If the station falls below 200 km, it will begin to fall rapidly and burn up in the atmosphere within a week. **NASA has asked you to determine when the ISS will fall to the Earth.**

a. **Enter the data** from the table above into a spreadsheet and build a scatter plot of the data on a graph.

b. **Fit a line** to the data and display the equation on the graph. ***Print the graph and attach it to this assignment.***

c. Use that equation to **predict the altitude for each year after 2011**.

d. Find the year at which the altitude falls below 200 km: ________________

Source: National Aeronautics and Space Administration: http://spacemath.gsfc.nasa.gov/algebra1.html

3. Because the ISS was pushed back into a higher orbit twice, your linear model may not be accurate.

   a. Replot your graph using only the years 2002 – 2011.

   b. Fit a line to the data and display the equation on the chart.

   c. Use that equation to predict the altitude for each year after 2011.

   d. Find the year at which the altitude falls below 200 km: _________________

   e. Repeat these steps using only the years 2008 – 2011. Find the year at which the altitude falls below 200 km: _________________

4. Which of the three predictions do you choose as your final one? _____________
   Why did you choose this one?

   ____

   ____

   ____

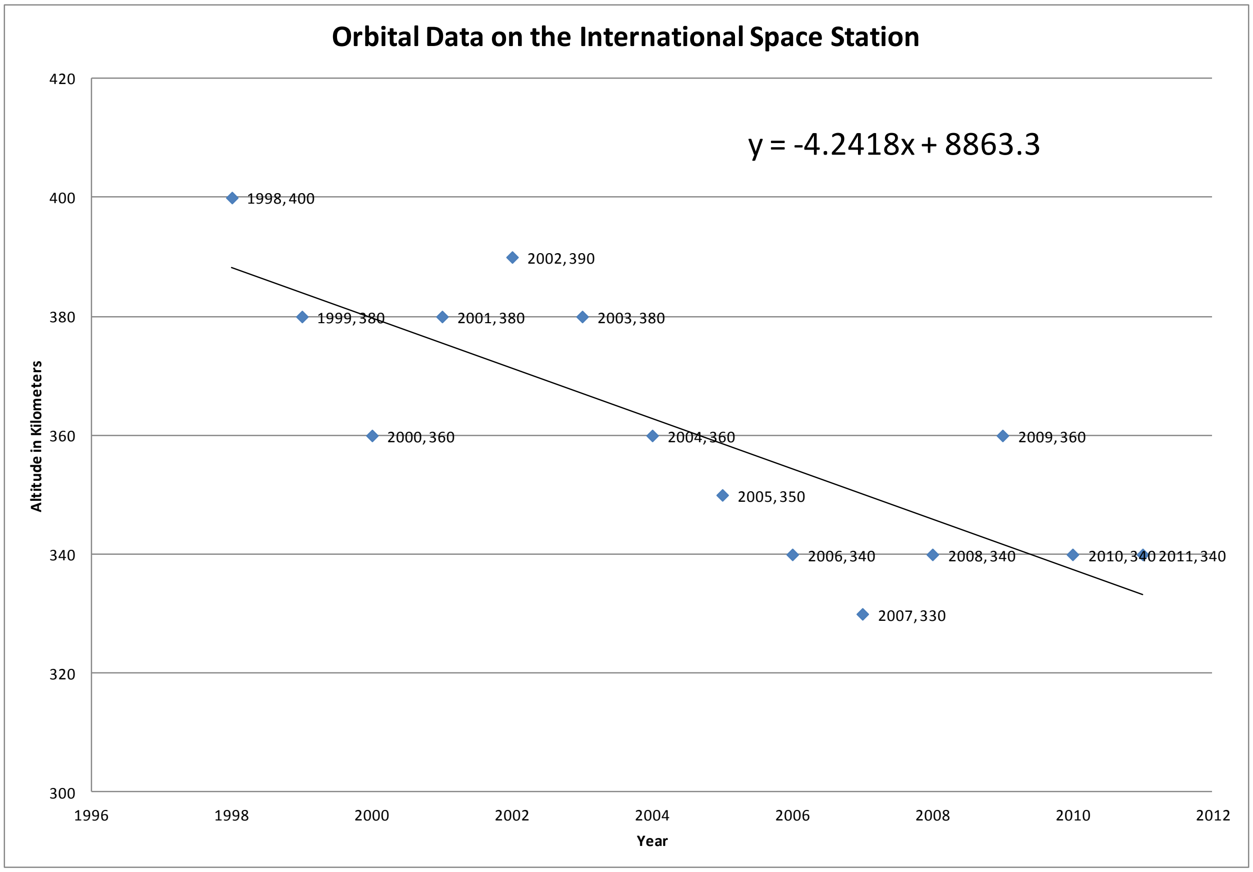
5. The ISS is expected to fall an additional 50 km in 2014 due to a phenomenon called Sunspot Maximum. How will this change your prediction?

   ____

   ____

   ____

Source: National Aeronautics and Space Administration: http://spacemath.gsfc.nasa.gov/algebra1.html



Source: National Aeronautics and Space Administration: http://spacemath.gsfc.nasa.gov/algebra1.html

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

“Geometry of the Ship”

8th / 9th Grade Geometry Unit Plan

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Introduction

Math literacy and economic access are how we are going to give hope to the young generation.
(Robert P. Moses)

I share Robert Moses' vision; it is what inspires me to pursue a career as a mathematics teacher. Further, I learned during my first career that familiarity with mathematics provides a rich additional dimension to one's understanding of the world. I want to help my students to achieve that familiarity so that their understanding of the world helps them to lead more meaningful and useful lives in their communities.

As a student, I found mathematics uninteresting. It wasn't until I studied a field of applied mathematics as a graduate student, following four years in my chosen profession surrounded by math, that I learned to appreciate the utility and beauty of mathematics. I want to impart that appreciation to my students at a much earlier age so that they may explore the potential of mathematics knowledge and learn to use it in their lives. It is this desire that motivates me to design instruction that makes use of real-world examples. This unit plan, while designed to meet national and state's expectations for achievement in geometry by 8^{th} grade students, supports my students' learning by linking theory with a real-world application, the loading and sailing of a commercial cargo container ship. The question I pose is "Why is Geometry relevant?" It is one I hope I and my students will explore together by exploring some aspects of the mathematics of cargo stowage and movement aboard merchant ships that today sail the world's oceans.

This unit plan has been developed in accordance with national and state expectations for content mastery at the 10^{th} grade level. These expectations are listed in each lesson plan. The unit also builds upon foundations established in previous grades and earlier in the year. In terms of Rhode Island expectations, essential foundations are:

1. **M(G&M)–7–2:** Applies theorems or relationships (triangle inequality or sum of the measures of interior angles of regular polygons) to solve problems.

2. **M(G&M)–7–6:** Demonstrates conceptual understanding of the area of circles or the area or perimeter of composite figures (quadrilaterals, triangles, or parts of circles), and the surface area of rectangular prisms, or volume of rectangular prisms, triangular prisms, or cylinders using models, formulas, or by solving related problems. Expresses all measures using appropriate units.

3. **M(G&M)–7–10**: Demonstrates conceptual understanding of spatial reasoning and visualization by sketching three-dimensional solids; and draws nets of rectangular and triangular prisms, cylinders, and pyramids and uses the nets as a technique for finding surface area.

4. **M(F&A)–8–1:** Identifies and extends to specific cases a variety of patterns (linear and nonlinear) represented in models, tables, sequences, graphs, or in problem situations; and generalizes a linear relationship (non-recursive explicit equation); generalizes a linear relationship to find a specific case; generalizes a nonlinear relationship using words or symbols; or generalizes a common nonlinear relationship to find a specific case.

5. **M(F&A)–8–2:** Demonstrates conceptual understanding of linear relationships ($y = kx$; $y = mx + b$) as a constant rate of change by solving problems involving the relationship between slope and rate of change; informally and formally determining slopes and intercepts represented in graphs, tables, or problem situations; or describing the meaning of slope and intercept in context; and distinguishes between linear relationships (constant rates of change) and nonlinear relationships (varying rates of change) represented in tables, graphs, equations, or problem situations; or describes how change in the value of one variable relates to change in the value of a second variable in problem situations with constant and varying rates of change.

6. **M(F&A)–8–4**: Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., $d = rt$; $\frac{d}{r} = t$); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context.

In designing this instruction, I have been guided by Rhode Island's Professional Teacher Standards, specifically:

1. **Standard 1: Teachers create learning experiences using a broad base of general knowledge that reflects an understanding of the nature of the world in which we live.**

This is reflected in the use of a real-world scenario that, while its relevance may not be immediately obvious, *is very relevant* to how students live their daily lives.

2. **Standard 2: Teachers have a deep content knowledge base sufficient to create learning experiences that reflect an understanding of central concepts, vocabulary, structures, and tools of inquiry of the disciplines/content areas they teach.** Lesson plans are aligned with national and state standards **(2.2)**, incorporate technological resources at appropriate times **(2.3)**, use a variety of explanations and representations **(2.4)**, and use multiple methods **(2.5)**.

3. **Standard 3: Teachers create instructional opportunities that reflect an understanding of how children learn and develop.** All lessons involve some combination of group work and hands-on activities, many with manipulatives. This instructional design is, therefore, well aligned with how students at the middle school level best learn.

4. **Standard 4: Teachers create instructional opportunities that reflect a respect for the diversity of learners and an understanding of how students differ in their approaches to learning.** Accommodations for individual student learning needs **(4.4)** are addressed in individual lessons.

5. **Standard 5: Teachers create instructional opportunities to encourage all students' development of critical thinking, problem solving, performance skills, and literacy across content areas.** All lessons are inquiry-based with Socratic dialog incorporated into instruction **(5.1, 5.2)**. The lesson plans guide me in making appropriate instructional decisions **(5.3)**, and engage students through multiple tasks **(5.4, 5.5)**.

6. **Standard 6: Teachers create a supportive learning environment that encourages appropriate standards of behavior, positive social interaction, active engagement in learning, and self-motivation.** Middle School classroom management is easier and more effective if students are organized into groups for collaborative effort with time allocated for exploration and discovery **(6.1, 6.5)**. All lessons involve this organization and do provide this time.

7. **Standard 7: Teachers work collaboratively with all school personnel, families and the broader community to create a professional learning community and environment that supports the improvement of teaching, learning and student achievement.** This unit was developed following consultation with my cooperating teacher. To a certain degree, it leverages science and social studies content providing me the opportunity to collaborate more closely with team colleagues when I implement the plan.

8. **Standard 8: Teachers use effective communication as the vehicle through which students explore, conjecture, discuss, and investigate new ideas.** Socratic dialog, multiple modes of communication, technology, and group interaction (encouraging sharing and listening) are incorporated into all lessons **(8.1, 8.2, 8.3, 8.4, 8.5)**.

9. **Standard 9: Teachers use appropriate formal and informal assessment strategies with individuals and groups of students to determine the impact of instruction on learning, to provide feedback, and to plan future instruction.** Informal assessment is incorporated into every lesson – particularly through numerous hands-on-activities that involve both inductive and deductive reasoning. The summative assessment at the conclusion of the unit brings together all of the concepts students have learned.

P C H S

CURRICULUM MAPPING: Middle and High School Geometry – Geometry of the Ship

CENTRAL QUESTION(S)/THEMES

3. What is the practical application of accurately measuring segments and angles?
4. When will I ever need to know how to find the surface area and volume of a shape?

TEXT(S) AND RESOURCES

4. Clark, I. C. *Ship Dynamics for Mariners.*
5. Hooyer, H. H. *Behavior and Handling of Ships.*
6. Lewis, *Principles of Naval Architecture.*
7. NCTM *Illuminations.* Popcorn Prisms Anyone?
8. NCTM *Illuminations.* Scaling Away.

STUDENTS WILL KNOW AND BE ABLE TO . . .

Content, Skills, & Standards to be assessed by rubric(s) in this unit (Common Core, GSE, HOL and PCHS Expectations)

1. **M(G&M)–10–2:** Makes and defends conjectures, constructs geometric arguments, uses geometric properties, or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g., Pythagorean Theorem, Triangle Inequality Theorem).

2. **M(G&M)-10-5:** Applies the concepts of similarity by solving problems within mathematics or across disciplines or contexts.

3. **M(G&M)-10-6:** Solves problems involving perimeter, circumference, area, surface area, and volume.

PROJECT/PRODUCT & PUBLIC DEMONSTRATION

1. Container Scale Model Project.
2. Loading Container Project.
3. Ship Model Construction Project.
4. Ship Stability Project.

OTHER EVIDENCE OF STUDENT LEARNING

3. Diagnostic Pre-assessment.
4. Pencil and Paper Post-assessments.

SCAFFOLDED TEACHING AND LEARNING ACTIVITIES

4. **"Layout of a Ship" Activity.** Students identify and define parts of a ship on a general layout diagram and relate those parts to geometric shapes, defining those as well.

5. **"Where Did That Product Come From" Activity.** Builds on student knowledge, learned in middle school, of how to find the surface area and volume of a rectangular prism to illustrate application in shipping cargo around the world.

6. **"Building a Scale Model of a Container" Project.** Building on review of rectangular prisms and how to find their dimensions, students will:

   A. Accurately design a scale model of a standard 20 ft. CEU shipping container.
   B. Build a scale model to design dimensions.
   C. Determine the surface area and volume of the scale model and discover the relationship between the model's dimensions and the actual container's dimensions.

7. **"Stowing and Loading of a Ship" Project.** Using the scale model of the container they have built and various shapes, students will determine the optimum loading of the container to maximize the shippers profit.

8. **Model Shipbuilding Project.** Using a simple naval architect's drawing of the contours of a ship's hull, students will:

   A. Create a geometric reflection of the forward and aft contours.
   B. Build a model of the ship using athwart ship frames traced from the contour drawing.

9. **Ship Stability Project.** Students will measure weights and dimensions of various shapes and the angle of heel of a floating platform when those weights are moved various distances from the centerline.

10. **Coastal Navigation Exercise.** Students will apply the Pythagorean Theorem and distance measures to develop piloting plans in coastal waters.

Unit Outline and Table of Contents

This unit comprises eight lessons. Given that students learn at varying rates, instruction will likely take place over 3 – 5 weeks. The outline of the plan is as follows:

<table>
<tr><th>Page</th><th>Lesson Topics</th><th>Geometry Topics</th></tr>
<tr><td>GS-1</td><td>Understanding the General Layout of a Ship and its Relationship to Geometry. (1 day)

1. Product movement – how, why.
2. Types of ships.
3. Definitions: Ship construction components.

Activity: General Layout of a Ship.</td><td rowspan="2">A. Basic Geometry.

1. Definitions of segments, rays, angles, planes.
2. Basic measurement.
3. Scaling and scale factors as they apply to basic measurement.

B. Measuring segments, rays, angles, planes.</td></tr>
<tr><td>GS-6</td><td>Where Did That Product Come From? (1 day)

Activity:

1. Shipping Costs.
2. Popcorn Prisms Anyone?</td></tr>
<tr><td>GS-19</td><td>Building and Filling a Scale Model of a Container (6 days).

Project: Building a Scale Model of a Container.

Project: Filling a Container.</td><td rowspan="5">A. Surface Area and Volume of Rectangular Prisms.

B. Scaling and Scale Factors as They Apply to Rectangular Prisms.

C. Finding the Surface Area and Volume of Cylinders.

D. Finding the Surface Area and Volume of Regular Polygons.</td></tr>
<tr><td>GS-45</td><td>Scaling (2 days).

Activity: Scaling Away.</td></tr>
<tr><td>GS-62</td><td>Fueling / Loading the Ship (1 day).

Project: Loading and Stability Lab.</td></tr>
<tr><td>GS-69</td><td>Cylinders and the General Formula for Prisms (3 – 4 days).</td></tr>
<tr><td>GS-102</td><td>Building a Ship Model (14 Days).

Project: The Geometry of the Ship.</td></tr>
<tr><td>GS-120</td><td>A. Navigation and a Rescue at Sea (3 days).

Activities:

1. Pythagorean Review Activity.
2. Rentz activity or
3. Coast Guard Cutter activity.</td><td>A. Angle and Side Measure of a Triangle.

B. Calculating the Area of Odd Shapes.</td></tr>
</table>

Unit Goals

Students will:

1. Place learning in a real-world context by exploring the connections between basic geometry and the design and operation of merchant ships at sea.

2. Design and build accurate models of three dimensional shapes.

3. Derive equations for surface area and volume of rectangular and triangular prisms and cylinders through inductive reasoning.

4. Accurately evaluate changes in surface area and volume of three dimensional shapes when one or more of the dimensions is changed.

5. Convert volume measurements into liquid measurements in both the English and metric systems.

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:___________________________________

Advisor:_________________________________

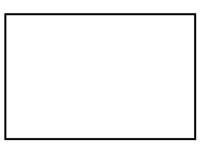
Date:____________________________________

Geometry Unit Pre-Assessment

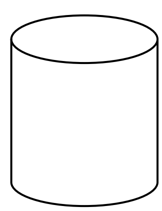
I. Definitions. Match the term by placing its letter next to its definition on the right:

Term	***Definition***
A. Radius of a Circle:	___C__ The sum of the lengths of the sides of a polygon.
B. Rectangle:	___A__ Any line segment from its center to its perimeter.
C. Perimeter:	___G__ The amount of space a solid body occupies.
D. Chord:	___E__ The size of the region enclosed by a figure.
E. Area:	___F__ The ratio of the circumference of a circle to its diameter.
F. Pi:	___D__ Line segment whose endpoints both lie on the circle.
G. Volume:	___B__ A quadrilateral with opposite sides of equal lengths and with four right angles.

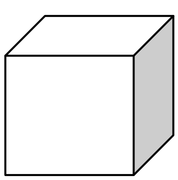
II. Identify each of the following shapes:



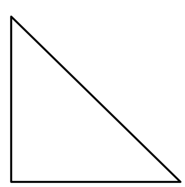
Rectangle



Cylinder

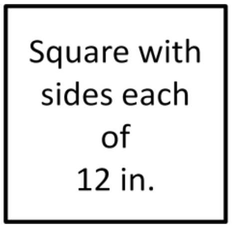


Rectangular Prism/ Cube



Triangle / Right Triangle

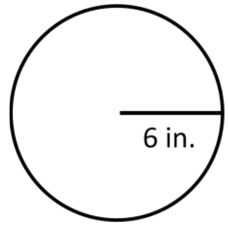
III. What is the **area** and the **perimeter** of each of the following shapes?



Area**: 144 sq. in.**

$Length\ =\ width = 12\ in.$
$Area = 12\ in. \times 12\ in. =$
$144\ sq.in.$

$Length\ =\ width = 12\ in.$
$Perimeter = 12 + 12 + 12 + 12\ =\ 48\ in.$

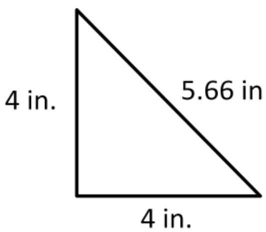


Area: **113.094 sq. in.**

$A = \pi \times r^2 = 3.1415 \times 36\ in.^2$
$= 113.094\ in.^2$

Perimeter: **37.698 in.**

$P = 2 \times \pi * r = 2 \times 3.1415 \times 6\ in.$
$= 37.698\ in.$

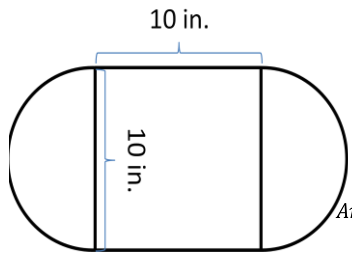


Area**: 8 sq. in.**

$A = \frac{1}{2} \times (l \times w) = \frac{1}{2} \times 16\ in.^2$
$= 8\ in.^2$

Perimeter: **13.66 in.**

$P = 4\ in. + 4\ in. + 5.66\ in. = 13.66\ in.$



Area**: 178.54 sq. in.**

$Area\ of\ rectangle = (l \times w) = 100\ in.^2$

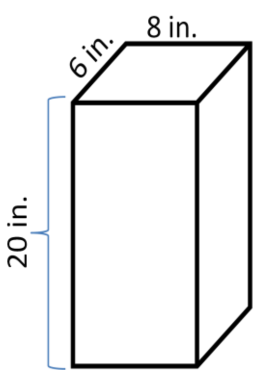
$Area\ of\ half\ circles = 2 \times \left(\frac{1}{2} \times (\pi \times 5^2)\right) = 2 \times$
$\left(\frac{1}{2} \times (3.1415 \times 25)\right) = 78.54\ in.^2$

$Area = 100\ in.^2 + 78.54\ in.^2 = 178.54\ in.^2$

Perimeter**: 51.415 in.**

$P = 2 \times (\frac{1}{2} \times (2 \times 3.1414 \times 5\ in.) +$
$(2 \times 10\ in.) = 51.415\ in.$

IV. What is the **surface area** and **volume** of each of the following shapes?



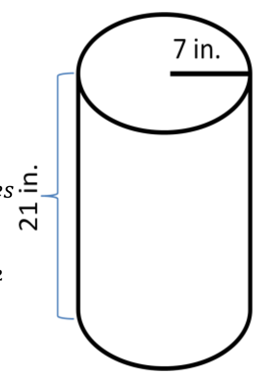
Surface Area**: 656 sq. in.**

$Surface\ Area$
$= the\ sum\ of\ the\ areas\ of\ the\ 6\ sides$

$= 2 \times (6\ in. \times 8\ in.) + 2 \times (6\ in. \times$
$20\ in.) + 2 \times (8\ in. \times 20\ in.) = 656\ in^2$

Volume**: 960 cu. in.**

$Volume = l \times w \times h = 8\ in. \times 6\ in. \times$
$20\ in. = 960\ in^3$



Surface Area**: 1231.47 sq. in.**

$Surface\ Area = 2 \times (3.1415 \times r^2) +$
$(2 \times \pi \times r) \times height$

$= 2 \times (3.1415 \times (7in.)^2) + (2 \times 3.1415 \times$
$7in.) \times 21 = 1231.47\ in.^2$

Volume**: 3232.6 cu. in.**

$Volume = Area\ of\ the\ cirle \times height$

$= \pi \times r^2 \times h = 3.1415 \times 49 \times 21 = 3232.6in^2$

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

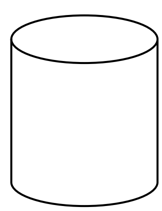
Name:__________________________________
Advisor:________________________________
Date:___________________________________

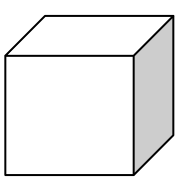
Geometry Unit Pre-Assessment

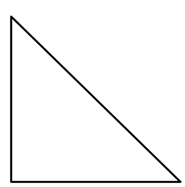
I. Definitions. Match the term by placing its letter next to its definition on the right:

Term	***Definition***
A. Radius of a Circle:	______ The sum of the lengths of the sides of a polygon.
B. Rectangle:	______ Any line segment from its center to its perimeter.
C. Perimeter:	______ The amount of space a solid body occupies.
D. Chord:	______ The size of the region enclosed by a figure.
E. Area:	______ The ratio of the circumference of a circle to its diameter.
F. Pi:	______ Line segment whose endpoints both lie on the circle.
G. Volume:	______ A quadrilateral with opposite sides of equal lengths and with four right angles.

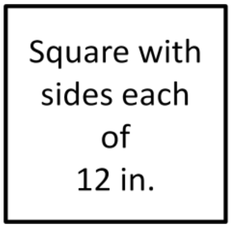
II. Identify each of the following shapes:





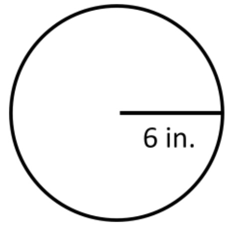


III. What is the **area** and the **perimeter** of each of the following shapes?



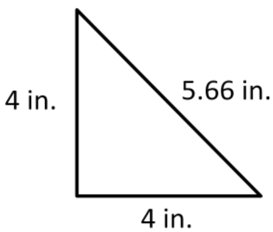
Area**:**

Perimeter**:**



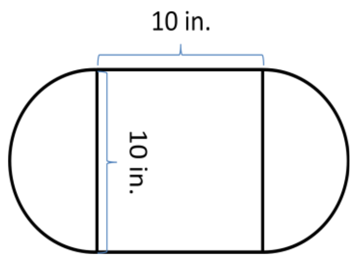
Area:

Perimeter:



Area**:**

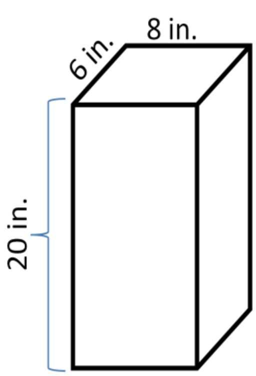
Perimeter:



Area**:**

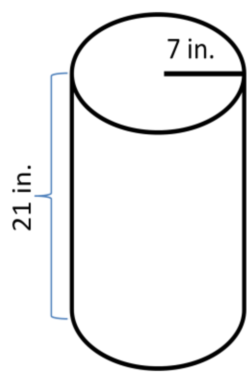
Perimeter**:**

IV. What is the **surface area** and **volume** of each of the following shapes?



Surface Area**:**

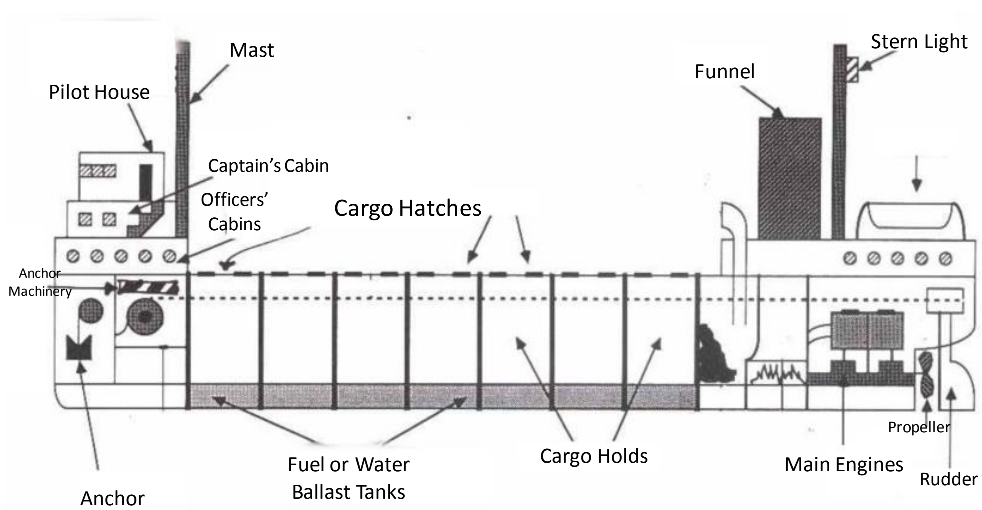
Volume**:**



Surface Area**:**

Volume**:**

Grade / Content Area	**8^{th} / 9^{th} Grade: Geometry**
Lesson Title	**Understanding the General Layout of a Ship and its Relationship to Geometry**
Guiding Question	***How do common components of a ship relate to geometric shapes?***
Content Standards	*NCTM Standards*: III. Instructional programs from prekindergarten through grade 12 should enable all students to— 1. Recognize reasoning and proof as fundamental aspects of mathematics; 2. Make and investigate mathematical conjectures; 3. Develop and evaluate mathematical arguments and proofs; **4.** Select and use various types of reasoning and methods of proof.
	Common Core Standards: High school students will: I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Student Learning Objectives	Students will be able to: I. Identify parts of a ship such as deck gear, maneuvering equipment, propulsion equipment, compartmentation. II. Relate these parts to geometric shapes.
Preparation	IV. *Classroom Organization*: Classroom Organization: Students will work in pairs. Desks will be rearranged to permit two students to work together with a common writing surface. V. *Differentiation*. A. Students will be required to identify on a simple diagram of a ship the components of that ship using proper terminology and then relate those components with geometric shapes; defining those as well. B. To accommodate learners at different levels and with different language abilities, I will: 1. Provide pictures of geometric shapes, allowing students to match those

	with the components. 2. Provide a list of definitions to selected students. VI. *Materials:* A. For each group of students: 1. “General Layout of a Ship” Diagram. 2. List of components of the ship to be labeled. 3. Definitions and pictures of geometric shapes. B. Additional materials: 1. Large classroom version of “General Layout of a Ship” diagram.
Instruction and Engagement	I. *Warm-up* (**10 minutes**). II. *Launch* (**5 minutes**). A. Introduce the idea that Geometry has practical application in the world by showing projected pictures of ships, especially dry-cargo, RO / RO and container ships. B. Ask the question, “What Geometry knowledge might be helpful to the crew of a cargo ship?” III. *Engagement* (**40 minutes**). A. Students working in pairs will label the diagram “General Layout of a Ship. When complete, it should look like this: 

B. Students will then relate these terms to geometric shapes and their definitions:

Part of Ship	Geometric Term	Definition
Captain's Cabin Windows	Square	A 4-sided regular polygon with all sides equal and all internal angles 90°.
Officers' Cabin Windows	Circle	A line forming a closed loop, every point on which is a fixed distance from a center point.
Cargo Holds	Rectangular Prism	A solid (3-dimensional) object which has six faces that are rectangles.
The intersection of two corners of a cargo hold.	Angle	A shape, formed by two lines or rays diverging from a common point (the vertex).
Funnel	Cylinder	A three-dimensional object that has two parallel ends (called bases) that are either circles or ellipses, connected by straight, parallel sides.
Rudder (the flat surface of the rudder)	Plane	Part of a flat surface that is infinitely large and with zero thickness.

C. Students will then answer the question, "How would an understanding of how to determine the area, surface area, and volume of these shapes help those who:

1. Design and build ships?
2. Move cargo in ships?"

IV. *Closing* (**10 minutes**). We will review how to determine the surface area and volume of rectangular prisms as a precursor to the following lesson. Homework will involve practice.

Assessment In-class worksheets and homework.

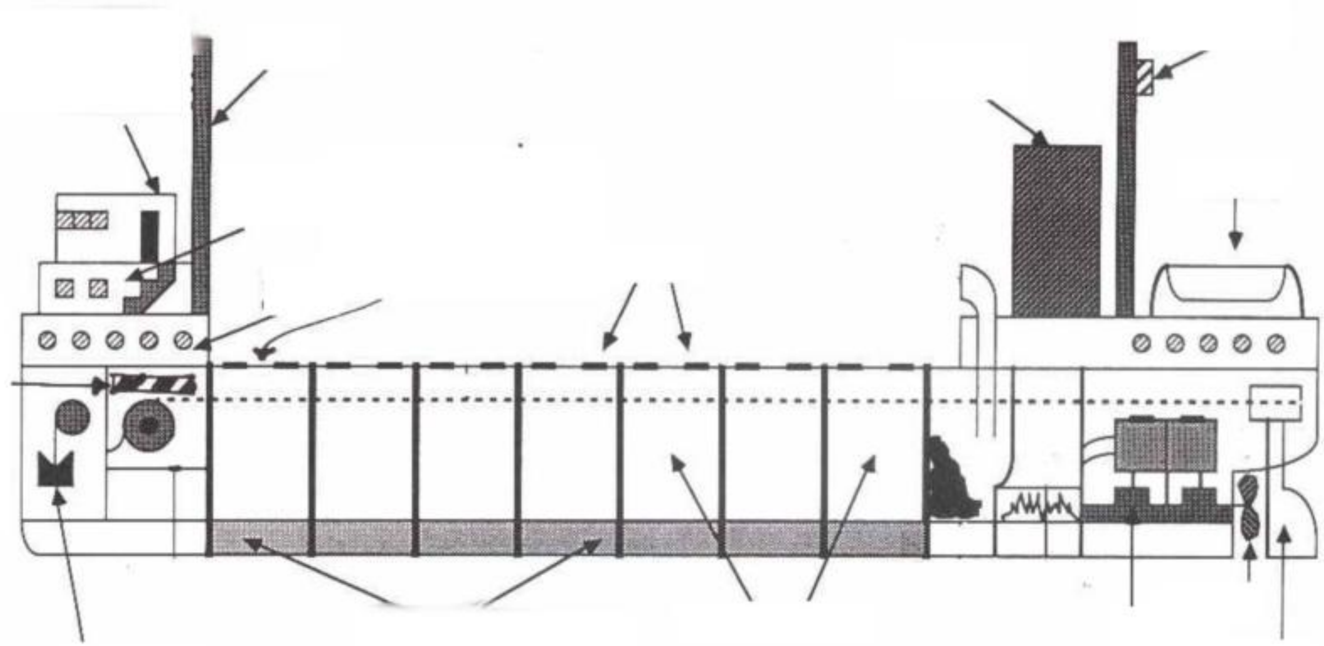
Part 1.

Directions:

I. Write the correct label next to the arrow pointing to that object on the ship diagram.

II. Underneath the label, write the geometric shape that most closely resembles the object you labeled.

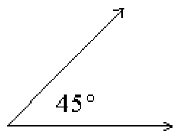
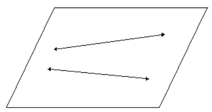
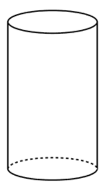
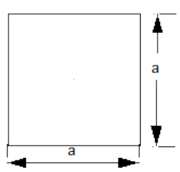
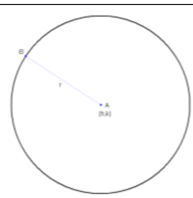
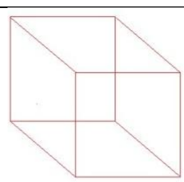
Labels
Captain's Cabin Windows
Officers' Cabin Windows
Funnel
Cargo Holds
Water Ballast Tanks
Main Engines
Anchor Machinery
Propeller
Rudder



Geometric Shapes
Angle
Rectangular Prism
Cylinder
Circle
Plane
Square

Part 2.

Directions: Draw a line to connect the term with its picture and its definition.

Picture	Geometric Term	Definition
	Square	A line forming a closed loop, every point on which is a fixed distance from a center point.
	Circle	A 4-sided regular polygon with all sides equal and all internal angles 90°.
	Rectangular Prism	A flat surface that is infinitely large and with zero thickness.
	Angle	A three-dimensional object that has two parallel ends (called bases) that are either circles or ellipses, connected by straight, parallel sides.
	Cylinder	A shape, formed by two lines or rays diverging from a common point (the vertex).
	Plane	A solid (3-dimensional) object which has six faces that are rectangles.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ______________________________

Date: _______________________________

Advisor: ____________________________

Geometry of the Ship: Practice 1: Perimeter and Area of 2 – Dimensional Shapes

Formulas: Use the following formulas to complete this practice exercise:

I. Square of side length *s*:

A. **Perimeter:** $\boldsymbol{P = s + s + s + s = 4s}$
B. **Area:** $\boldsymbol{A = s \times s = s^2}$

II. Rectangle of length *l* and width *w*:

A. **Perimeter:** $\boldsymbol{P = l + w + l + w = 2l + 2w}$
B. **Area:** $\boldsymbol{A = l \times w}$

III. Circle of radius *r*:

A. **Perimeter:** $\boldsymbol{P = 2 \times \pi \times r = 2\pi r}$
B. **Area:** $\boldsymbol{A = \pi \times r \times r = \pi r^2}$

IV. Triangle of base length *b* and height *h*

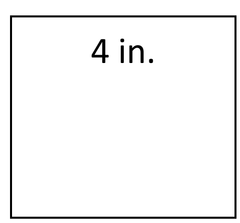
A. **Area:** $\boldsymbol{A = \left(\frac{1}{2}\right) bh}$

Solve the following problems. Show all work and answers on separate sheets of paper attached to this one.

1. Use the following example as a guide to solving the problems on the next page.

   Example: A square of side length $s = 4\ in.$

   $P = 4s$ | $A = s^2$
   $P = 4 \times 4\ in.$ | $A = (4\ in.)^2$
   $P = 16\ in.$ | $A = 16\ in.^2$



2. Find the perimeter and area of the following 2 – dimensional shapes. Draw each shape, using ruler and compass, labeling the dimensions. The drawings do not have to match the exact dimensions.

   a. A square of side length $s = 7\ cm$.

   b. A square of side length $s = 12\ mm$.

   c. A square of side length $s = 22\ ft$.

   d. A rectangle with length $l = 6\ in$. and width $w = 4\ in$.

   e. A rectangle with length $l = 13\ in$. and width $w = 6\ in$.

   f. A rectangle with length $l = 15\ m$. and width $w = 7.5\ m$.

   g. A rectangle with length $l = 22\ m$. and width $w = 110\ cm$.

   h. A rectangle with length $l = 18\ ft$. and width $w = 108\ in$.

   i. A circle of radius $r = 2\ in$.

   j. A circle of radius $r = 17\ in$.

   k. A circle of radius $r = 19\ ft$.

   l. A circle of diameter $d = 12\ in$.

   m. A circle of diameter $d = 42\ cm$.

   n. A circle of diameter $d = 1\ in$.

   o. A triangle of base length $b = 2\ cm$. and height $h = 3\ cm$. (Area only)

   p. A triangle of base length $b = 12\ m$. and height $h = 10\ m$. (Area only)

   q. A triangle of base length $b = 15\ ft$. and height $h = 144\ in$. (Area only)

Grade / Content Area	**8th / 9th Grade: Geometry**
Lesson Title	**Where Did That Product Come From? (1 day)**
Guiding Question	***What factors do merchants consider when shipping products across great distances?***
Content Standards	*NCTM Standards:* I. Instructional programs from prekindergarten through grade 12 should enable all students to— A. Recognize reasoning and proof as fundamental aspects of mathematics; B. Make and investigate mathematical conjectures; C. Develop and evaluate mathematical arguments and proofs; D. Select and use various types of reasoning and methods of proof.
	Common Core Standards: None for this lesson.
Student Learning Objectives	I. Gain insight into to the various factors involved in shipping products overseas. II. Make the connection between cargo shipment and Geometry.
Preparation	I. *Classroom Organization*: Classroom Organization: Students will work in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface. II. *Differentiation*. III. *Materials:* A. For each group of students: 1. Worksheet, camera box. B. Additional materials: None.
Instruction and Engagement	I. *Warm-up* (**10 minutes**). II. *Launch* (**5 minutes**).

A. I will start with, *"In middle school, you learned about the surface area and volume of a rectangular prism. Who can tell me the formulas for each?"*

$$V = length \times width \times height = Bh$$

$$SA = 2((l \times w) + (l \times h) + (w \times h) = 2B + Ph$$

I will do an example on the board, introducing how $B = l \times w$ and $P = 2l + 2w$.

B. *"Who can give me an example of why it is important to understand this?"* Introduce the concept of shipping in a box using the digital camera box.

C. **Group Work:** Since this will be the first time students work in groups, we will discuss what makes for good group work:

1. Work quietly – do not distract other groups.
2. Every member has a role: leader, timekeeper, recorder, moderator.
3. Every member works.
4. Conduct discussions respectfully – one member should not dominate. Listen more than talk.

III. *Engagement* (**50 minutes**).

A. **First Activity** (**10 minutes**): Students, working in groups, will begin to work on the attached worksheet, asking themselves several questions including:

1. *When you buy an electronic device, what is most important to you: what it looks like, what it can do, how much it costs, or how big it is?* Give them a couple of minutes to answer the question.

   Each group should report group consensus. Answers will vary. What do students value most?

2. *As the shipping company, if you are paid a fixed amount of money for every camera shipped, what is your goal?* Give them a couple of minutes to answer the question and come to consensus.

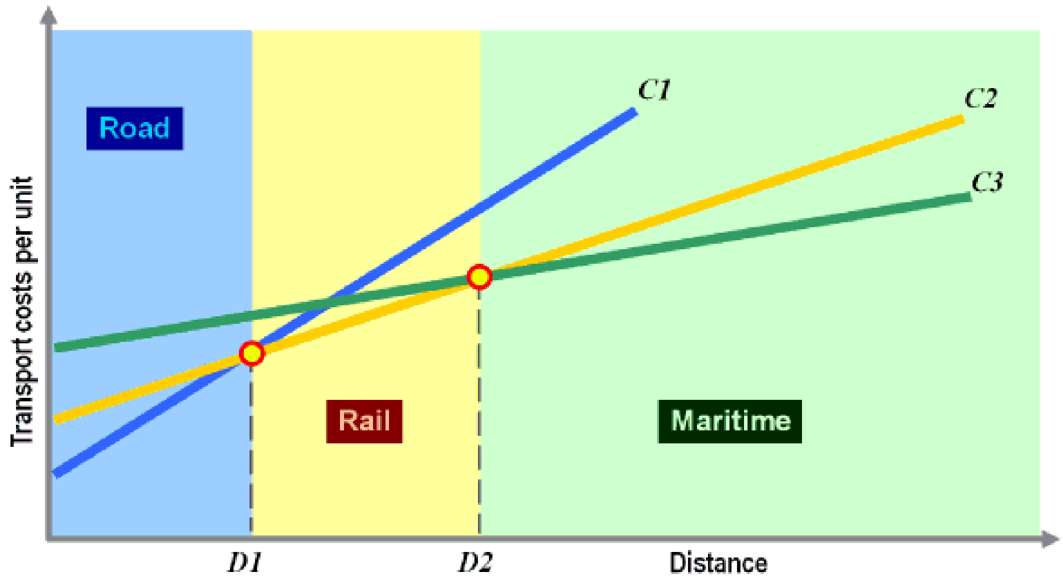
   Each group should report group consensus. Again, answers will vary. What do students consider the most important factors?

3. *How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each*

mode (plane, truck, train, ship) can carry.

Student answers most likely will be based on understanding of how much each mode can carry, how fast it is, how safe, and how expensive.

B. **Second Activity** (**10 minutes**): Students, in groups will be asked to evaluate the meaning of the following graph and information:



1. C1 is transport costs per unit for transport by road – i.e. truck.
2. C2 is transport costs per unit by rail – i.e. train.
3. C3 is transport costs per unit by maritime – i.e. ship.

4. D1 is a distance of 300 – 450 miles.
5. D2 is a distance of 1000 miles.

I may have to give some support such as indicating what the *x*-axis and *y*-axis mean and what the distances mean. The following more detailed example might be used:

Distance (100's miles)	Transportation Cost by Ship	Transportation Cost by Train	Transportation Cost by Truck
0	$500.00	$250.00	$200.00
1	$520.00	$287.50	$250.00
2	$540.00	$325.00	$300.00
3	$560.00	$362.50	$350.00
4	$580.00	$400.00	$400.00
5	$600.00	$437.50	$450.00
6	$620.00	$475.00	$500.00
7	$640.00	$512.50	$550.00
8	$660.00	$550.00	$600.00
9	$680.00	$587.50	$650.00
10	$700.00	$625.00	$700.00
11	$720.00	$662.50	$750.00
12	$740.00	$700.00	$800.00
13	$760.00	$737.50	$850.00
14	$780.00	$775.00	$900.00
15	$800.00	$812.50	$950.00
16	$820.00	$850.00	$1,000.00
17	$840.00	$887.50	$1,050.00
18	$860.00	$925.00	$1,100.00
19	$880.00	$962.50	$1,150.00
20	$900.00	$1,000.00	$1,200.00

C. **Third Activity** (**10 minutes**): Students in groups will be asked to reevaluate their answer to the question:
"How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each mode (plane, truck, train, ship) can carry," given the chart and the following

	information: 1. Factory to Ocean: 700 miles. 2. Across the Ocean: 6000 miles. 3. Ocean to Store: 3000 miles. Students will be asked to come to group consensus and report their consensus. D. **Fourth Activity** (**10 minutes**): I will state, *“Let’s keep in mind our choices and consider that, regardless of how we ship, we will need to ship in a large container. As we said before, the standard container measures 20 ft. by 8 ft. by 8 ft. Another type of container measures 40 ft. by 8 ft. by 8 ft. In your groups, consider the questions on the worksheet.”* Students will answer the questions and report their findings. I will then ask, *“What equation did you use to solve the problems concerning the amount of cameras that could be fit into the container box.?”* Answer: $V = lwh = Bh$ *“How can we prove that equation is always true?”* E. **Fifth Activity** (**10 minutes**): Finally, I will ask students to consider and reflect on the question: *“What does all this have to do with Geometry?”* ...asking them to list some specific Geometric concepts that might apply to the movement of cargo. Students will be asked to share their reflections. IV. *Closing* (**5 minutes**). I will state, *“We’ve considered the real world considerations of cargo movement, we’ve considered how they relate to Geometry – specifically how cargo is moved in geometric shapes such as rectangular prisms. Next we will consider how to prove the simple formulae we use to calculate the volume of these shapes”*
Assessment	1. Worksheets and class participation. 2. Homework Worksheet: Surface Area and Volume of Rectangular Prisms

Name: ________________________

Where Did That Product Come From?

When you buy a cell phone or other electronic device, list, in order from 1 (most important) to 4 (least important) the things you consider when making your choice:

What it can do.	1. ____
How big it is.	2. ____
How much it costs.	3. ____
What it looks like.	4. ____

You are CEO a shipping company whose job it is to transport products made in China for sale in the United States. You've just received an order to ship a large order of digital cameras that come packed in small boxes like the one I will show you.

Consider the following questions:

1. As the shipping company, if you are paid a fixed amount of money for every camera shipped, what is your goal?

2. How do you think your digital cameras should be shipped at each stage of their journey? Consider cost, speed, and how much each mode (plane, truck, train, ship) can carry. Circle your choice on the picture below and tell why you chose it in the space below:

	From factory to Ocean	Across the Ocean	From Ocean to Store	
				
Why did you choose ..... Consider: 1. Cost 2. Speed 3. How much it can carry				

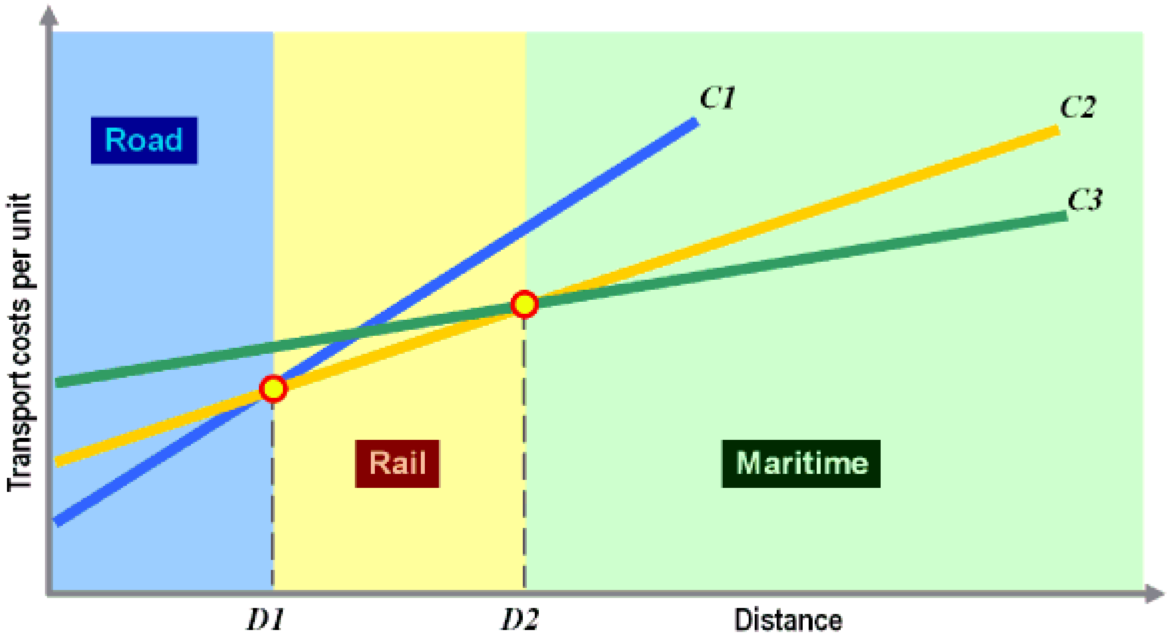
3. Explain which mode of transport at each stage is best and why it is best.

   a. From Factory to Ocean: ____

   b. Across the Ocean: ____

   c. Ocean to Store: ____

4. Now look at the following graph. Decide what this graph means and be prepared to tell the class what it means.



a. C1 is transport costs per unit for transport by road – i.e. truck.
b. C2 is transport costs per unit by rail – i.e. train.
c. C3 is transport costs per unit by maritime – i.e. ship.

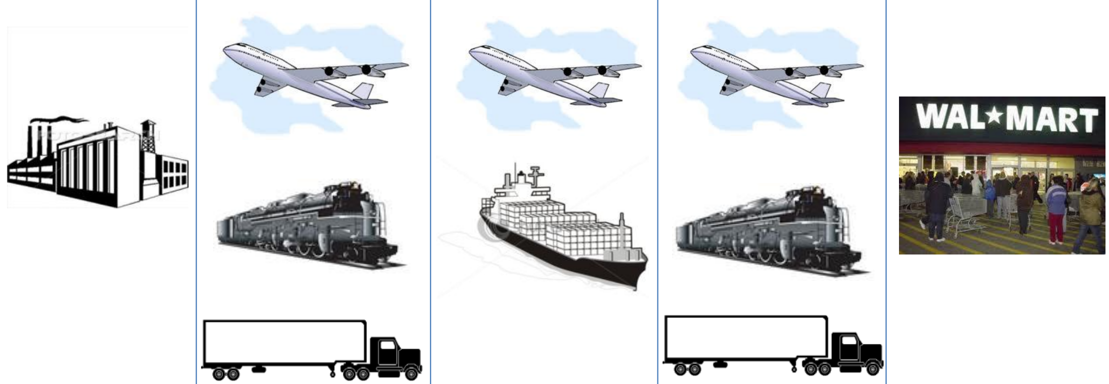
d. D1 is a distance of 300 – 450 miles.
e. D2 is a distance of 1000 miles.

5. The following are distances from factory to ocean, across the ocean, and from the ocean to the store.

   a. Factory to Ocean: 700 miles.
   b. Across the Ocean: 6000 miles.
   c. Ocean to Store: 3000 miles.

6. Now, which would you pick:

	From factory to Ocean	Across the Ocean	From Ocean to Store



Why did you choose .....

Consider:
1. Cost
2. Speed
3. How much it can carry

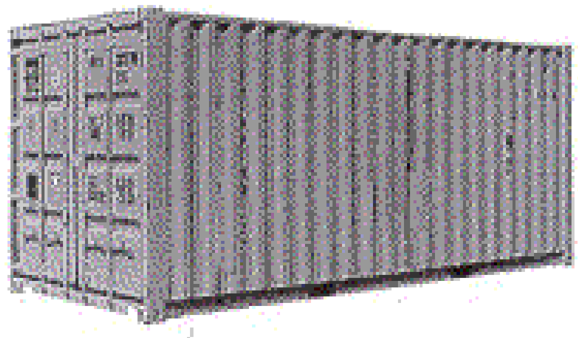
7. Decide which mode of transport at each stage is best and why it is best.

   a. From Factory to Ocean: ______________________________

   b. Across the Ocean: ______________________________

   c. Ocean to Store: ______________________________

8. Commercial cargo is normally shipped in containers like the one pictured below.



9. There are two standard sizes:

   $B = 20\ ft. \times 8\ ft.$ and $h = 8\ ft.$

   $B = 40\ ft. \times 8\ ft.$ and $h = 8\ ft.$

   a. If our digital camera box is $12\ in.\ x\ 6\ in.\ x\ 6\ in.$, how many can fit into each type of container?

      $B = 20\ ft. \times 8\ ft.$ and $h = 8\ ft.$ ____________________

      $B = 40\ ft. \times 8\ ft.$ and $h = 8\ ft.$ ____________________

   b. What if the cameras came in cylindrical containers with radius 4 in. and height __________ in.?

      Would you be able to ship more or fewer?

      How many more or fewer?

10. Reflect on what we have discussed and answer the question (in your own words) **"What does all this have to do with Geometry?"**

    List some specific Geometric concepts that might apply to the movement of cargo.

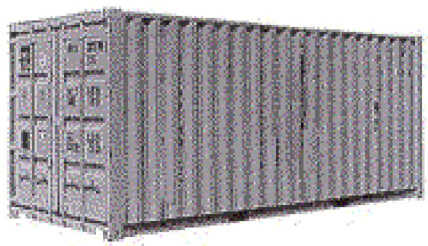
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:______________________
Advisor: __________________

Homework Worksheet: Surface Area and Volume of Rectangular Prisms

(WRITE ALL FORMULAS AND SHOW ALL WORK ON SEPARATE SHEETS OF PAPER)

1. Standard cargo shipping containers are of two types. Answer the following questions about each.



   A. The most common container is 20 ft. long, 8 ft. wide, and 8 ft. high. Find the following:

   Surface Area: **768 sq. ft.**

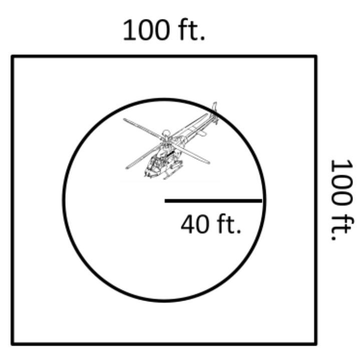
   Volume: **1280 cu. ft.**

   B. The other standard container is also 8 ft. wide and 8 ft. wide and has a total volume of 2560 $ft.^3$ Find the following:

   Length of the container: **40 ft.**

   Surface Area: **1408 sq. ft.**

2. The U. S. Navy Hospital Ship *Comfort* (which is providing relief in Haiti) has a helicopter flight deck (on top of the ship) with dimensions of 100 feet long and 100 feet wide. The helicopters land on a circular landing pad with radius of 40 feet.



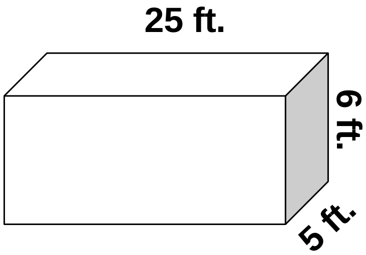
   A. What is the area of the circular landing pad?

   $\pi \times 40 \times 40 = 5027\ sq.ft.$

   B. What is the area of the ***remainder*** of the flight deck?

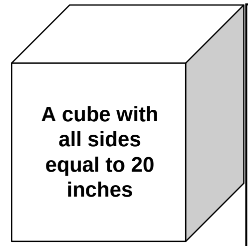
   $10000 - 5027 = 4973\ sq.ft.$

Find the surface area and volume of the following shapes:



$SA = 2B + Ph$
$SA = 2(25 \times 5) + 2(25 + 5) \times 6$
$SA\ =\ 610\ ft.^2$

$Volume\ = B \times h$
$Volume\ = 25\ ft. x\ 5\ ft. x\ 6\ ft.$
$=\ 750\ ft.^3$

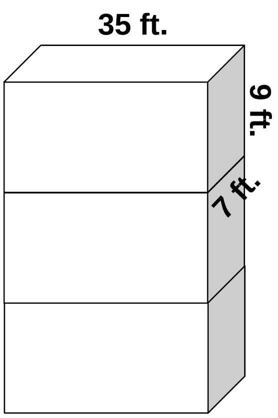


$SA = 2B + Ph$
$SA = 2(20^2) + (4 \times 20)20$
$SA = 2400\ ft.^2$

$Volume\ =\ Bh$
$Volume\ =\ 20x20x20$
$=\ 8000\ in.^3$

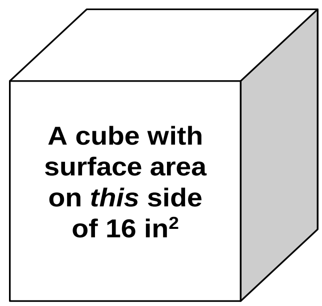
3.a Surface Area: **610 sq. ft.**
3.b. Volume: **750 cu. ft.**

4.a Surface Area: **2400 sq. ft.**
4.b Volume: **8000 cu. ft.**



$SA = 2B + Ph$
$SA = 2(35 \times 7) + 2(35 + 7) \times 9$
$SA\ =\ 610\ ft.^2$

$Volume\ = B \times h$
$Volume\ = 35\ ft. \times\ 7\ ft. \times\ 9\ ft.$
$=\ 6615\ ft.^3$



$SA = 2B + Ph$
$SA = 2(4^2) + (4 \times 4)4$
$SA = 2400\ ft.^2$

$Volume\ =\ Bh$
$Volume\ =\ 4^3\ =\ 64\ in.^3$

Each box is 35 x 7 x 9 ft.

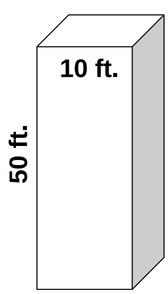
5.a Surface Area: **2506 sq. ft.**

5.b. Volume: **6615 cu. ft.**

6.a Surface Area: **96 sq. in.**

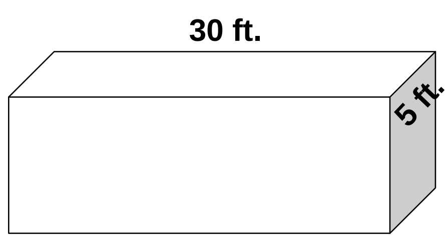
6.b Volume: **64 cu. in.**

A. Given the volume of the shape and two of the dimensions, find the third dimension.



Volume = 4000 ft³

$4000/(50x10)\ =\ 8\ ft.$



$1500/(30x5)\ =\ 10\ ft.$

Volume = 1500 ft³

6.a. **8 ft.**

6.b. **10 ft.**

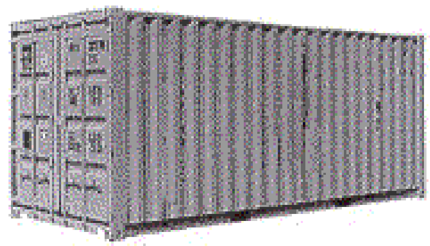
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:____________________
Advisor: ____________________

Homework Worksheet: Surface Area and Volume of Rectangular Prisms

(WRITE ALL FORMULAS AND SHOW ALL WORK ON SEPARATE SHEETS OF PAPER)

1. Standard cargo shipping containers are of two types. Answer the following questions about each.



A. The most common container is 20 ft. long, 8 ft. wide, and 8 ft. high. Find the following:

Surface Area:

Volume:

B. The other standard container is also 8 ft. wide and 8 ft. wide and has a total volume of 2560 ft.^{3} Find the following:

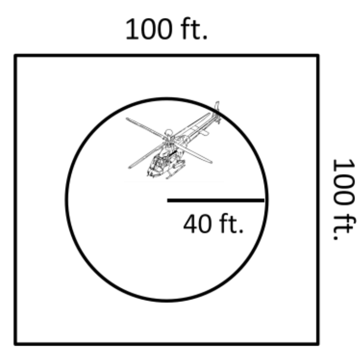
Length of the container:

Surface Area:

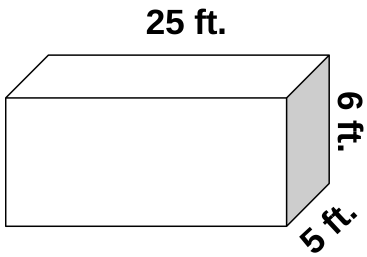
2. The U. S. Navy Hospital Ship *Comfort* (which is providing relief in Haiti) has a helicopter flight deck (on top of the ship) with dimensions of 100 feet long and 100 feet wide. The helicopters land on a circular landing pad with radius of 40 feet.

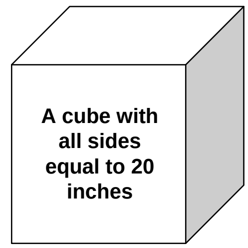
A. What is the area of the circular landing pad?

B. What is the area of the ***remainder*** of the flight deck?



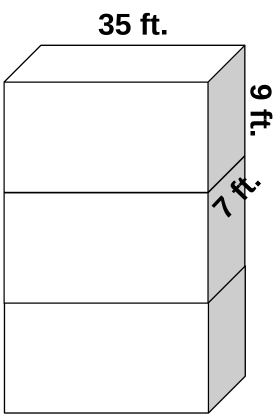
Find the surface area and volume of the following shapes:



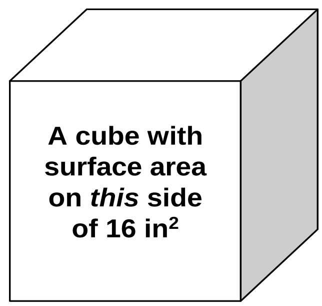


3.a Surface Area:
3.b. Volume:

4.a Surface Area:
4.b Volume:



Each box is 35 x 7 x 9 ft.



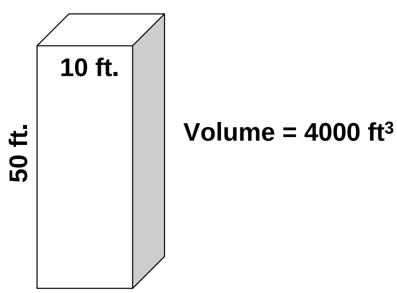
5.a Surface Area:

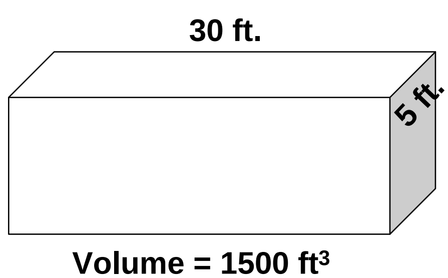
5.b. Volume:

6.a Surface Area:

6.b Volume:

B. Given the volume of the shape and two of the dimensions, find the third dimension.





6.a.

6.b.

<table>
<tr><td>Grade / Content Area</td><td>8th / 9th Grade Geometry</td></tr>
<tr><td>Lesson Title</td><td>Building and Filling a Scale Model of a Container (6 days)</td></tr>
<tr><td>Guiding Question</td><td>What is a scale model and how to I build one?
Why is it built to these dimensions?</td></tr>
<tr><td>Content Standards</td><td>State Content Standards:
I. M(G&M)–8–5: Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local)

II. M(G&M)-10-6: Solves problems involving perimeter, circumference, area, surface area, and volume.

III. M(G&M)-10-7: Uses units of measure appropriately and consistently when solving problems across content strands; makes conversions within or across systems and makes decisions concerning an appropriate degree of accuracy in problem situations involving measurement in other GSE's.</td></tr>
<tr><td></td><td>NCTM Standards: Students should:
I. Analyze characteristics: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

II. Use visualization: draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.</td></tr>
<tr><td></td><td>Common Core Standards: High school students will:
I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).</td></tr>
<tr><td>Student Learning Objectives</td><td>Goals: Students will be able to:

I. Accurately design a scale model of a standard 20 ft. CEU shipping</td></tr>
</table>

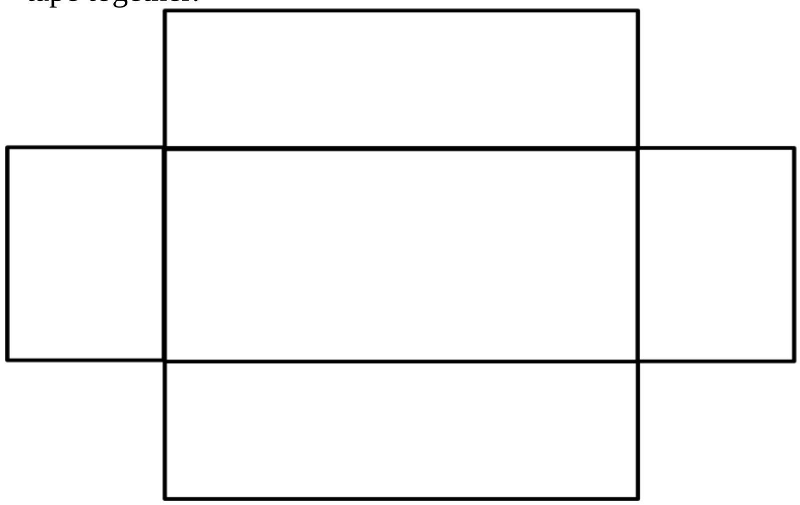
	container. II. Build a scale model to design dimensions. III. Determine the surface area and volume of the scale model and discover the relationship between the model's dimensions and the actual container's dimensions.
Preparation	I. *Classroom Organization*: Students will work in groups of three or four the first day and groups of two the second day. Desks will be rearranged to permit students to work together with a common work surface. II. *Differentiation*. There are many things in this lesson that will appeal to the multiple intelligences of the students: new terminology, worksheets, and lots of hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each group of students)*:* A. Day 1 – 5: 1. Posterboard. 2. Material to strengthen sides of box (plastic or heavy paper). 3. Glue / glue guns. 4. Tape. 5. Washable paint (many basic colors). 6. Brushes. 7. Cleaning buckets (two). 8. Cubic inch blocks to fill the box. 9. Rulers. 10. Calculators. B. Day 6 – 7: 1. NCTM *Illuminations* handout "Popcorn Prisms Anyone?" (http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AS-Prisms.pdf) 2. Two sheets of 8 ½ x 11 inch construction paper per student. 3. Tape.

	4. Lots of popcorn.
Instruction and Engagement	I. *Launch.* **Day 1** (**15 minutes**): A. We will begin with a review of concepts learned previously, specifically the following definitions (these will be pre-posted so students can copy them in their vocabulary notes. 1. *Surface area:* The sum of all the areas of all the shapes that cover the surface of the object. 2. *Volume:* The amount of "space" a three-dimensional shape occupies. “It is he room in the box” (we will generalize this definition in this lesson). 3. *Scale Model:* A representation or copy of an object that is usually smaller than the actual size of the object, which seeks to maintain the relative proportions (the scale factor) of the physical size of the original object. Very often the scale model is used as a guide to making the object in full size. II. *Engagement.* **Day 1 – 5:** A. Students working in groups will design and construct a durable scale model of a container box. 1. The dimensions of the real container are: $20' \times 8' \times 8'$ $12(20)'' \times 12(8)'' \times 12(8)''$ $240'' \times 96'' \times 96''$ 2. The model should be precisely scaled. 3. The model should have a removable top. 4. The model should have hinged doors on one side. 5. The model should be large enough to place some objects inside. 6. The model should be painted, illustrating as closely as possible an actual shipping container (show some pictures to students and suggest they look online). B. Construction involves several steps for which students may need prompting or help: 1. Choose the length of one dimension of the model in inches – for example: 20". 2. Discover the lengths of the other dimensions using the proportional

relationship:

$$\frac{20''}{240''} = \frac{x}{96''} \Rightarrow \frac{20'' \times 96''}{240''} = 8''$$

3. Draw an outline of the box on a piece of poster board. Cut out and tape together.



C. Decoration: Students can research online the appearance of container boxes. I will provide some examples:

III. **Launch (Day 6):**

A. We will begin with the question, *"Why is a container box built to the dimensions of* $20 \times 8 \times 8$*?"*

B. Answer is that the dimensions are an ideal balance between the need to fit as much into a box as possible (reducing cost – recall last lesson) and need to stack them efficiently on a ship.

IV. **Engagement (Day 6):** We will explore the relationship between dimension and volume, arriving at some rather surprising conclusions. This exercise was developed by Jamie Chaikin and can be found on the NCTM *Illuminations* website at http://illuminations.nctm.org/Lessons/Popcorn/Popcorn-AS-Prisms.pdf.

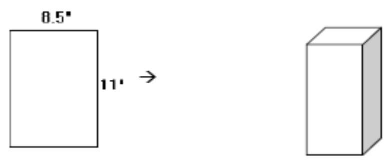
Answer Key – Popcorn Prisms Anyone?

For this activity you will be comparing the volume of 2 prisms created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

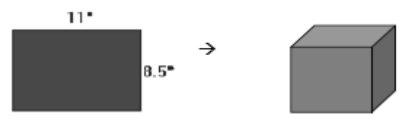
Materials:

- 8.5 inch by 11 inch white paper
- 8.5 inch by 11 inch colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and fold it in half the long way. Do this a 2nd time. You are forming a baseless rectangular prism that is tall and narrow. Do not overlap the sides. Tape along the edge. Measure the length, width, and height of each dimension with a ruler. Record your data below and on the rectangular prism. Label it Prism A.



Take the colored paper and fold it in half the wide way. Do this a 2nd time. You are forming a baseless rectangular prism that is short and stout. Do not overlap the sides. Tape along the edge. Measure the length, width, and height of each dimension with a ruler. Record your data below and on the rectangular prism. Label it Prism B.



1.

DIMENSION	PRISM A	PRISM B
LENGTH (in.)	[2.125 in]	[2.75 in]
WIDTH (in.)	[2.125 in]	[2.75 in]
HEIGHT (in.)	[11 in]	[8.5 in]

2. Do you think the two prisms will hold the same amount? Do you think one will hold more than the other? Which one? Why?

Answers will vary.

3. Place Prism B on the paper plate with Prism A inside it. Use your cup to pour popcorn into Prism A until it is full. Carefully, lift Prism A so that the popcorn falls into Prism B. Describe what happened. Is Prism B full, not full, or overflowing?

Prism B is not full. There is still room in the prism for more popcorn.

As you share your popcorn snack, answer the questions below.

4. a) Was your prediction correct? How do you know?

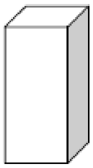
Answers will vary.

b) If your prediction was incorrect, describe what actually happened.

Prism B has a greater volume than Prism A.

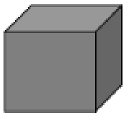
5. a) State the formula for finding the volume of a prism.

b) Calculate the volume of Prism A? Label the dimensions in the figure.



$V = lwh = (2.125)(2.125)(11) \approx 49.7 \text{ in}^3$

c) Calculate the volume of Prism B? Label the dimensions in the figure.



$V = lwh = (2.75)(2.75)(8.5) \approx 64.3 \text{ in}^3$

d) Explain why the prisms do or do not hold the same amount. Use the formula for the volume of a prism to guide your explanation.

The prisms have different dimensions, so the volumes are different.

6. a) What do you notice about the length and the width?

They are the same.

b) Rewrite the formula with only two variables to reflect this observation.

$V = l^2h$ or $V = w^2h$

7. By how much would you have to decrease the height of Prism B to make the volumes of the two prisms equal?

$$V_A \approx 49.7 \text{ in}^3$$
$$V_B \approx 49.7 \text{ in}^3 = (2.75)(2.75)(h)$$
$$h \approx 6.6 \text{ in}$$
The height would need to be decreased by about $8.5 - 6.6 \approx 1.9$ in .

ILLUMINATIONS Resources for Teaching Math

A. Together we will construct the rectangular prisms. I will then let the students begin work on the questions as I walk around the room helping as necessary.

B. After students answer Question 2, I will pass out cooked popcorn (ideally prepared in the classroom or, at least, just moments before) to be used for the rest of the exercise. There should be enough to enjoy a

snack as well as to conduct the exercise. Classroom management will be a bit of a challenge. Ideally, I should not do this on a Friday or just before a holiday.

C. Some students are likely to be surprised that the two prisms do not hold the same amount, given they were constructed from identical sheets of construction paper. As they complete the exercise and consider the volume formula as it applies to either prism:

length *l* * width *w** height *h* = Volume *V*

So, the taller prism is:

$$\left(\frac{8.5\ in.}{4}\right)^2 * 11\ in. = 49.6719\ cu.in.$$

The, shorter, wider prism is:

$$\left(\frac{11\ in.}{4}\right)^2 * 8.5\ in. = 64.28143\ cu.in.$$

WHY DOES THIS HAPPEN?

D. Algebraically, it can be seen that the value $\frac{11}{4}$ is greater than $\frac{8.5}{4}$. When they are both squared, the $\left(\frac{11}{4}\right)^2$ is 1.67 times greater than $\left(\frac{8.54}{4}\right)^2$ whereas 11 is only 1.29 times greater than 8.5. Therefore, the magnitude of the length and width together has greater impact than the magnitude of the height.

V. *Closing.* **Day 6:**

A. Upon completion of the exercise and in closing, I will ask the question, *"So what kind of box would you want your furniture to go into?"* Probably one more like the shorter, broader prism because it can carry more and probably fit bigger things.

B. I will then ask them to think about our shipping container because it is into that which we will need to fit our shipping crates. I will leave them with the following question, *"Remember, you want to get as much into that container as possible. What are our limitations on the dimensions of our shipping crates?"*

Assessment	I. The container building project will be graded using the following rubric:

	4	**3**	**2**	**1**
Accuracy of Scale		Model is scaled such that dimensions are proportional to those of a $20' \times 8' \times 8'$ container box.		Model is not accurately scaled.
Working Parts		Model has durable hinged doors and removable top.	Model has durable hinged doors or removable top.	Model has no working parts.
Decoration	Model is decorated in great detail – realistically and thoroughly depicting an actual container box.	Model is decorated in sufficient detail to make it an acceptable depiction of an accurate container box but is lacking in some details.	Model is decorated neatly but not in sufficient detail to visually represent an accurate container box.	Model is not decorated.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:________________________________
Date:________________________________
Advisor: ______________________________

Container Scale Model Project

Project:

You will construct a model of a shipping container. These are pictures of standard shipping containers.


4 Locking Bars
Waterproof Gasket Seals Around Doors
5 Vents Per Side
Double Doors For Easy Storage Access
CBCU 200061 5
22G1
M.G.W.
TARE
NET
Lockbox For Lock Protection
Corten Steel Panels
CU.CAP.
Waist High Locking Handles
Fork Lift Pockets

Directions:

1. The dimensions of the real container are:

$$20' \times 8' \times 8'$$

2. The model should be precisely scaled.
3. The model should have a removable top.
4. The model should have hinged doors on one of the smaller sides.
5. The model should be large enough to place some objects inside.
6. The model should be painted, illustrating as closely as possible an actual shipping container (you can find pictures online).

Grading: Your model will be graded using the following rubric:

	4	**3**	**2**	**1**
Accuracy of Scale		Model is scaled such that dimensions are proportional to those of a $20' \times 8' \times 8'$ container box.		Model is not accurately scaled.
Working Parts		Model has durable hinged doors and removable top.	Model has durable hinged doors or removable top.	Model has no working parts.
Decoration	Model is decorated in great detail – realistically and thoroughly depicting an actual container box.	Model is decorated in sufficient detail to make it an acceptable depiction of an accurate container box but is lacking in some details.	Model is decorated neatly but not in sufficient detail to visually represent an accurate container box.	Model is not decorated.

Project Proposal

1. My container model will have the following dimensions (show your work below the table):

	Actual Container Dimensions in Feet	Actual Container Dimensions in Inches	Scale Factor	Model Container Dimensions in Inches
Length	20 feet			
Width	8 feet			
Height	8 feet			

2. Attached are pictures of my model on separate sheets of paper. The lengths, widths, and heights of the drawings should match the dimensions I have selected for my model. These pictures include

   a. A picture of the top of the container.
   b. A picture of one of the longer sides of the container.
   c. A picture of one of the shorter sides of the container.
   d. Each drawing is illustrated as I will decorate my model.

Submitted:

____

Student Signature

Project Proposal Approved:

____

Teacher Signature

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:______________________________

Date:______________________________

Advisor: ______________________________

Directions:

1. Find the dimensions and volume of each 3-D shape:
2. Using a **drawing** of your container box, determine how many of each shape you will place in the container box to **fill it completely at maximum profit to you**.
3. Fill your container box with the 3-D shapes to determine if your fill plan is correct.

Green Cube: Length: _____ Width: _____ Height: _____

Volume: __________________

Price: $0.25

Yellow Rectangle: Length: _____ Width: _____ Height: _____

Volume: __________________

Price: $0.80

Yellow Square: Length: _____ Width: _____ Height: _____

Volume: __________________

Price: $3.25

Orange Rectangle: Length: _____ Width: _____ Height: _____

Volume: __________________

Price: $0.85

Orange Square: Length: _____ Width: _____ Height: _____

Volume: __________________

Price: $3.40

Light Orange Rectangle: Length: _____ Width: _____ Height: _____

Volume: ___________________

Price: $3.50

Yellow Prism: Length: _____ Width: _____ Height: _____

Volume: ___________________

Price: $13.50

Orange Prism: Length: _____ Width: _____ Height: _____

Volume: ___________________

Price: $14.00

Light Orange Prism Length: _____ Width: _____ Height: _____

Volume: ___________________

Price: $14.50

Large Wood Prism Length: _____ Width: _____ Height: _____

Volume: ___________________

Price: $16.00

Shape	**Total Number**	**Total Volume**	**Total Price**
Your Container Box			
Green Cube			
Yellow Rectangle			
Yellow Square			
Orange Rectangle			
Orange Square			
Light Orange Rectangle			
Yellow Prism			
Orange Prism			
Light Orange Prism			
Large Wood Prism			
Total			

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

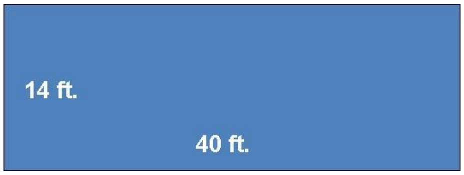
Name:________________________________
Date:________________________________
Advisor: ______________________________

Problem Solving Strategy: *Draw a Picture*[9]

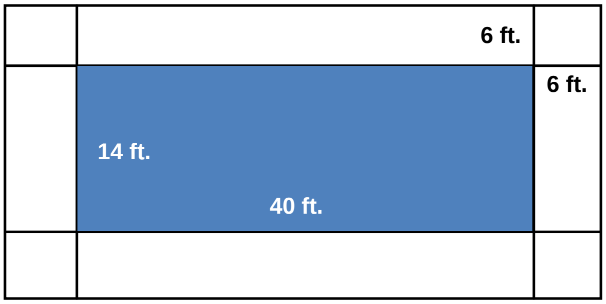
Curly dug his own swimming pool by hand with a shovel. He figured he needed a pool because digging it was hard work, and he could use it to cool off after working on it all day. He also planned a rectangular concrete deck around the pool that would be six feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

1. Draw a picture. What do we know?

   a. The pool is 14 feet by 40 feet:



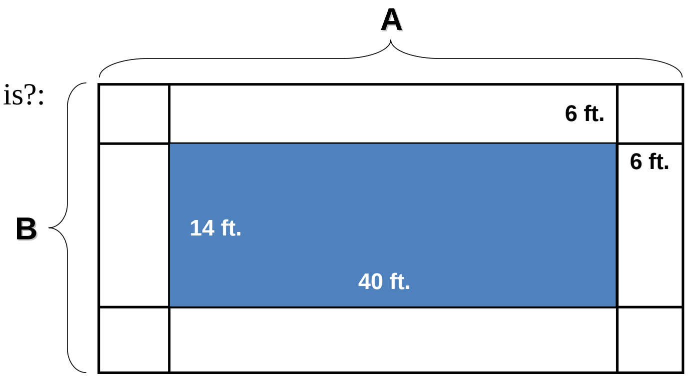
   b. The deck is 6 feet wide around the edges:



2. So what do you think the length of *A* is? *B* is?:

   A = 40 + 6+ 6 = 52 feet

   B = 14 + 6 + 6 = 26 feet



3. What is the area of the pool? *Area of pool = 40 * 14 = 560 sq. ft.*

4. What is the area of the pool and the deck? *Area = 52 * 26 = 1352 sq. ft.*

5. What is the area of the deck? *Area of deck = Area of pool & deck – Area of pool.*
1352 – 560 = 792 sq. ft.

[9] From Herr and Johnson, *Problem Solving Strategies: Crossing the River With Dogs*, pp. 16 – 18.

1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________________________
Date:_____________________________________
Advisor: ____________________________________

Problem Solving Strategy: *Draw a Picture* [10]

Curly dug his own swimming pool by hand with a shovel. He figured he needed a pool because digging it was hard work, and he could use it to cool off after working on it all day. He also planned a rectangular concrete deck around the pool that would be six feet wide at all points. The pool is rectangular and measures 14 feet by 40 feet. What is the area of the deck?

1. Draw a picture. What do we know?

   a. The pool is 14 feet by 40 feet:

   b. The deck is 6 feet wide around the edges:

2. So what do you think the length of *A* is? *B* is?:

3. What is the area of the pool?

4. What is the area of the pool and the deck?

5. What is the area of the deck?

[10] From Herr and Johnson, *Problem Solving Strategies: Crossing the River With Dogs*, pp. 16 – 18.

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

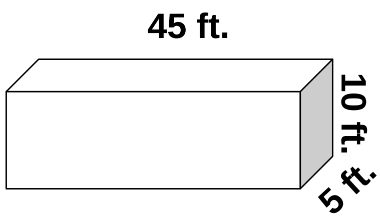
Name:____________________

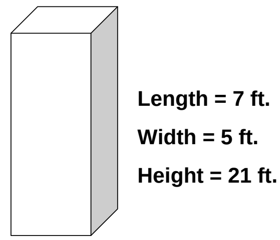
Date:____________________

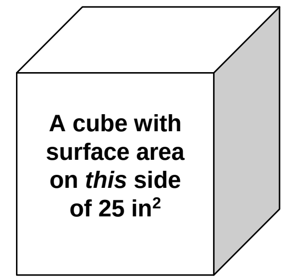
Advisor: ____________________

Assessment Review: Surface Area and Volume of Rectangular Prisms

1. Find the surface area and volume (if asked for) of the following shapes:

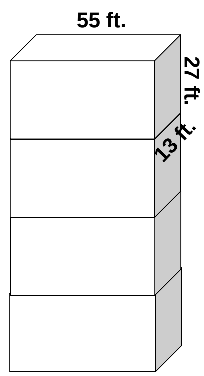


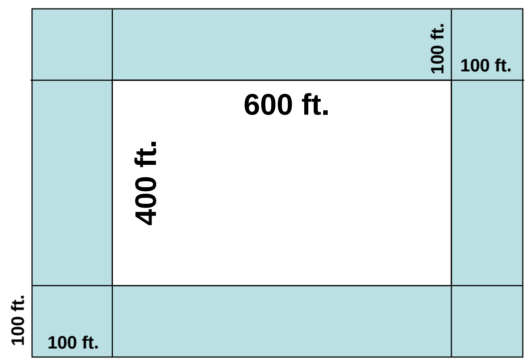


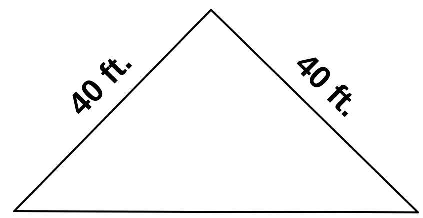


1.a Surface Area: ________ 1.c Surface Area: ________ 1.e Surface Area: ________

1.b Volume: ________ 1.d Volume: ________ 1.f Volume: ________







1.g Surface Area: ________ 1.i Entire Surface Area ________ 1.k Surface Area: ________

1.h Volume: ________ 1.j Surface Area of Shaded Area ________

2. John wants to frame a photograph of his shipmates on the cargo ship SS *Cape Mohican*. The photograph measures 10 inches long by 8 inches wide. He wants to put a frame around the picture that extends 2 inches from each side of the photograph.

   a. Draw a picture of the photograph and the frame showing all the measurements you have been given.

   b. How much wall space will the picture and the frame take up (what is the total surface area of the picture and frame)?

   ____

   c. What is the surface area of the frame alone? ____________________________

3. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Perimeter: ________ The amount of space, inside and out, a solid body occupies.

b. Surface Area: ________ A quadrilateral with opposite sides of equal lengths and with four right angles.

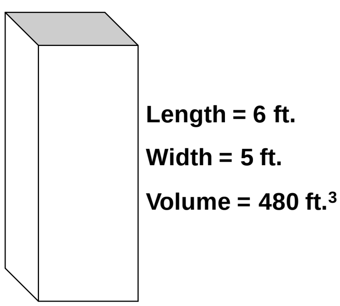
c. Rectangle: ________ A polygon with four sides and four corners.

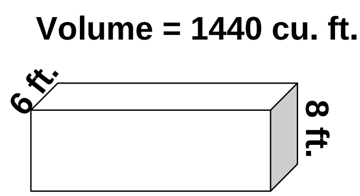
d. Volume: ________ The sum of the areas of the surfaces of a three-dimensional object.

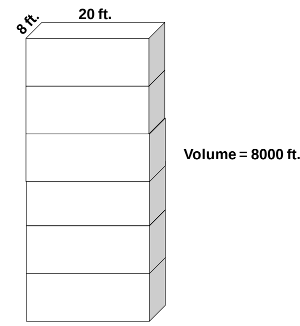
e. Square: ________ The sum of the lengths of the sides of a polygon.

f. Quadrilateral: ________ A quadrilateral with four equal sides and four right

4. Given two dimensions and the volume of the figure, find the third dimension.







4.a Height: ________ 4.b Length: ________ 4.c Height: ________

5. The cargo bay of the Space Shuttle *Discovery* is a rectangular prism that measures 60 ft. x 15 ft. x 30 ft. *Discovery's* mission is to carry as many cargo containers for the International Space Station in its cargo bay as possible on each flight. Each cargo container measures 30 ft. x 20 ft. x 10 ft.

a. What is the surface area of *Discovery's* cargo bay? ____________________

b. What is the volume of *Discovery's* cargo bay? ____________________

c. What is the surface area of one cargo container? ____________________

d. What is the volume of one cargo container? ____________________

e. ***EXTRA CREDIT*** How many cargo containers can *Discovery* carry? ____________________

Review: Answer Key

1.a. 1450 ft.^{2}

1.b. 2250 ft.^{3}

1.c. 574 ft.^{2}

1.d. 735 ft.^{3}

1.e. 150 ft.^{2}

1.f. 125 ft.^{2}

1.g. 16,118 ft.^{2}

1.h. 77,220 ft.^{3}

1.i. 480,000 ft.^{2}

1.j. 240,000 ft.^{2}

1.k. 800 ft.^{2}

2.a.

2.b. 168 in^2

2.c. 88 in^2

3.a. The sum of the lengths of the sides of a polygon.

3.b. The sum of the areas of the surfaces of a three-dimensional object.

3.c. A quadrilateral with opposite sides of equal lengths and four right angles.

3.d. The amount of space, inside and out, a solid body occupies.

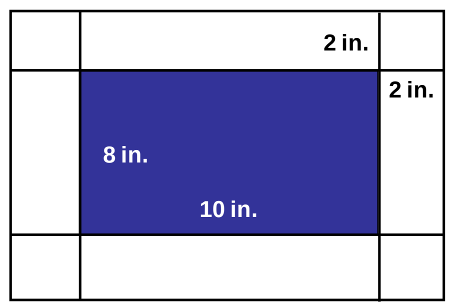
3.e. A quadrilateral with four equal sides and four right angles.

3.f. A polygon with four sides and four corners.

4.a. 16 ft.

4.b. 30 ft.

4.c. 50 ft.



5.a. 6300 ft.^{2}

5.b. 27,000 ft.^{3}

5.c. 2200 ft.^{2}

5.d. 6000 ft.^{3}

5.e. 3

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

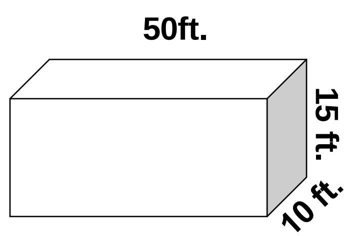
Name:____________________

Date:_____________________

Advisor: __________________

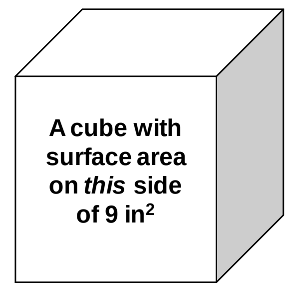
Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes:



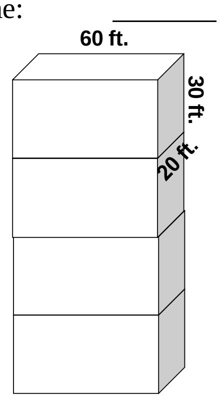
1.a Surface Area: ________

1.b Volume: ________



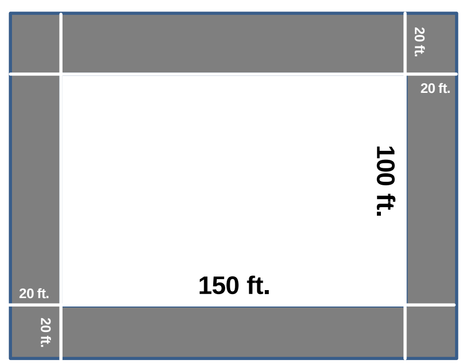
1.c Surface Area: ________

1.d Volume: ________



1.e Surface Area: ________

1.f Volume: ________



1.g Entire Surface Area ________

1.h Surface Area of Shaded Area ________

2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral: _________ A quadrilateral with four equal sides and four right angles.

b. Rectangle: _________ A polygon with four sides and four corners.

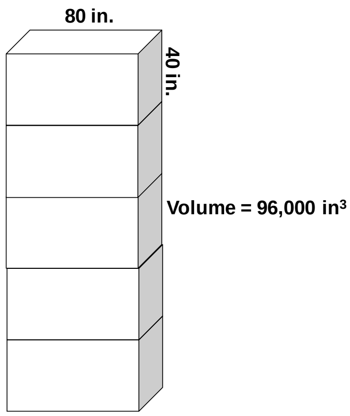
c. Square: _________ The sum of the lengths of the sides of a polygon.

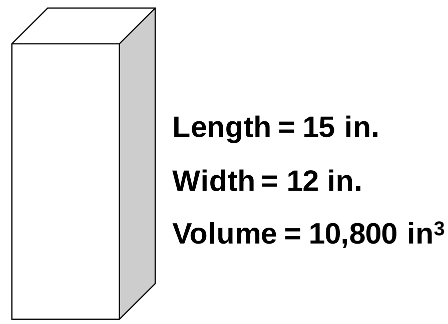
d. Perimeter: _________ The amount of space, inside and out, a solid body occupies.

e. Surface Area: ________ A quadrilateral with opposite sides of equal lengths and with four right angles.

f. Volume: ________ The sum of the areas of the surfaces of a three-dimensional object.

3. Given two dimensions and the volume of the figure, find the third dimension.





3.a Width: _______ 3.b Height: _______

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of C-5 Galaxy's cargo bay? ____________________

b. What is the volume of C-5's Galaxy's cargo bay? ____________________

c. What is the surface area of one cargo container? ____________________

d. What is the volume of one cargo container? ____________________

How many cargo containers can the C-5 Galaxy carry? ____________________

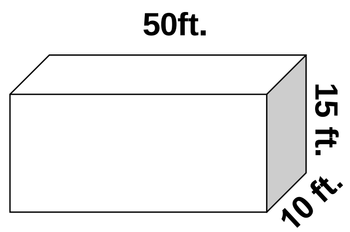
1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Differentiated Exam

Name:____________________
Date:_____________________
Advisor: __________________

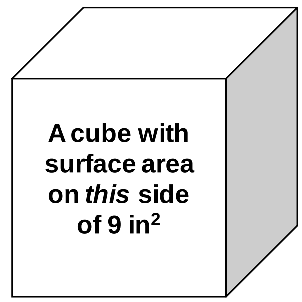
Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes **(5 points apiece)**:



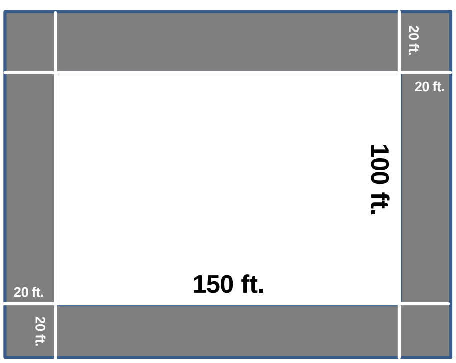
1.a Surface Area: _______

1.b Volume: _______



1.c Surface Area: _______

1.d Volume: _______



1.e Entire Surface Area _______

1.f Surface Area of Shaded Area _______

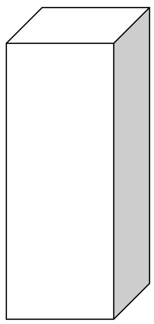
2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral:	________	A quadrilateral with four equal sides and four right angles.
b. Rectangle:	________	A polygon with four sides and four corners.
c. Square:	________	The sum of the lengths of the sides of a polygon.
d. Perimeter:	________	The amount of space, inside and out, a solid body occupies.

e. Surface Area: ________ A quadrilateral with opposite sides of equal lengths and with four right angles.

f. Volume: ________ The sum of the areas of the surfaces of a three-dimensional object.

3. Given two dimensions and the volume of the figure, find the third dimension.



Length = 15 in.

Width = 12 in.

Volume = 10,800 in^3

3.a Height: _______

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of C-5 Galaxy's cargo bay? ____________________

b. What is the volume of C-5's Galaxy's cargo bay? ____________________

c. What is the surface area of one cargo container? ____________________

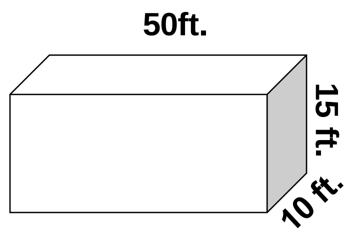
d. What is the volume of one cargo container? ____________________

e. ***EXTRA CREDIT*** How many cargo containers can The C-5 Galaxy carry? ____________________

Answer Key

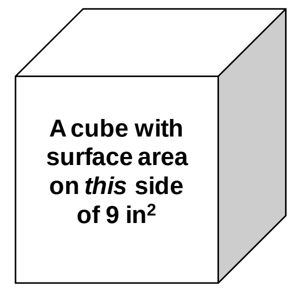
Assessment: Surface Area and Volume of Rectangular Prisms
SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Find the surface area and volume (if asked for) of the following shapes:



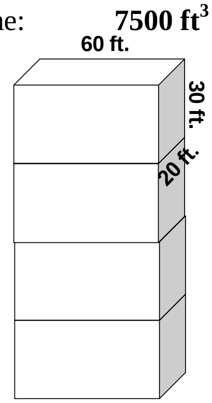
1.a Surface Area: **2800 ft^2**

1.b Volume: **7500 ft^3**



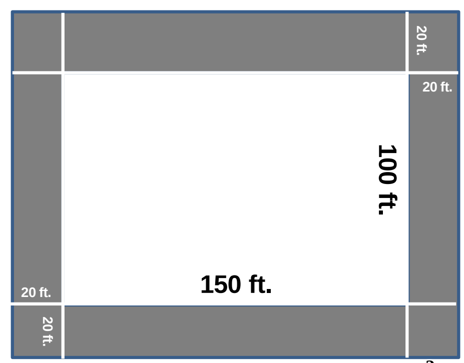
1.c Surface Area: **54 in^2**

1.d Volume: **27 in^3**



1.e Surface Area: **21,600 ft^2**

1.f Volume: **144,000 ft^3**



1.g Entire Surface Area **26,600 ft^2**

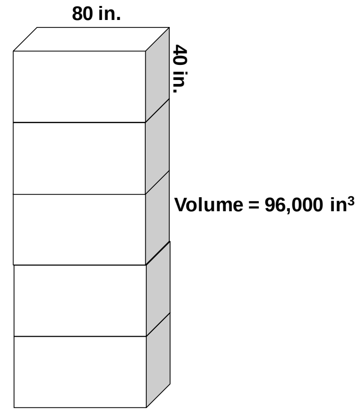
1.h Surface Area of Shaded Area **11,600 ft^2**

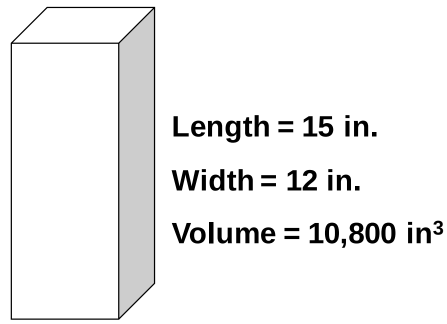
2. **Definitions**. Match the term by placing its letter next to its definition on the right:

a. Quadrilateral:	**c**	A quadrilateral with four equal sides and four right angles.
b. Rectangle:	**a**	A polygon with four sides and four corners.
c. Square:	**d**	The sum of the lengths of the sides of a polygon.
d. Perimeter:	**f**	The amount of space, inside and out, a solid body occupies.
e. Surface Area:	**b**	A quadrilateral with opposite sides of equal lengths and with four right angles.
f. Volume:	**e**	The sum of the areas of the surfaces of a three-dimensional object.

Answer Key

3. Given two dimensions and the volume of the figure, find the third dimension.





3.a Width: **6 in.**

3.b Height: **60 in.**

4. One of the world's largest cargo aircraft is the U. S. Air Force C-5 Galaxy. Its cargo bay is a rectangular prism that measures 120 ft. x 19 ft. x 13 ft. The C-5's mission is to lift people and cargo by air into war zones and into areas that need disaster relief. Among other things, the C-5 can carry cargo containers measuring 20 ft. x 8 ft. x 8 ft.

   a. What is the surface area of C-5 Galaxy's cargo bay? **8174 ft^2**

   b. What is the volume of C-5's Galaxy's cargo bay? **29,640 ft^3**

   c. What is the surface area of one cargo container? **768 ft^2**

   d. What is the volume of one cargo container? **1280 ft^3**

   e. ***EXTRA CREDIT*** How many cargo containers can The C-5 Galaxy carry? **12**

Grade / Content Area	8th / 9th Grade Geometry
Lesson Title	**Scaling (2 days)**
Guiding Question	*If we change one, two, or all three dimensions of an object, how does its surface area and volume change?*
Content Standards	*State Content Standards*: I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: Middle and high school students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: I. Verify experimentally the properties of dilations given by a center and a scale factor:

	A. Dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. B. The dilation of a line segment is longer or shorter in the ratio given by the scale factor."
Student Learning Objectives	I. Students will identify both empirically and algebraically the impact of changing the dimensions of rectangular prisms on surface area and volume
Preparation	I. *Classroom Organization.* Students will work together in groups of two or four. Desks will be rearranged to permit two or four students to work together with a common writing surface. II. *Differentiation.* Again, this lesson has much to appeal to multiple intelligences including the use of manipulatives as an aid to deriving and understanding mathematical results. As always, I will be mindful of students having difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each group of students): A. **Day 1:** 1. Container models built in class or 16 – 20 "Kleenex" boxes. 2. Worksheet for warm-up and at home exercise. 3. Ruler, pencil, paper. B. **Day 2:** 1. Worksheet for warm-up exercise.
Instruction	I. *Warm-up* (**10 minutes**): II. *Launch* **Day 1:** (**10 minutes**). I will open with a warm-up exercise which will ask students to determine the surface area and volume of a rectangular prism and then ask them to determine the dimensions of the shape if we double its volume. I expect that some of the students will speculate that all three dimensions are doubled if the volume is doubled. Others will determine that only one dimension is doubled. This will set the stage for the lesson.

III. *Engagement* **Day 2:**

A. (10 minutes) Students will be divided into groups of two to four and asked to compare their answers in the warm-up and make a group prediction or hypothesis – writing their findings and sharing with the class. I will compile these hypotheses for the entire class.

B. (10 minutes) Students will then be given two "Kleenex" boxes and then asked to prove their prediction or explain why they now believe their prediction is wrong.

C. (10 minutes) If / when a group demonstrates that doubling only one dimension is necessary, they will be asked to demonstrate the impact of doubling two and then three dimensions. Groups will be given additional "Kleenex" boxes to experiment and evaluate.

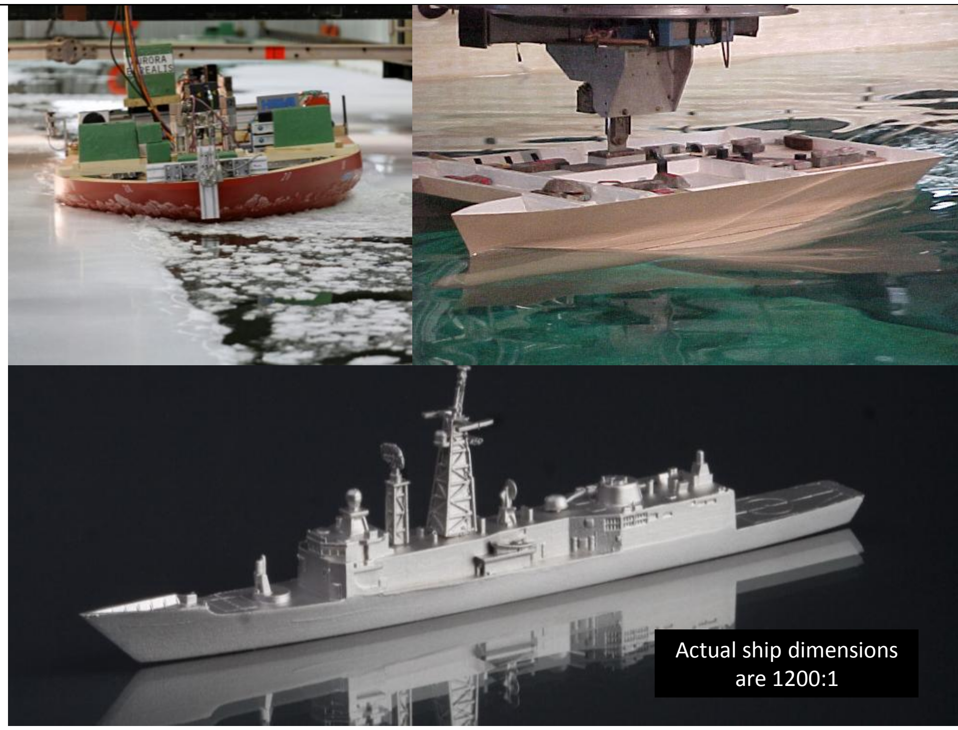
IV. *Closing* **Day 1** (**10 minutes**)**:** Now I will ask the question, *"OK, we have determined that for a rectangular prism, we need to double one dimension to double the volume, double, double two dimensions to increase the volume by* 2^2*, and double all three to increase the volume by* 2^3*.*

Double:	Volume increases by:	Or by:
One dimension	Factor of two	2^1
Two dimensions	Factor of four	2^2
Three dimensions	Factor of eight	2^3

Note how the exponent equals the number of dimensions we have doubled. Now what happens if we triple the dimensions.

V. *Warm-up* **Day 2** (**10 minutes**):

VI. *Launch* **Day 2** (**10 minutes**)**:** I will start by asking students to consider the ships we discussed earlier. "*How do you think they are tested to ensure they can operate safely at sea – in storms or icy waters, etc.?"* I will then show some pictures of scale models of ships:



"To build the actual ship from the model, one multiplies all the dimensions by the scale factor. For my ship, U.S.S. Rentz, *the builders would take this model and multiply all of the dimensions by 1200.*

VII. *Engagement* **Day 2** (**40 minutes**): We will now consider scaling and particularly its relationship to the volume and surface area of scaled objects. We will use a worksheet from the NCTM Illuminations lesson "Scaling Away" by Rhonda Naylor. I will ask the following questions:

A. Question: *What happens to the volume when an object is enlarged by a given scale factor? What happens to the surface area?*

Answer: *Both the surface area and volume increase when an object is enlarged.*

B. Question: *What is the ratio of the surface area of the original object to the surface area of the model? What is the ratio of the volumes? How does this compare to the ratio of the side lengths?*

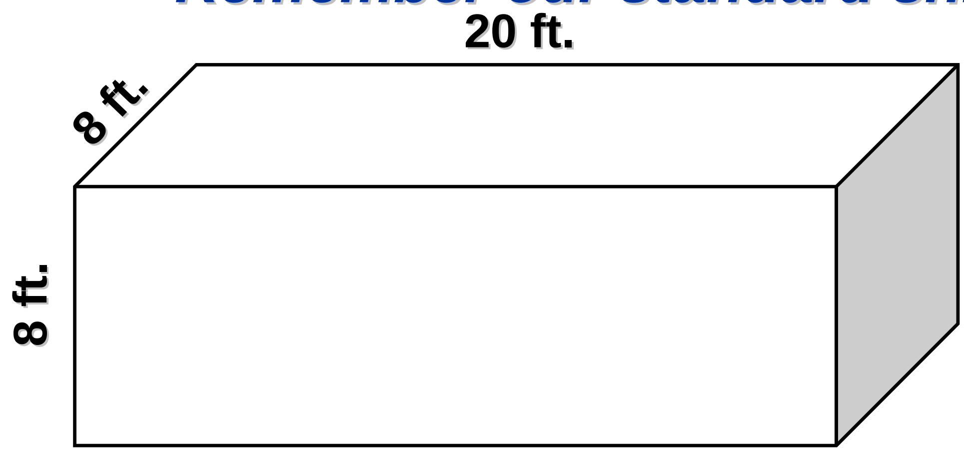
Answer: *When an object is enlarged (or, for that matter, shrunk) by a scale factor n, the resulting surface area is n2 times the original surface area, and the resulting volume is n3 times the original volume. Consequently, the ratio of side lengths is 1:n, the ratio of surface areas*

	is 1:n^2, and the ratio of volume is 1:n^3. VIII. *Closing* **Day 2** (**10 minutes**): Algebraically: If the original object was a prism, it had dimensions *l*, *w*, and *h*. The dimensions of the model are equal to the dimensions of the object multiplied by the scale factor *n*, which are *nl*, *nw*, and *nh*. **<u>Original surface area:</u>** $SA = 2lw + 2lh + 2wh = 2B + Ph$ **<u>New surface area:</u>** $2(nl)(nw) + 2(nl)(nh) + 2(nw)(nh) = 2n^2lw + 2n^2lh + 2n^2wh$ $= \boldsymbol{n^2}\,(2lw + 2lh + 2wh)$ **<u>Original volume:</u>** *lwh* **<u>New volume:</u>** $(nl)(nw)(nh) = \boldsymbol{n^3}(lwh)$ Try it.
Assessment	I. In this lesson, I will assess: A. Readiness to learn material covered in each day's lesson through the warm-up exercise. B. Understanding of the concepts of this lesson through observation of group work, report out, and completion of the worksheets. C. Final comprehension through a graded assessment.

Warm-up

March 2nd, 2010

Remember our standard shipping container…



Volume: ______________

What are the new dimensions when you double the volume?

Doubled Volume: ____________

Length: ____________

Width: ____________

Height: ____________

Hint: There is more than one correct answer!

Prove your work using formulas here and a calculator.

Why do you think these are the *new* dimensions?

1. List all formulas.
2. Show all work on separate sheets of paper.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:_____________________
Date:______________________
Advisor: ________________ __

Scaling of Rectangular Prisms I

1. Refer to your proposal for a model of a container box. Record the dimensions of your container box model.

Length	Width	Height	Surface Area	Volume

2. Now, consider two identical container boxes with the dimensions of your model. Draw a picture of the two boxes with one placed atop the other. If one were placed atop the other, determine the dimensions, surface area and volume.

Length	Width	Height	Surface Area	Volume

a. Does the volume double when you put the two boxes together? **YES** **NO**

b. How does each of the dimensions change when you put the two boxes together?

	Length	Width	Height	Volume
One container box				
Two container boxes				

c. So to double the volume, how many dimensions change and by what percent? Explain.

3. What do you think will happen to the volume when all three dimensions are doubled? Explain your answer.

Warm-up

March 3rd, 2010

"overstock.com" sells 36-box cartons of "Kleenex" tissue. If each "Kleenex" box measures

4.75 in. x 4 in. x 9 in.

1. **What is the volume of each "Kleenex" box?**

   ____

2. **What is the total volume of the carton?**

   ____

3. **What might be the dimensions of the carton?**

   ____

4. **If "overstock.com" wants to sell 72-box cartons, what might be the dimensions of the new carton.**

   ____

Scaling Away

NAME ______________________

Select an object that is a rectangular prism or a cylinder. Record all of your work below.

Object: ______________________________

Shape: ______________________________

1. What do you think will happen to the volume when you enlarge a common object by your given scale factor? What do you think will happen to the surface area? Write your **hypothesis**.

   If I multiply each dimension by the scale factor, then I think...

   the surface area will ______________________________

   and the volume will ______________________________

   because ______________________________.

2. Measure and Record the dimensions of your object. Include the correct units.

   Length ____________ Width ____________ Height ____________

   – or –

   Diameter ____________ Height ____________

3. Compute the surface area of your object. Include the correct units.

4. Compute the volume of your object. Include the correct units.

Imagine you are going to enlarge your object by building a scale model. To do this, you will multiply each dimension by a number known as the **scale factor**.

Choose a scale factor (from 3 to 8): ____________

5. Multiply each dimension by the scale factor, and record the new dimensions below.

   Length _______________ Width _________________ Height _______________

   – or –

   Diameter _____________ Height ________________

6. Compute the surface area of the model. Include the correct units.

7. Compute the volume of model. Include the correct units.

8. Determine the ratio of the surface area of the original object to the surface area of the model.

9. Determine the ratio of the volume of the original object to the volume of the model.

10. Was your hypothesis correct? Why or why not? Explain what you have discovered about multiplying a side length by a scale factor. What happens to the surface area? What happens to the volume?

11. If you had used a scale factor of 8, by what factor would the surface area have increased? By what factor would the volume have increased?

Warm-up

March 4th, 2010

	Dimensions (inches)	Volume (in^3)	Number of doubled from dimensions of Box 1	Number of times volume increased over Box 1
Box 1	5 x 3 x 2	30 in^3		
Box 2	5 x 6 x 2			
Box 3	5 x 6 x 4			
Box 4	10 x 6 x 4			

If you double one dimension, the volume increases by a factor of _____________ or 2—.

If you double two dimensions, the volume increases by a factor of _____________ or 2—.

If you double three dimensions, the volume increases by a factor of _____________ or 2—.

Warm-up

March 3rd, 2010

"overstock.com" sells 36-box cartons of "Kleenex" tissue. If each "Kleenex" box measures

4.75 in. x 4 in. x 9 in.

1. **What is the volume of each "Kleenex" box?**

   ____

2. **What is the total volume of the carton?**

   ____

3. **What might be the dimensions of the carton?**

   ____

4. **If "overstock.com" wants to sell 72-box cartons, what might be the dimensions of the new carton.**

   ____

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ____________________
Date: ____________________
Advisor:____________________

Scaling of Rectangular Prisms II

1. For a rectangular prism, if you:

   double one dimension, the volume increases by a factor of ____________ or 2—,

   double two dimensions, the volume increases by a factor of ____________ or 2—.

   double three dimensions, the volume increases by a factor of ____________ or 2—.

2. Standard shipping containers come in various sizes. In the table below, fill in the missing information for the three types of containers:

Type	Length	Width	Height	Volume
Type 1	20 ft.	________	8 ft.	1280 ft.^{3}
Type 2	_________	8 ft.	_________	2560 ft.^{3}
Type 3	45 ft.	8 ft.	_________	3456 ft.^{3}

3. Complete the following table:

	Dimensions (inches)	Volume (in^3)	Number of doubled dimensions from Box 1	Number of times volume increased over Box 1
Box 1	10 x 7 x 3	______		
Box 2	10 x 14 x 3	420 in^3	__________	2
Box 3	_________	______	2	_____________
Box 4	20 x 14 x 6	______	3	_____________

Be sure to check your answers!

4. When you scale all three dimensions of an object such as a box or a cylinder by a scaling factor, n, the volume of the object increases by a factor of:

____

5. "Kleenex" boxes are sold in packages of 10. If each "Kleenex" box has the following dimensions:

9 in. x 4.5 in. x 4 in.

a. What is the volume of each "Kleenex" box" ____________

b. What is the total volume of a package of 10 "Kleenex" boxes? ____________

c. List two possible sets of dimensions for the package? ____________

____

d. If the seller wants to sell 20-box packages, what is volume of the new 20-box package? ____________

e. If the seller wants to sell 20-box packages, list two possible sets of dimensions for the new 20-box package. ____________

____

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ____________________
Date: ____________________
Advisor: ____________________

Scaling of Rectangular Prisms II

1. For a rectangular prism, if you:

   double one dimension, the volume increases by a factor of 2 or 2^1.

   double two dimensions, the volume increases by a factor of 4 or 2^2.

   double three dimensions, the volume increases by a factor of 8 or 2^3

2. Standard shipping containers come in various sizes. In the table below, fill in the missing information for the three types of containers:

Type	Length	Width	Height	Volume
Type 1	20 ft.	8 ft.	8 ft.	1280 $ft.^3$
Type 2	40 ft.	8 ft.	8 ft.	2560 $ft.^3$
Type 3	45 ft.	8 ft.	9.6 ft.	3456 $ft.^3$

3. Complete the following table:

	Dimensions (inches)	Volume (in^3)	Number of doubled dimensions from Box 1	Number of times volume increased over Box 1
Box 1	10 x 7 x 3	210 in^3		
Box 2	10 x 14 x 3	420 in^3	1	2 or 2^1
Box 3	10 x 14 x 6 20 x 14 x 3	840 in^3	2	4 or 2^2
Box 4	20 x 14 x 6	1680 in^3	3	8 or 2^3

Be sure to check your answers!

4. When you scale all three dimensions of an object such as a box or a cylinder by a scaling factor, n, the volume of the object increases by a factor of:

$$n^3$$

5. “Kleenex” boxes are sold in packages of 10. If each “Kleenex” box has the following dimensions:

9 in. x 4.5 in. x 4 in.

a. What is the volume of each “Kleenex” box” **162 in.^{3}**

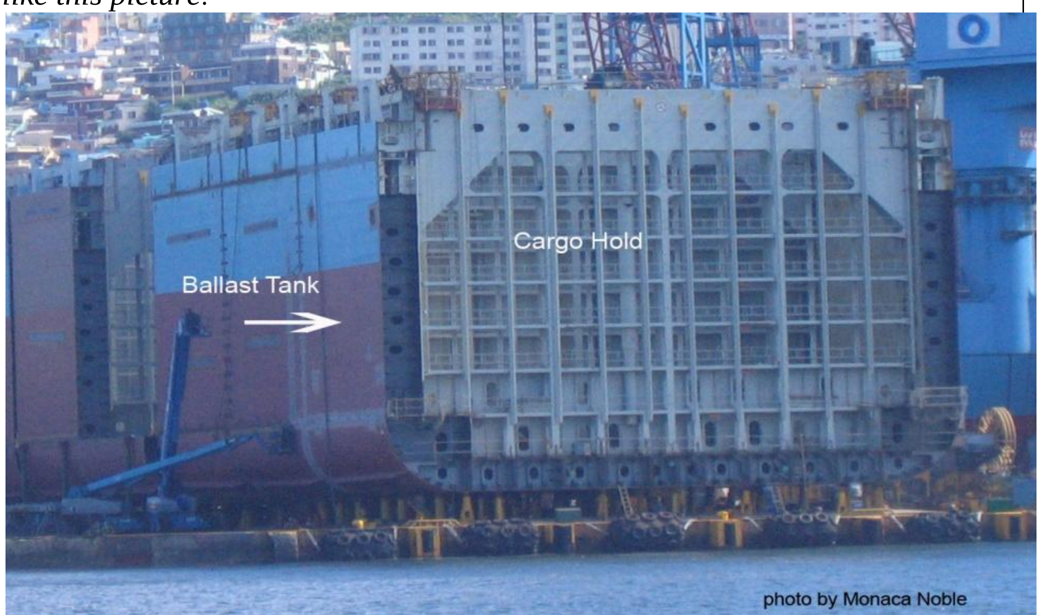
b. What is the total volume of a package of 10 “Kleenex” boxes? **1620 in.^{3}**

c. List two possible sets of dimensions for the package? **9 in. x 8 in. x 22.5 in.**
9 in. x 4 in. x 45 in.
18 in. x 4 in. x 22.5 in.

d. If the seller wants to sell 20-box packages, what is volume of the new 20-box package? **3240 in.^{3}**

e. If the seller wants to sell 20-box packages, list two possible sets of dimensions for the new 20-box package. **18 in. x 8 in. x 22.5 in.**
9 in. x 8 in. x 45 in.
18 in. x 4 in. x 45 in.

Grade / Content Area	**8th / 9th Grade Geometry**
Lesson Title	**Fueling / Loading the Ship (1 day)**
Guiding Question	*How much liquid is needed to fill a three-dimensional shape?*
Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: Middle and high school students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Planning	I. *Classroom Organization.* Students will work together in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface.

	II. *Differentiation.* For my students with special needs, and guided by my collaborating special needs teacher (if I have one) I will prepare a worksheet which incorporates hints and some answers filled in to provide these students with a starting point helpful to their learning. As always, I will be mindful of those students who have difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each student)*:* A. Liquid load worksheet. B. Calculator.
Student Learning Objectives	I. Students will determine the volume of fuel necessary to fill a tank. II. Students will demonstrate ability to calculate the volume of a triangular prism. III. Students will demonstrate ability to use metric measures including conversion from measures of volume to liquid measures.
Instruction and Engagemen t	I. *Warm-up* (**10 minutes**): II. *Launch* (**10 minutes**): I will open with the following, *"We've been talking about filling cargo containers with solid materials but much of what is loaded onto our ship is liquid; primarily fuel. The interior of a container ship looks like this picture:"*  *"Fuel is normally loaded into tanks just below the cargo hold* (point to where the fuel tanks are on the "General Layout of a Ship" diagram). *How can we*

calculate how much fuel we need to fill the tanks?" At this point I will let students speculate on the answer. If necessary, I will prompt them with, *"Well, we can calculate the volume of the tanks, can't we? Does this give us our answer?"* If the answer is yes, I will prompt further thinking with the following question and answer discussion:

A. Question: *So, do you fill your car with cubic feet or cubic inches of gasoline?"*

   Answer: *No, we fill it with gallons of gasoline.*

B. Question: *So, is there a way to convert the volume of a space like a container or a fuel tank into liquid measures?* After they ponder this for a while, I will ask the final question.

C. Question: *What do you notice about the shape of some of these tanks?* Answer is that some are not prisms. In today's exploration, we will find how to calculate the surface area and volume of these shapes and how to convert the those measures into the amount of liquid we can put into the tanks.

D. **Vocabulary:** I will introduce the following:

   1. **Triangular Prism:** a prism whose bases are triangles.

      Surface area of a regular triangular prism:

$$SA = 2\left(\frac{1}{2}bh\right) + PH$$

      Volume of a regular rectangular prism:

$$V = (\frac{1}{2}bh) \times H$$

      - *b* is the length of one side of the base triangle.
      - *h* is the height of the base triangle.
      - *P* is the perimeter of the base.
      - *H* is the height of the prism

   2. **Liquid Conversions:**

$$1\ foot^3 = 7.48051948\ gallons$$

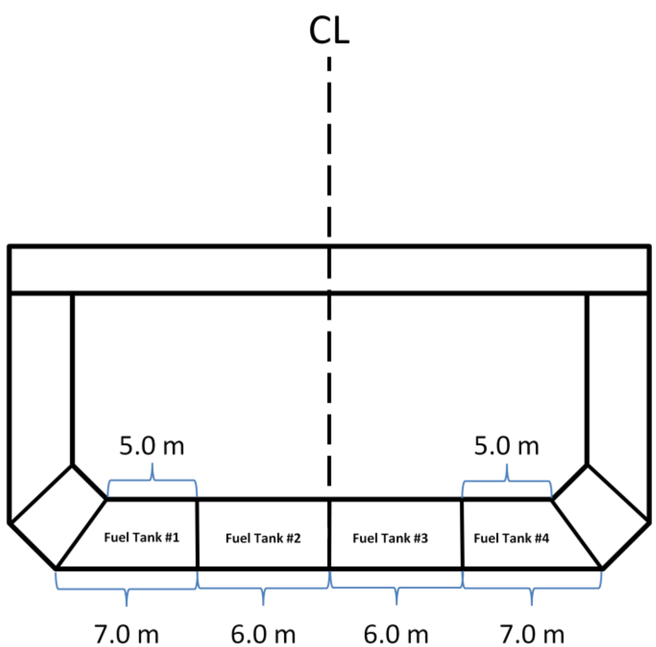
$$1\ meter^3 = 1000\ liters$$

	III. *Engagement* (**30 minutes**). Students will fill in the worksheet (see below). I will explain the diagram to them, relating it to the photograph of a container ship cross-section I showed them before. Then I will let them begin work. At first, the exercise may appear to be straightforward but it might prove more challenging when they try to find the volume of fuel tanks #1 and #4 because they are not rectangular prisms. If students are struggling, I will offer the following prompts, as necessary, group-by-group: A. Question: *Why don't you try drawing the side of the tank and separating it into two smaller shapes, what would they be?* Answer: *We can divide it into a triangle and a rectangle.* B. Question: *Can you find the area of these two shapes?* Answer: *We can find the area of the rectangle, it is 20 square meters.* C. Question: *What about the triangle? Does it look like something or half of something?* Answer: *Sure! It looks like one half of a rectangle. So if I multiply the lengths of the sides and divide by 2 I will get the area?* D. Question: *That's right, so do you know enough now to compute the volume of the tank?* Answer: *I can add the areas of the rectangle and triangle and multiply by the height.* IV. *Closing* (**10 minutes**): We will share our findings and share how we were able to find the volume of the tanks that our not rectangular prisms. I will then ask the following question, *"Which tanks should be loaded first and why?"* I would like them to visualize the problem of stability – how a weight added to the ship will heel it over more as the weight is moved from the center line of the ship. There is a geometrical relationship to be considered – beyond the scope of this lesson but worthy of some discussion, especially for the more inquiring minds.
Assessment	Completed worksheets will serve as informal assessments of student ability to compute volume, convert volume to liquid measure, and determine the volume of a non-rectangular prism.

MAERSK LINE

Container Ship Liquid Load Planning Sheet

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1					
2					
3					
4					



Height of all tanks is 4.0 m
Length of all tanks is 20.0 m
1 cu. m. = 1000 liters

Container Ship Liquid Load Planning Sheet

Version 2

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1	20.0 m	5.0 m / 7.0 m	4.0 m		
2	20.0 m	6.0 m	4.0 m		
3	20.0 m	6.0 m	4.0 m		
4	20.0 m	5.0 m / 7.0 m	4.0 m		

Surface Area and Volume Formulas

1. Surface area of a rectangular prism:

$$SA = 2((l \times w) + (l \times h) + (w \times h))$$

2. Volume of a rectangular prism:

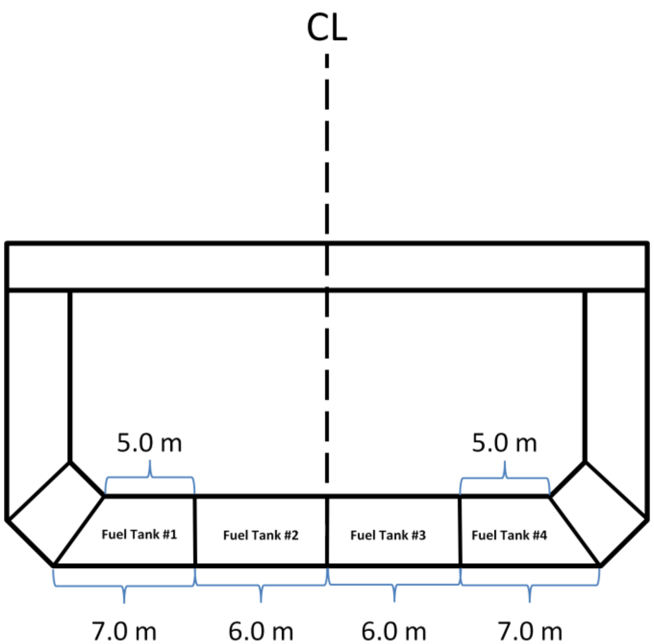
$$V = l \times w \times h$$

3. Surface area of a triangular prism:

$$SA = 2\left(\frac{1}{2}bh\right) + PH$$

4. Volume of a triangular prism:

$$V = \left(\frac{1}{2}bh\right) \times H$$



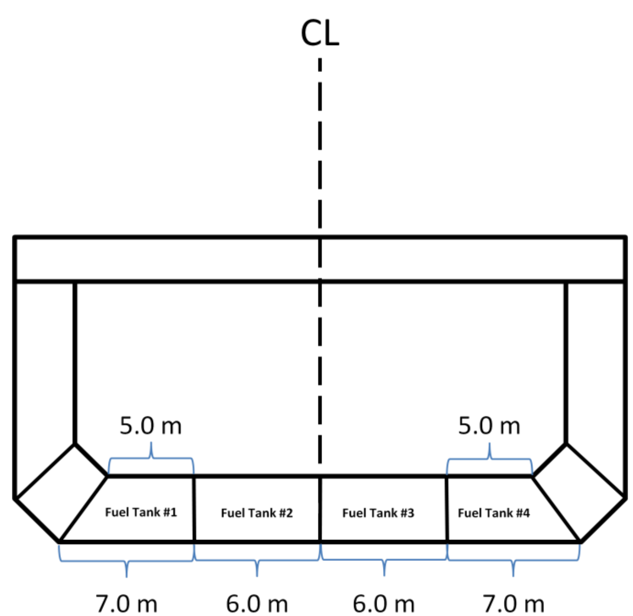
Height of all tanks is 4.0 m
Length of all tanks is 20.0 m
1 cu. m. = 1000 liters

MAERSK LINE

Container Ship Liquid Load Planning Sheet

(Answer Key)

Tank Number	Tank Length	Tank Width	Tank Height	Tank Volume	Fuel Necessary to Fill Tank
1	20.0 m	5.0 m / 7.0 m	4.0 m	$(20*5*4)+(0.5*20*2*4)=480\ cu.m.$	$Fuel\ needed=480*1000=480{,}000\ litres$
2	20.0 m	6.0 m	4.0 m	$(20*7*4)=560\ cu.m.$	$Fuel\ needed=560*1000=560{,}000\ litres$
3	20.0 m	6.0 m	4.0 m	$(20*7*4)=560\ cu.m.$	$Fuel\ needed=560*1000=560{,}000\ litres$
4	20.0 m	5.0 m / 7.0 m	4.0 m	$(20*5*4)+(0.5*20*2*4)=480\ cu.in.$	$Fuel\ needed=480*1000=480{,}000\ litres$



Height of all tanks is 4.0 m

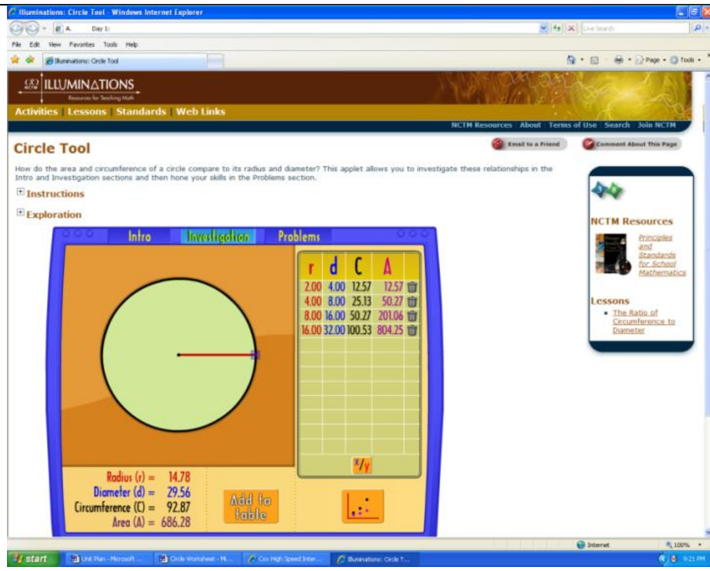
Length of all tanks is 20.0 m

1 cu. m. = 1000 liters

Grade / Content Area	8th / 9th Grade Geometry
Lesson Title	**Cylinders and the General Formula for Prisms (3 – 4 days)**
National and State Content Standards	*State Content Standards:* I. **M(G&M)–8–5:** Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate. (Local) II. **M(G&M)–8–6:** Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units. (Local)
	NCTM Standards: In grades 6 – 8 all students should: I. **Analyze characteristics:** precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship. II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
Goals, Context of the Lesson, and Rationale	I. *Goals:* My goal in this lesson is to help students make the connection between physically filling a cylinder and the mathematical equation for the volume of a cylinder. II. *Context and Rationale:* This is the third lesson in this unit on Geometry, expanding our focus on surface area and volume to cylindrical objects. In the

	last lesson, we derived the equation for the volume of a rectangular prism. In a similar fashion and with similar objects, we will derive the equations for the volume of a cylindrical object. We will then explore how varying the dimensions of cylindrical objects produces some surprising results – just as we discovered with prisms.
Opportunities to Learn	I. *Classroom Organization*: Students will work in groups of two at computer workstations for the first activity and then in groups of three or four. Desks will be rearranged to permit students to work together with a common work surface. II. *Differentiation*. There are many things in this lesson that will appeal to the multiple intelligences of the students: new terminology, worksheets, and lots of hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials* (for each group of students): A. Day 1: 1. A soup can. 2. One computer workstation for each group of two students. 3. *Illuminations* Circle Tool Applet: (http://illuminations.nctm.org/ActivityDetail.aspx?ID=116). 4. Circle worksheet. B. Day 2: 1. Cylindrical objects (each student will be asked to bring one). 2. Construction paper. 3. Rulers and pencils. C. Day 3: 1. 8.5 x 11 in. white paper 2. 8.5 x 11 in. colored paper.

	3. Tape. 4. Popcorn. 5. Plate. 6. Cup. 7. Ruler.
Student Learning Objectives	I. Students derive the formula for surface area of a circle and, from that, derive the formula for the volume of a cylindrical shape. II. Students compare the volumes of cylindrical shapes by filling them with liquids / solids as well as algebraically to understand the relationship between a cylinder's dimensions and its volume.
Timing	I. Circle Tool Applet Lesson: **45 minutes**. II. In-class group practice on worksheet: **30 minutes**. III. Measuring cylinders: **45 minutes.** IV. Popcorn cylinders: **45 minutes.** **Total: 165 minutes.**
Instructional Procedures	I. *Opening.* **Day 1:** I will begin by pointing out that not all of the shapes that will go into our shipping container will be rectangular. Another common shape for shipping goods, particularly liquids, is a cylinder. In our previous lesson, we derived the equation to calculate the volume of a rectangular box. Now we will do the same for a cylinder. A. I will hold up a soup can and ask the question, *"Can anyone tell me the volume of this can?"* B. Given that students should have learned the equation for the surface area of a circle in the seventh grade, it is possible that someone will make the connection with the previous lesson and state that the volume equals the surface area of the base circle times the height. C. Regardless of whether someone does or not, we will conduct an activity in groups of two at computer workstations to derive the equation. II. *Engagement.* **Day 1:** In groups of two, students will work at computer stations with internet access, using the *Illuminations* Circle Tool Applet:



A. Using the worksheet below, students will use the applet to determine the diameter, circumference, and area of circles of radius 2, 4, 8, and 16. Using the applet's ratio tool (x/y), students will derive the formulas for the circumference and area of a circle.

Finding the Area and Circumference of a Circle

1. Using the Circle Tool (http://illuminations.nctm.org/ActivityDetail.aspx?ID=116), set the radius at r = 2 and record the diameter, circumference, and area of the circle. Do the same for r = 4, r = 8, and r = 16.

Radius (r)	Diameter (d)	Circumference (C)	Area (A)	d/r	C/d	C/r	C
2	4	12.57	12.57	2	3.14	2*3.14	2*3.14*r
4	8	25.13	50.27	2	3.14	2*3.14	2*3.14*r
8	16	50.27	201.06	2	3.14	2*3.14	2*3.14*r
16	32	100.53	804.25	2	3.14	2*3.14	2*3.14*r

2. Calculate the relationship between the diameter and the radius.

   Ans. d/r is always 2 so, solving for d,

$$d = 2r$$

3. Calculate the relationship between the circumference and the diameter.

Ans. C/d is always 3.14 so, solving for C,

$$C = 3.14d$$

…and, since d = 2r,

$$C = 2 * 3.14 * r$$

*So, the circumference C of a circle equals 2*radius*3.14. Now let's try to derive a formula for the area of the circle.*

4. Calculate the relationship between Area and Circumference.

   Ans. Area divided by the circumference equals one half the radius. So:

$$\frac{A}{C} = \frac{A}{2 * 3.14 * r} = 0.5 * r$$

   Solving for A, we get:

$$A = (2 * 3.14 * r) * 0.5 * r = (2 * 0.5) * 3.14 * (r * r) = 1 * 3.14 * r^2$$

5. Do you remember what 3.14 is? ***Ans. 3.14 is an estimate for the value of π which is 3.1415926535898…so the area of a circle is*** $\boldsymbol{A = \pi * r^2}$

B. We will review the students' findings on the worksheets, ensuring everyone understands how to derive the formula. I will then conduct the following dialog:

1. Question: *Do you recall how we found the volume of a rectangular prism, like a box?*

2. Answer: *We measured the surface area of the base and multiplied it by the height of the box.*

3. Question: *Then what do you think is the volume of the cylinder? Now that we know the area of the base (the circle) what should we do?*

4. Answer: *The same thing, multiply the surface area of the base by the height of the cylinder.*

5. Question: *Now, remember we discovered yesterday that the volume of a box varies with its dimensions and that two boxes formed from the same sheet of construction paper can have different dimensions. Does this work for cylinders too? Let's find out.*

At this point, I will produce two cylinders each formed from identical sheets of construction paper. I will ask two students to come forward and fill both

with popcorn, demonstrating that the same thing happens with cylinders.

III. *Closing.* **Day 1:** I will conclude with, "*So now we know that we can find the volume of a cylinder using the surface area of the base circle and the height of the cylinder. We also know that cylinders and rectangular prisms have similar properties with regard to their volume. What we don't know yet is how to calculate the surface area of a cylinder. We will explore tomorrow how we can find the surface area of the cylinder. Try to think about how to find the surface area of the cylinder tonight. Oh, and by the way, I would like each of you to bring a cylindrical object to school tomorrow – a food can would be best.*"

IV. *Opening.* **Day 2:** We will begin with a review of the previous day's lesson. I will have the *Illuminations* applet we used in that lesson and review how we derived the equation of the circle and then calculated the volume of the sphere. Then I will ask if anyone can tell me how to calculate its surface area. Chances are, someone will know or have some idea – i.e. he / she might know that it is the sum of the areas of the base, the top, and the "sides" but may not know how to calculate the area of the "sides." In our exercise today, we will derive that formula.

V. *Engagement.* **Day 2:** In groups of four and with the cylindrical objects they brought to class, students will determine the surface area of the objects by calculating the surface area of the base and the top using the equation we derived and, using the construction paper and rulers, the surface area of the side. They will enter their observations into the following table. Students will also calculate the volume of the can using our equation and then *speculate on how many such cans could fit into the shoe box whose volume we calculated previously* (this will set the stage for a follow-on lesson).

Can	**Surface Area of Top**	**Surface Area of Base**	**Surface Area of Side**	**Surface Area of Can**	**Volume of Can**	**How Many Cans Fit Into a Shoe Box?**

A. **Question**: *We know how to calculate the area of the two circles that make up the top and the base. How can we calculate the area of the side?*

Answer: *We can use our construction paper. By wrapping it around the cylinder, trimming the excess paper, unwrapping the paper, and then*

measuring the length of the sides, we can determine the area of the paper sufficient to wrap around the cylinder and, thus, the surface area of cylinder itself.

B. **Question**: *That does work but can we derive an equation from this? Wrap the paper around the cylinder again. Look at the side that wraps around the cylinder. That side has become what?*

Answer: *A circle! So, if we compute the circumference of the circle as one of the sides and multiply it by the length of the other side, we have the surface area. The equation is:*

$$Side\ Surface\ Area = (2 * \pi * r) * h$$

C. **Question**: *Are we done?*

Answer: *No. We need to add the surface areas of the top and the base. The complete equation is:*

$$Surface\ Area\ of\ Cylinder = (2 * \pi * r) * h + 2 * (\pi * r^2)$$
$$General\ Formula\ for\ Surface\ Area = 2B + PH$$

VI. *Closing.* **Day 2:** Now we will explore the final question, *"How many cans fit into the shoe box?"* Perhaps some students will simply divide the volume of their can into the volume of the shoe box to get the answer. Perhaps others will realize that the answer is more complicated. We will speculate on possible answers to this question to conclude the lesson.

VII. *Opening.* **Day 3:** Just as we did with the "Popcorn Prisms", we will explore the relationship between dimension and volume, arriving at some rather surprising conclusions. This exercise was developed by Jamie Chaikin and can be found on the NCTM Illuminations website at http://illuminations.nctm.org/LessonDetail.aspx?ID=L797.

VIII. *Engagement.* **Day 3:**

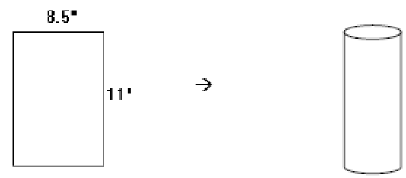
Answer Key – Popcorn Cylinders Anyone?

For this activity you will be comparing the volume of 2 cylinders created using the same sheet of paper. You will be determining which can hold more popcorn. To do this, you will have to find a pattern for the dimensions for containers.

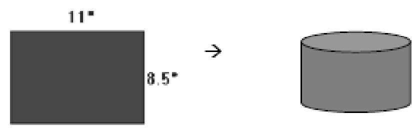
Materials:

- 8.5 inch by 11 inch white paper
- 8.5 inch by 11 inch colored paper
- Tape
- Popcorn
- Plate
- Cup
- Ruler

Take the white paper and roll it up along the longest side to form a baseless cylinder that is tall and narrow. Do not overlap the sides. Tape along the edge. Measure the dimensions with a ruler. Record your data below and on the cylinder. Label it Cylinder A.



Take the colored paper and roll it up along the shorter side to form a baseless cylinder that is short and stout. Do not overlap the sides. Tape along the edge. Measure the height and diameter with a ruler. Record your data below and on the cylinder. Label it Cylinder B.



1.

Dimension	Cylinder A	Cylinder B
Height (in.)	[11 in]	[8.5 in]
Diameter (in.)	[~2.7 in]	[~3.5 in]
Radius (in.)	[~1.4 in]	[~1.8 in]

2. Do you think the two cylinders will hold the same amount? Do you think one will hold more than the other? Which one? Why?

Answers will vary.

6. Which measurement impacts the volume more: the radius or the height? Work through the example below to help you answer the question.

   a) Assume that you have a cylinder with a radius of 3 inches and a height of 10 inches. Increase the radius by 1 inch and determine the new volume. Then using the original radius, increase the height by 1 inch and determine the new volume.

CYLINDER	RADIUS	HEIGHT	VOLUME
ORIGINAL	3 in	10 in	[~282.7 in^3]
INCREASED RADIUS	[4 in]	[10 in]	[~502.7 in^3]
INCREASED HEIGHT	[3 in]	[11 in]	[~311.0 in^3]

   b) Which increased dimension had a larger imact on the volume of the cylinder? Why do you think this is true?

Increasing the radius increased the volume more than increasing the height. This is because the radius is squared to find the volume, which increases its impact on the volume.

7. By how much would you have to decrease the height of Cylinder B to make the volumes of the two prisms equal?

$$V_A \approx 67.7 \text{ in}^3$$
$$V_B \approx 67.7 \text{ in}^3 = \pi(1.8)^2(h)$$
$$h \approx 6.7 \text{ in}$$

The height would need to be decreased by about $8.5 - 6.7 \approx 1.8$ in .

8. Compare and contrast your results from the prism activity and the cylinder activity. What conclusions can you make about the relationship between dimensions, area, and volume?

Answers will vary. Students may point out the similarity in the volume formulas $V = l^2h$ and $V = \pi r^2 h$ and how this effected their results.

3. Place Cylinder B on the paper plate with Cylinder A inside it. Use your cup to pour popcorn into Cylinder A until it is full. Carefully, lift Cylinder A so that the popcorn falls into Cylinder B. Describe what happened. Is Cylinder B full, not full, or overflowing?

Cylinder B is not full. There is still room in the cylinder for more popcorn.

As you share your popcorn snack, answer the questions below.

4. a) Was your prediction correct? How do you know?

Answers will vary.

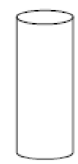
b) If your prediction was incorrect, describe what actually happened.

Cylinder B has a greater volume than Cylinder A.

5. a) State the formula for finding the volume of a cylinder.

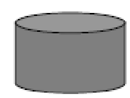
$V = \pi r^2 h$

b) Calculate the volume of Cylinder A? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi(1.4)^2(11) \approx 67.7 \text{ in}^3$

c) Calculate the volume of Cylinder B? Label the dimensions in the figure.



$V = \pi r^2 h \approx \pi(1.8)^2(8.5) \approx 86.5 \text{ in}^3$

d) Explain why the cylinders do or do not hold the same amount. Use the formula for the volume of a cylinder to guide your explanation.

The cylinders have different radii and heights, so the volumes are different.

ILLUMINATIONS Resources for Teaching Math

A. Together we will construct the cylindrical prisms. I will then let the students begin work on the questions as I walk around the room helping as necessary.

B. After students answer Question 2, I will pass out cooked popcorn (ideally prepared in the classroom or, at least, just moments before) to be used for the rest of the exercise. There should be enough to enjoy a snack as well as to conduct the exercise. Classroom management will be a bit of a

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challenge. Ideally, I should not do this on a Friday or just before a holiday.

C. Some students are likely to be surprised that the two cylinders do not hold the same amount, given they were constructed from identical sheets of construction paper. As they complete the exercise and consider the volume formula as it applies to either prism:

$$Volume = B \times h = \pi \times r^2 \times h$$

So, the taller prism is:

$$V = \pi \times r^2 \times h = \pi \times (1.4)^2 \times 11 \cong 67.7 in^3$$

The, shorter, wider prism is:

$$V = \pi \times r^2 \times h = \pi \times (1.8)^2 \times 8.5 \cong 86.5\, in^3$$

WHY DOES THIS HAPPEN?

D. Algebraically, it can be seen that the radius of the second cylinder squared is approximately 1.7 times greater than that of the first cylinder. Therefore, the magnitude of the area of the base circle has greater impact than the magnitude of the height.

IX. *Closing.* **Day 3.** I will ask students to think about our shipping container because it is into that which we will need to fit our cylinders. I will leave them with the following question, *“Remember, you want to get as much into that container as possible. Which of the two cylinders would you want to use to load cargo into the container?”*

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Assessment

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I will assess progress through a number of instruments to include in-class and at-home activities and worksheets. There will be a formative assessment at the conclusion of this lesson.

1. *Circles and Rectangles*: a review of how to determine the circumference and area of a circle.
2. *Queen Mary 2 Menu*: more review and introduction to cylinders.
3. *Circles and Cylinders*: further review of the circumference and area of a circle as well as practice in calculating the surface are and volume of a cylinder.
4. *Popcorn Cylinders*: an in-class exercise from NCTM *Illuminations*.

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1. List all formulas.
2. Show all work on separate paper attached to this one.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________

Date:____________________

Advisor:____________________

Worksheet: Circles and Rectangles

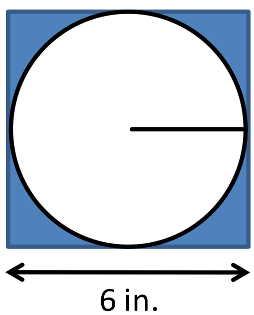
1. What are the equations for the circumference of a circle? ________________

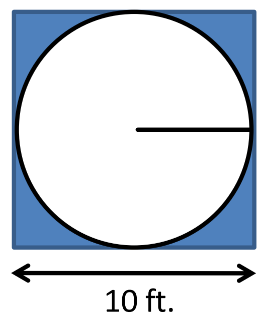
2. What is the equation for the area of a circle? ________________

3. What is the value of pi? ________________

4. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





Circumference: ______________ Circumference: ______________

Circle area: ______________ Circle area: ______________

Area of square: ______________ Area of square: ______________

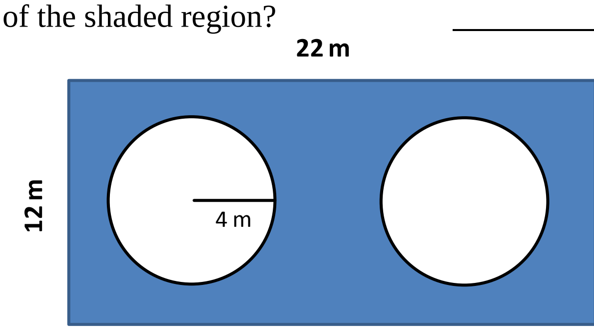
Shaded region area: ______________ Shaded region area: ______________

5. In the following figure:

   A. What is the circumference of each small circle? ________________________

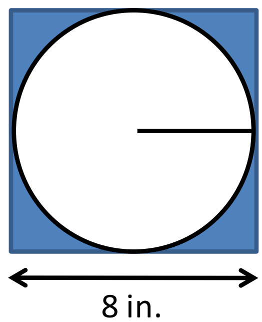
   B. What is the area of each small circle? ________________________

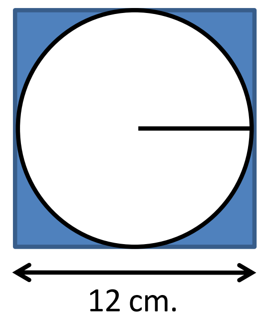
   C. What is the area of the shaded region? ________________________



6. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





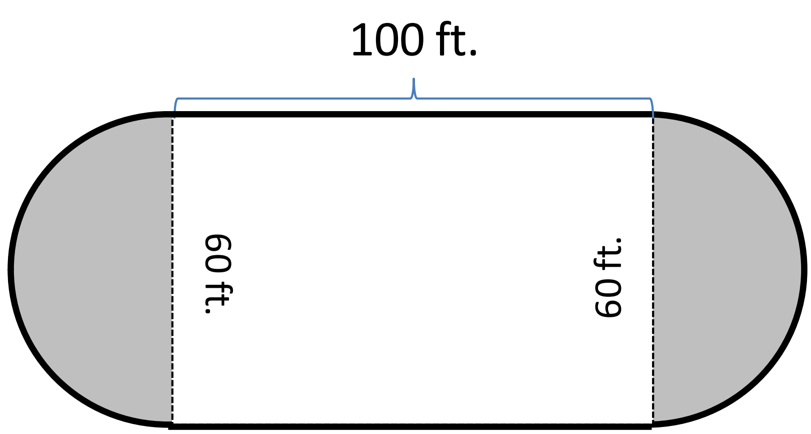
Circumference:	______________	Circumference:	______________
Circle area:	______________	Circle area:	______________
Area of square:	______________	Area of square:	______________
Shaded region area:	______________	Shaded region area:	______________

7. For the racetrack below:

   A. What is the distance around this track? ____________

   B. What is the area of the shaded regions? ____________

   C. What is the area of the complete region inside the track? ____________



1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Differentiated Worksheet

Name: ______________________
Date: _____________________
Class: ______________________

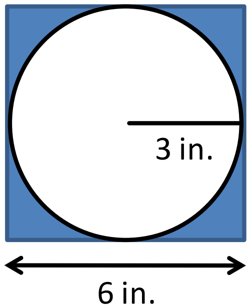
Worksheet: Circles and Rectangles

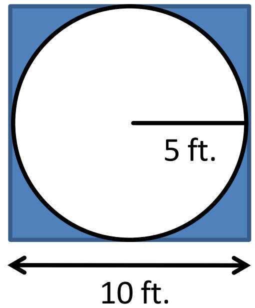
1. What are the equations for the circumference of a circle? __________________

2. What is the equation for the area of a circle? __________________

3. What is the value of pi? __________________

4. For each of the following:





A. What is the circumference of the circle inside the square?

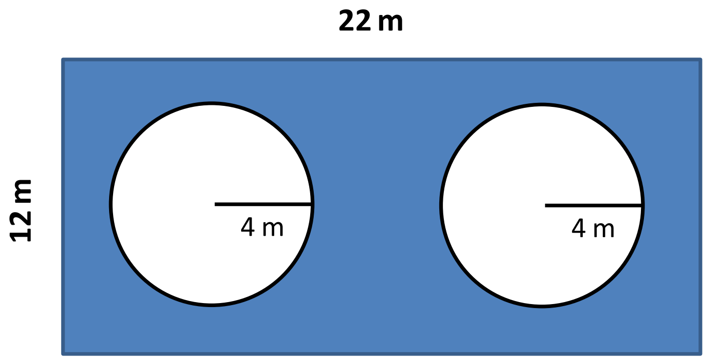
B. What is the area of the circle inside the square?

C. What is the total area of the square?

D. What is the area of the shaded region?

Circumference:	________________	Circumference:	________________
Circle area:	________________	Circle area:	________________
Area of square:	________________	Area of square:	________________
Shaded region area:	________________	Shaded region area:	________________

5. In the following figure:

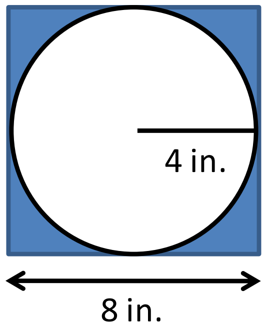


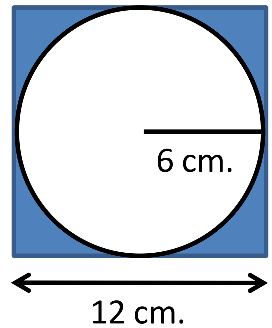
A. What is the circumference of each small circle? ____________________

B. What is the area of each small circle? ____________________

C. What is the area of the shaded region? ____________________

6. For each of the following:





A. What is the circumference of the circle inside the square?

B. What is the area of the circle inside the square?

C. What is the total area of the square?

D. What is the area of the shaded region?

Circumference: ________________

Circle area: ________________

Area of square: ________________

Shaded region area: ________________

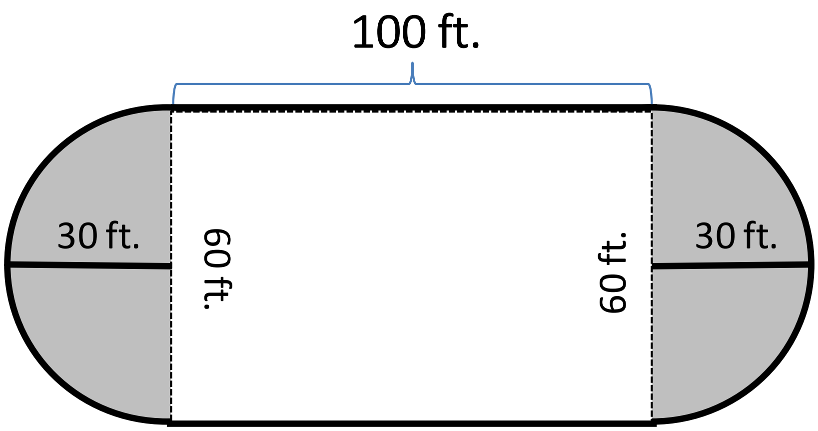
Circumference: ________________

Circle area: ________________

Area of square: ________________

Shaded region area: ________________

7. For the racetrack below:



A. What is the distance around this track? ____________

B. What is the area of the shaded regions? ____________

C. What is the area of the complete region inside the track? ____________

Circles and Rectangles (Answer Key)

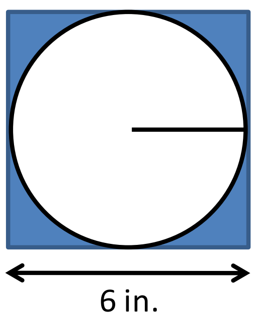
1. What are the equations for the circumference of a circle? $\mathbf{2 \times \pi \times r}$ $\mathbf{\pi \times d}$

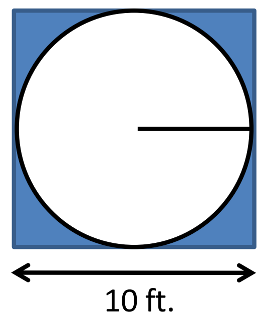
2. What is the equation for the area of a circle? $\mathbf{\pi \times r^2}$

3. What is the value of pi? **approx. 3.14**

4. For each of the following:

   A. What is the circumference of the circle inside the square?
   B. What is the area of the circle inside the square?
   C. What is the total area of the square?
   D. What is the area of the shaded region?





Circumference: **18.84 in.**

Circle area: **28.26 in.^{2}**

Area of square: **36 in.^{2}**

Shaded region area: **7.74 in.^{2}**

Circumference: **31.4 ft.**

Circle area: **78.5 ft.^{2}**

Area of square: **100 ft.^{2}**

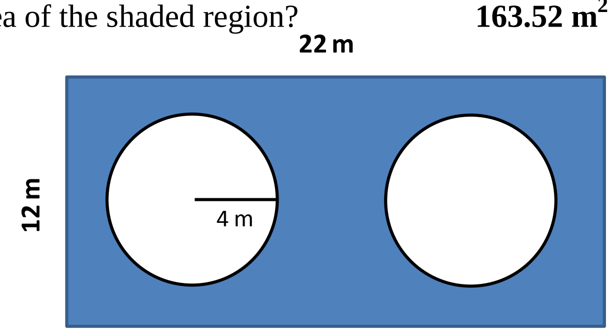
Shaded region area: **21.5 ft.^{2}**

5. In the following figure:

   A. What is the circumference of each small circle? **25.12 m**

   B. What is the area of each small circle? **50.24 m^2**

   C. What is the area of the shaded region? **163.52 m^2**



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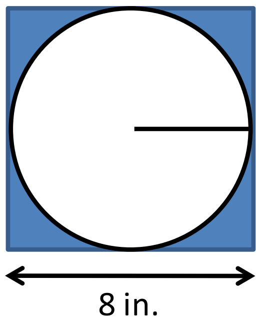
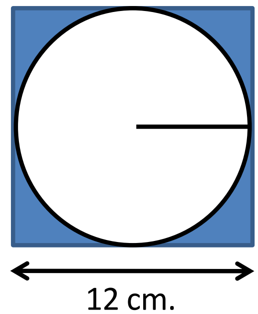
   D. What is the area of the shaded region?





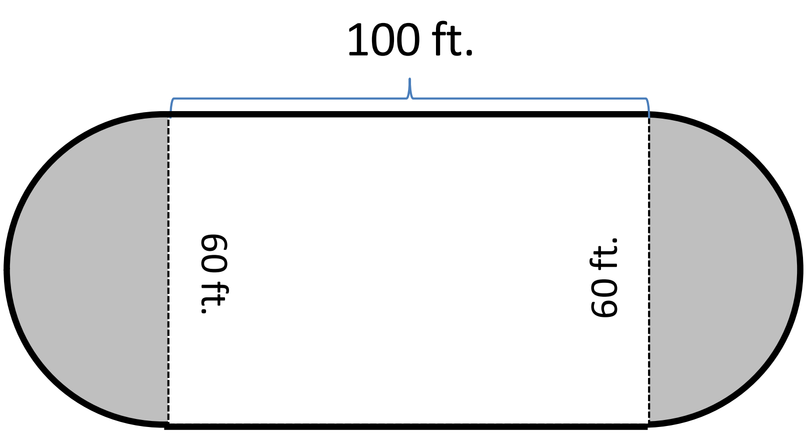
Circumference:	**25.12 in.**	Circumference:	**37.68 cm.**
Circle area:	**50.24 $in.^2$**	Circle area:	**113.04 $cm.^2$**
Area of square:	**64 $in.^2$**	Area of square:	**144 $cm.^2$**
Shaded region area:	**13.76 in^2**	Shaded region area:	**30.96 $cm.^2$**

7. For the racetrack below:

   A. What is the distance around this track? **388.4 ft.**

   B. What is the area of the shaded regions? **2826 $ft.^2$**

   C. What is the area of the complete region inside the track? **8826 $ft.^2$**

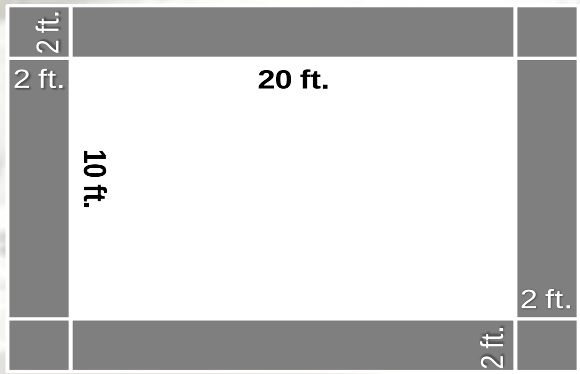


Royal Mail Steamer *Queen Mary 2*

Dinner, September 26th, 2010

Appetizer

Find the total area and the shaded area of the following shape:

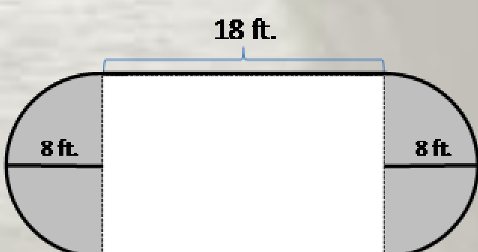


1.a Total Area: ______________ 1.b Shaded Area: ______________

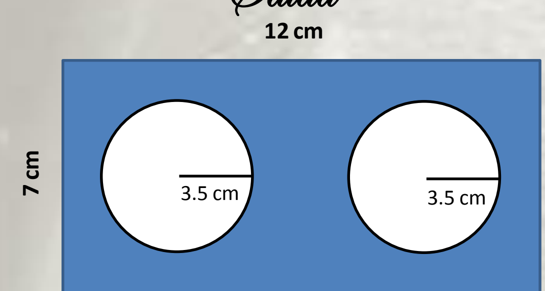
Soup or Salad

Choose just one and find the requested answers:

Soup



Salad



2.a Shaded region area: ______________

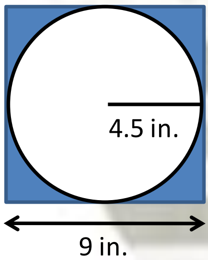
2.b Total Area: ______________

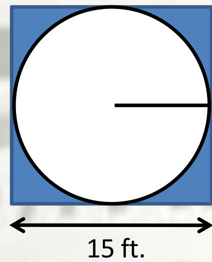
2.c Each circle area: ______________

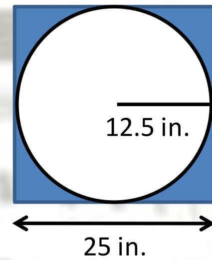
2.d Shaded area: ______________

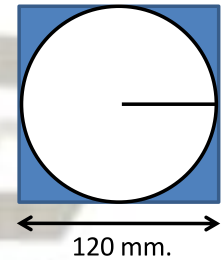
Main Course

Choose any two and find the requested answers:









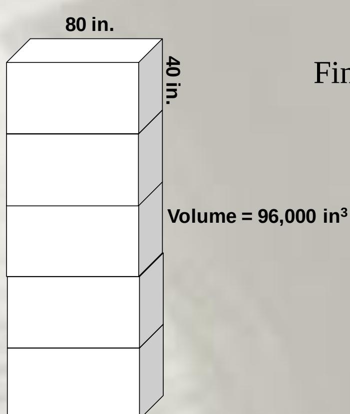
3.a Circle Area: _______
3.b Square Area: _______
3.c Shaded Area: _______

3.d Circle Area: _______
3.e Square Area: _______
3.f Shaded Area: _______

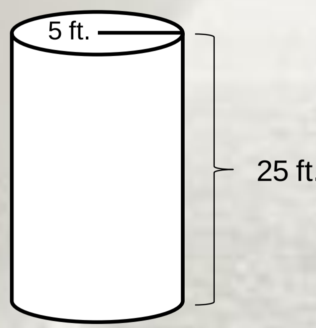
3.g Circle Area: _______
3.h Square Area: _______
3.i Shaded Area: _______

3.j Circle Area: _______
3.k Square Area: _______
3.l Shaded Area: _______

Dessert



Find the requested answers:



4.a Width: _______________

5.a Surface Area: _______________
5.b Volume: _______________

Instructions from the Captain's Table:

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

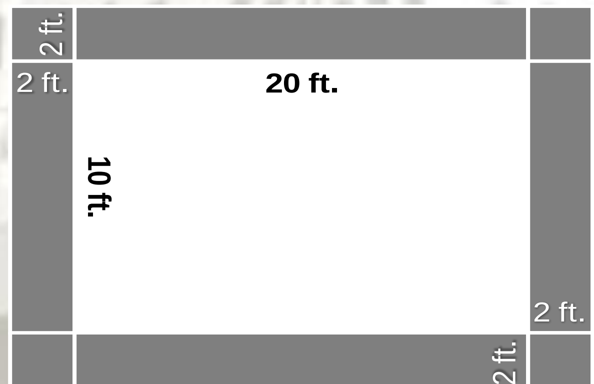
Answer Key

Royal Mail Steamer *Queen Mary 2*

Dinner, February 22nd, 2010

Appetizer

Find the total area and the shaded area of the following shape:

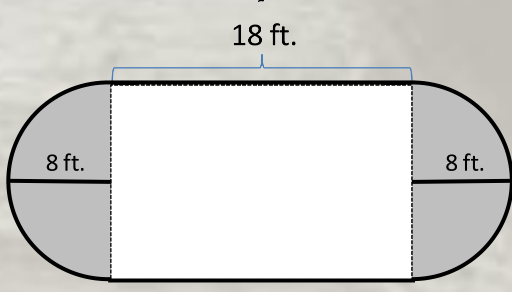


1.a Total Area: **336 ft.^{2}** 1.b Shaded Area: **136 ft.^{2}**

Soup or Salad

Choose just one and find the requested answers:

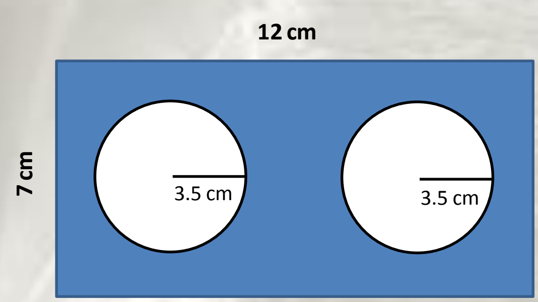
Soup



2.a Shaded region area: **200.96 ft.^{2}**

2.b Total Area: **488.96 ft.^{2}**

Salad

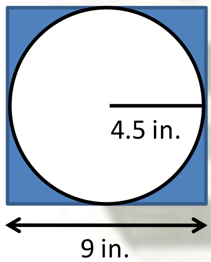


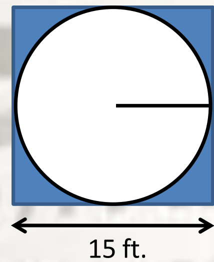
2.c Each circle area: **38.465 cm.^{2}**

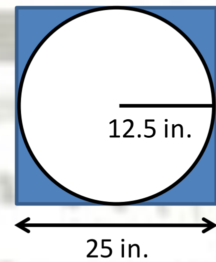
2.d Shaded area: **45.535 cm.^{2}**

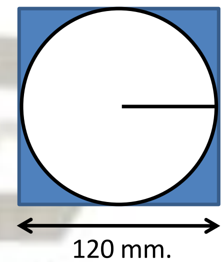
Main Course

Choose any two and find the requested answers:





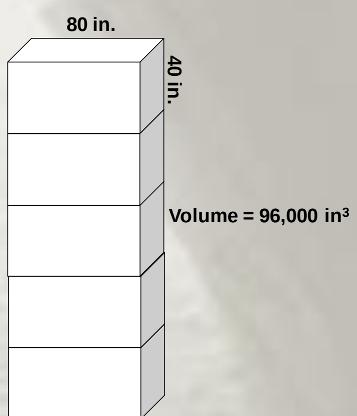


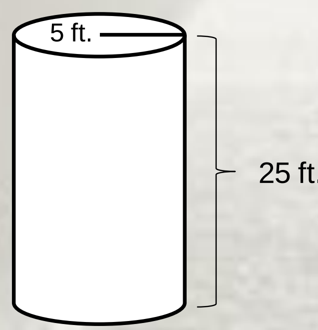


3.a Circle Area: **63.59 in.^{2}**	3.d Circle Area: **176.63 ft.^{2}**	3.g Circle Area: **490.63 ft.^{2}**	3.j Circle Area: **11304 mm.^{2}**
3.b Square Area: **81 in.^{2}**	3.e Square Area: **225 ft.^{2}**	3.h Square Area: **625 ft.^{2}**	3.k Square Area: **14400 mm.^{2}**
3.c Shaded Area: **17.41 in.^{2}**	3.f Shaded Area: **48.37 ft.^{2}**	3.i Shaded Area: **134.37 ft.^{2}**	3.l Shaded Area: **3096 mm.^{2}**

Dessert

Find the requested answers:





4.a Width: **6 ft.**

5.a Surface Area: **942 ft.^{2}**

5.b Volume: **1962.5 ft.^{2}**

Instructions from the Captain's Table:

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:_____________________
Class:_____________________

Circles and Cylinders

1. Definitions.

   a. Circumference: _____ Line segment whose endpoints lie on the circle.

   b. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: _____ The distance around a closed curve such as a circle.

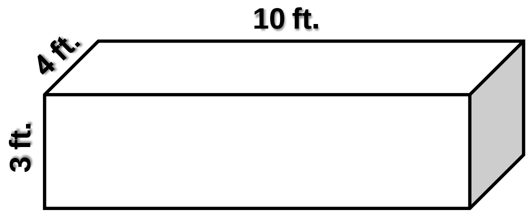
   d. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

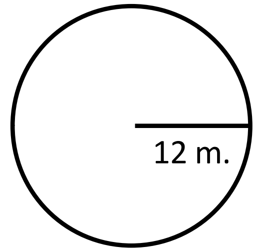
   e. Volume: _____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? _________________
3. What is the formula for the **area** of a circle? _________________
4. What is the formula for the **volume** of a rectangular prism? _________________
5. What is the formula for the **surface area** of a rectangular prism? _________________
6. What is the formula for the **volume** of a cylinder? _________________
7. What is the formula for the **surface area** of a cylinder? _________________
8. What is a general formula for the **volume** of a prism or cylinder? _________________
9. What is a general formula for the **surface area** of a prism or cylinder? _________________

10. For the following shapes, find the asked-for information:



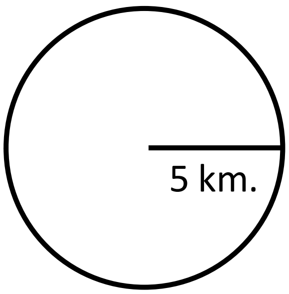


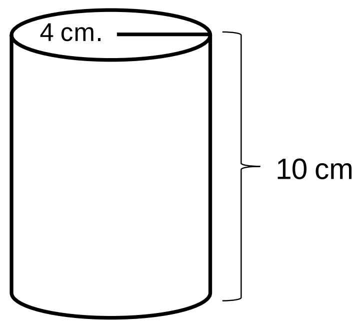
10.a Surface Area: ____________________

10.b Volume: ____________________

10.c Circumference: ____________________

10.d Area: ____________________



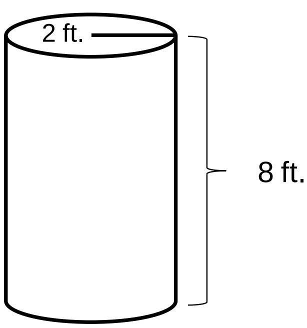


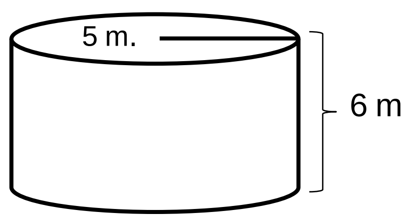
10.e Circumference: ____________________

10.f Area: ____________________

10.g Surface Area: ____________________

10.h Volume: ____________________





10.i Surface Area: ____________________

10.j Volume: ____________________

10.k Surface Area: ____________________

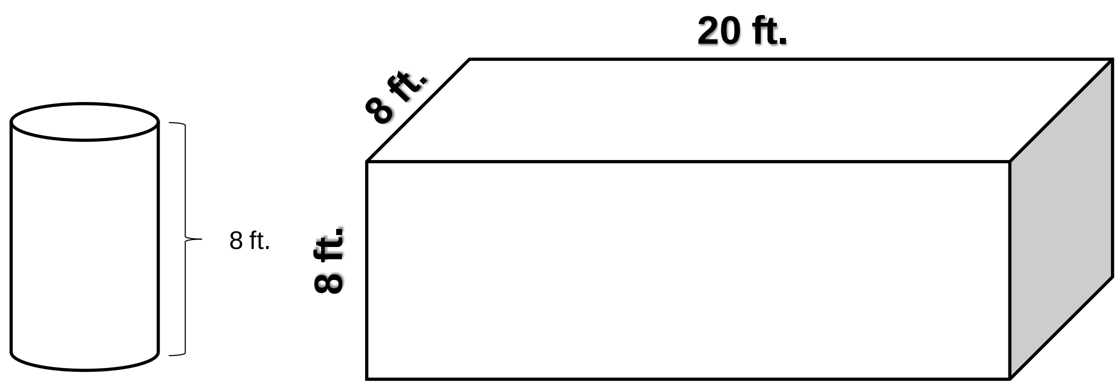
10.l Volume: ____________________

11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of the container? ________________

b. What is the volume of the container? ________________

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 $ft.^3$ How many cylinders do you think we could fit into our container? ***Think!*** ________________



Differentiated Version

Circles and Cylinders

1. Definitions.

    g. Circumference: _____ Line segment whose endpoints lie on the circle.

    h. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

    i. Diameter: _____ The distance around a closed curve such as a circle.

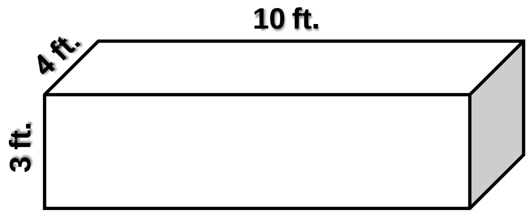
    j. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

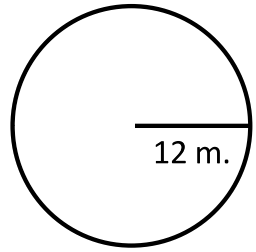
    k. Volume: _____ Any line segment from the center of a circle to its perimeter.

    l. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? ________________
3. What is the formula for the **area** of a circle? ________________
4. What is the formula for the **volume** of a rectangular prism? ________________
5. What is the formula for the **surface area** of a rectangular prism? ________________
6. What is the formula for the **volume** of a cylinder? ________________
7. What is the formula for the **surface area** of a cylinder? ________________
8. What is a general formula for the **volume** of a prism or cylinder? ________________
9. What is a general formula for the **surface area** of a prism or cylinder? ________________

10. For the following shapes, find the asked-for information:



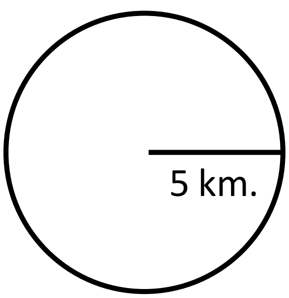


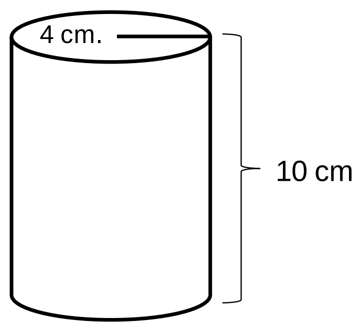
10.a Surface Area: ________________

10.b Volume: ________________

10.c Circumference: ________________

10.d Area: ________________





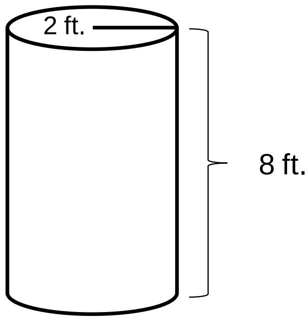
10.e Circumference: ________________

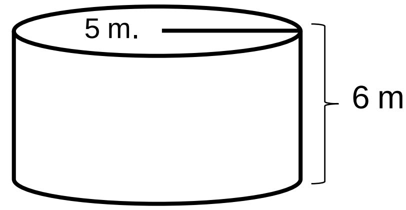
10.f Area: ________________

10.g Surface Area: ________________

10.h Volume: ________________

Differentiated Version





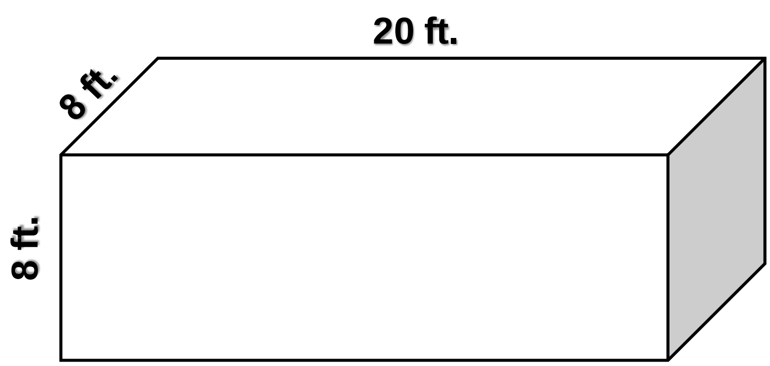
10.i Surface Area: ____________________

10.j Volume: ____________________

10.k Surface Area: ____________________

10.l Volume: ____________________

11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.

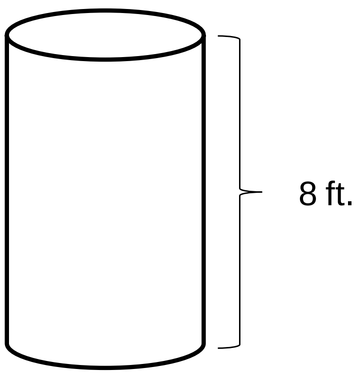


a. What is the surface area of the container? ____________________

Differentiated Version

b. What is the volume of the container? __________________

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 $ft.^3$ How many cylinders do you think we could fit into our container? ***Think!*** __________________



Worksheet: Circles and Cylinders (Answer Key)

1. Definitions.

   a. Circumference: **d**____ Line segment whose endpoints lie on the circle.

   b. Radius: **f**____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: **a**____ The distance around a closed curve such as a circle.

   d. Chord: **c**____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   e. Volume: **b**____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: **e**_____The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? $2\pi r$

3. What is the formula for the **area** of a circle? πr^2

4. What is the formula for the **volume** of a rectangular prism? Bh

5. What is the formula for the **surface area** of a rectangular prism? $2B + Ph$

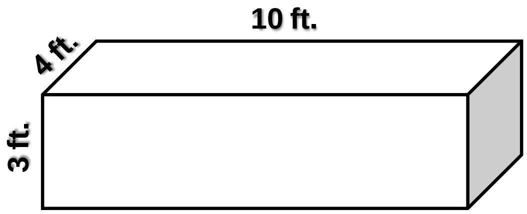
6. What is the formula for the **volume** of a cylinder? $\pi r^2 h$

7. What is the formula for the **surface area** of a cylinder? $2\pi r^2 + 2\pi rh$

8. What is a general formula for the **volume** of a prism or cylinder? Bh

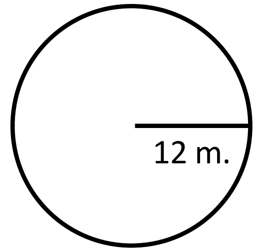
9. What is a general formula for the **surface area** of a prism or cylinder? $2B + Ph$

10. For the following shapes, find the asked-for information:



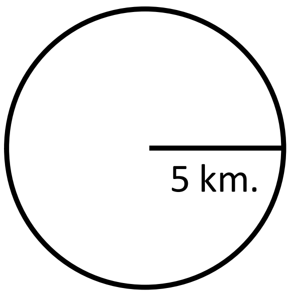
10.a Surface Area: **164 ft^2**

10.b Volume: **120 ft^3**



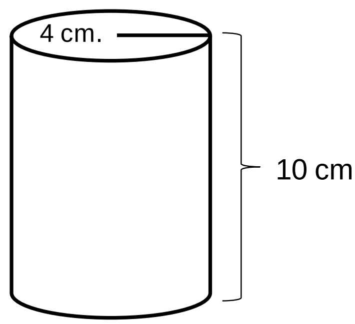
10.c Circumference: **75.36 m**

10.d Area: **452.16 m^2**



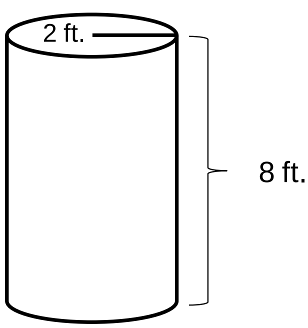
10.e Circumference: **31.4 km.**

10.f Area: **78.5 $km.^2$**



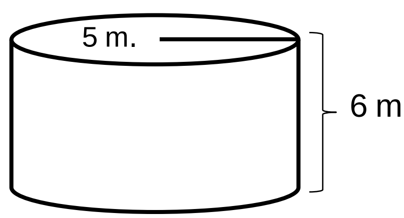
10.g Surface Area: **351.68 cm^2**

10.h Volume: **502.4 cm^3**



10.i Surface Area: **125.6 $ft.^2$**

10.j Volume: **100.48 $ft.^3$**



10.k Surface Area: **345.4 $ft.^2$**

10.l Volume: **471 $ft.^3$**

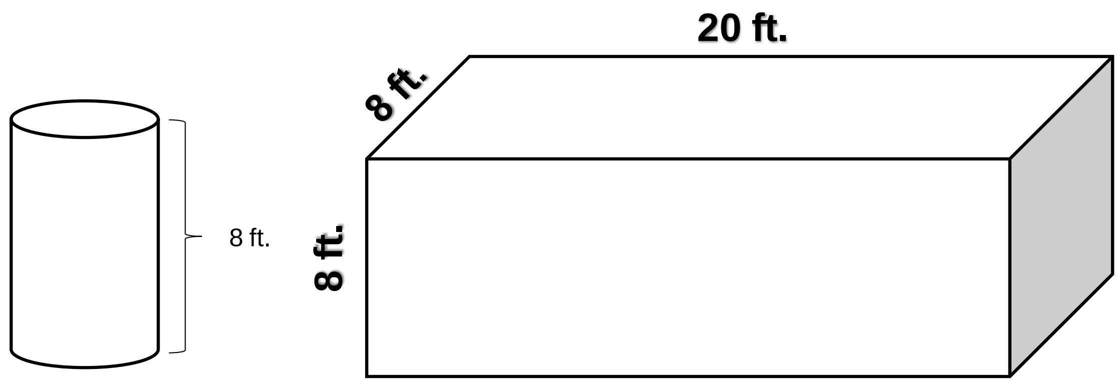
11. Remember our cargo container? Dimensions were 20 ft. x 8 ft. x 8 ft.

a. What is the surface area of the container? **768 ft.^{2}**

b. What is the volume of the container? **1280 ft.^{3}**

c. Suppose we had to ship cylinders each of which is 8 ft. high and has a volume of 25.12 ft.^{3} How many cylinders do you think we could fit into our container? ***Think!***

$\mathbf{25.12 \div 8 = 3.14\ \mathit{ft}.^2}$
$\mathbf{10 \times 4 = 40}$



1. List all formulas.
2. Show all work.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name: ______________________________

Date: ______________________________

Advisor: ____________________________

Geometry of the Ship: Practice 2: Surface Area and Volume of 3 – Dimensional Shapes

Formulas: Use the following formulas to complete this practice exercise:

I. Surface area and volume of a regular prism of base area *B*, base perimeter *P*, and height *H*.

A. **Surface area:** $SA = 2B + PH$
B. **Area:** $V = BH$

II. Square prism with side length *s*:

A. **Base Perimeter:** $P = s + s + s + s = 4s$
B. **Base Area:** $B = s \times s = s^2$

III. Rectangular prism with base length *l* and width *w*:

A. **Perimeter:** $P = l + w + l + w = 2l + 2w$
B. **Area:** $B = l \times w$

IV. Circular prism of radius *r*:

A. **Base Perimeter:** $P = 2 \times \pi \times r = 2\pi r$
B. **Base Area:** $B = \pi \times r \times r = \pi r^2$

V. Triangle of base length *b*, base side length *s*, and base height *h*

A. **Base Perimeter:** $P = 3s$
B. **Base Area:** $A = \left(\frac{1}{2}\right) bh$

Problems: Find the surface area and volume of the following 3 – dimensional shapes. Draw each shape, using ruler and compass, labeling the dimensions. The drawings do not have to match the exact dimensions. Show all work and answers on separate sheets of paper attached to this one.

1. A square prism of side length $s = 7\ cm$.

2. A square prism of side length $s = 12\ mm$.

3. A square prism of side length $s = 22\ ft$.

4. A rectangular prism with base length $l = 6\ in.$, base width $w = 4\ in.$, and height $H = 12\ in.$

5. A rectangular prism with base length $l = 13\ in.$, base width $w = 6\ in.$, and height $H = 7\ in.$

6. A rectangular prism with base length $l = 15\ m.$, base width $w = 7.5\ m.$, and height $H = 7\ m.$

7. A rectangular prism with base length $l = 22\ m.$, base width $w = 110\ cm.$, and height $H = 6.5\ m.$

8. A rectangular with base length $l = 18\ ft.$, base width $w = 108\ in.$, and height $H = 7\ ft.$

9. A circular prism of base radius $r = 2\ in.$ and height $H = 12\ in.$

10. A circular prism of base radius $r = 17\ in.$, and height $H = 40\ in.$

11. A circular prism of base radius $r = 19\ ft.$, and height $H = 360\ in.$

12. A circular prism of base diameter $d = 12\ in.$, and height $H = 12\ ft.$

13. A circular prism of base diameter $d = 42\ cm.$, and height $H = 12\ m.$

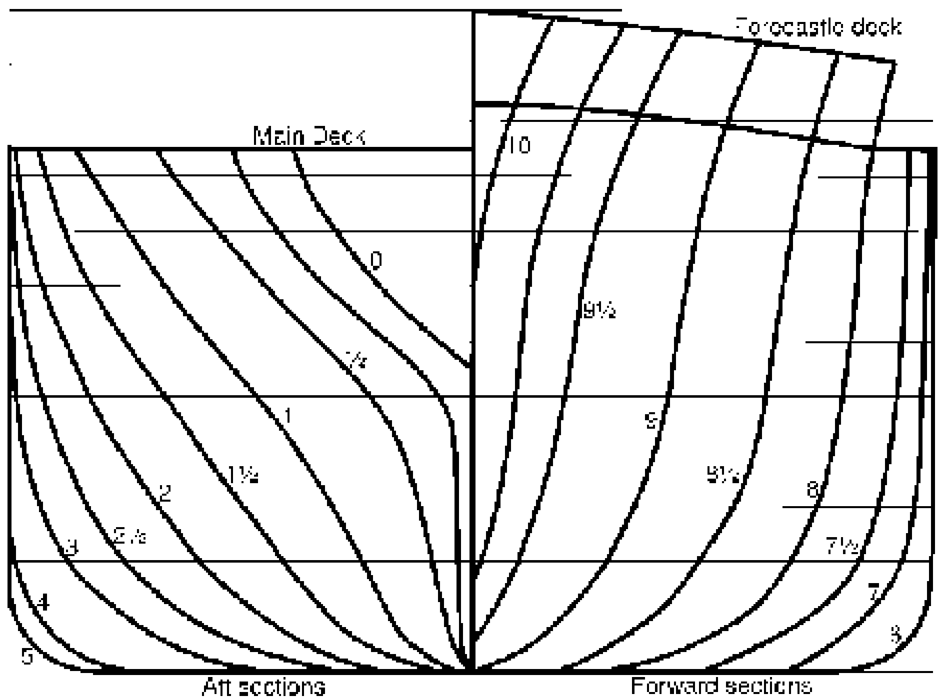
14. A circular prism of base diameter $d = 1\ in.$, and height $H = 12\ in.$

15. A regular triangular prism of base length / base side length $b = s = 2\ cm.$, base height $h = 1.73\ cm.$, and prism height $H = 10\ cm.$

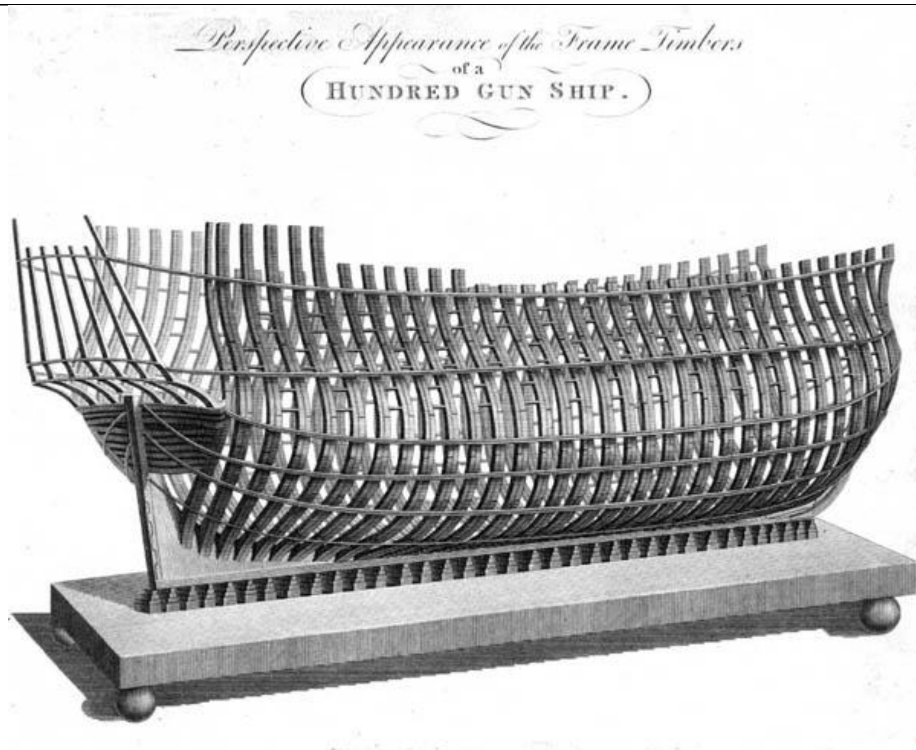
16. A regular triangular prism of base length / base side length $b = s = 12\ in.$, base height $h = 10.39\ in.$, and prism height $H = 10\ cm.$

Grade / Content Area	8^{th} / 9^{th} **Grade Geometry**
Lesson Title	**Building a Ship Model (2 weeks)**
Guiding Question	*How can we interpret a two dimensional plan and translate it into a three dimensional object.*
Content Standards	*State Content Standards:* III. **M(G&M)–10–10:** Demonstrates conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three-dimensional objects, or by solving related problems.
	Common Core Standards: I. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
Preparation	VII. *Classroom Organization:* Students will work individually on this project but can interact to assist each other. VIII. *Materials:* A. "The Geometry of the Ship" task sheets with ship contour drawings copied onto graph paper. B. Poster board. C. Wooden kabob skewers. D. Scissors. E. Glue guns and glue sticks. F. Colored construction paper. G. Newsprint and paste for making papier mache.
Student Learning Objectives	V. Students will draw a precise reflection of a two dimensional shape on graph paper. VI. Students will translate a two dimensional plan of a three dimensional object into a three dimensional model of that object.
Instruction and Engagement	VII. *Warm-up* (**each day for 10 minutes**). VIII. *Launch* (**first day – 15 minutes**). A. I will show the following drawing and ask students if they can determine

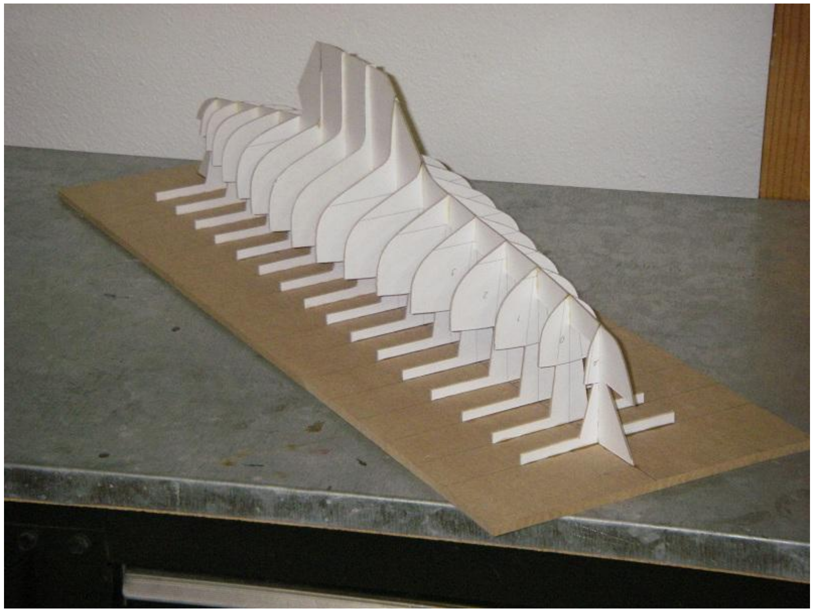
what it is:

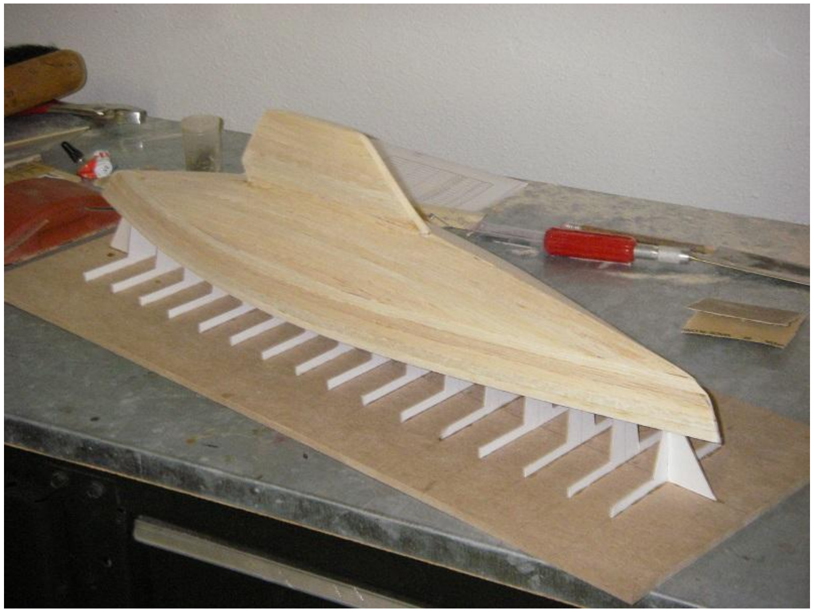


B. I will then show these pictures and tell them these are the three dimensional equivalents of the drawing:

	 C. The drawing is a naval architect's two dimensional drawing, the other pictures are the scale models he makes from the drawing. From the scale models, he makes the ship. IX. *Engagement* (**each day – 60 minutes**). Students will complete their drawings and construct their models in accordance with the attached task sheets.
Assessment	I. Students will meet the standard if they: A. Complete precise reflections of the forward and aft contours in the naval architect's drawing. B. Trace and cutout precise cutouts of the contours. C. Place the contour cutouts on the wooden skewers precisely equidistant. D. Cover the model with papier mache and paint it so that it looks like a ship model.

Body Plan to Model





Name: ____________________
Class: ____________________
Advisor: ____________________

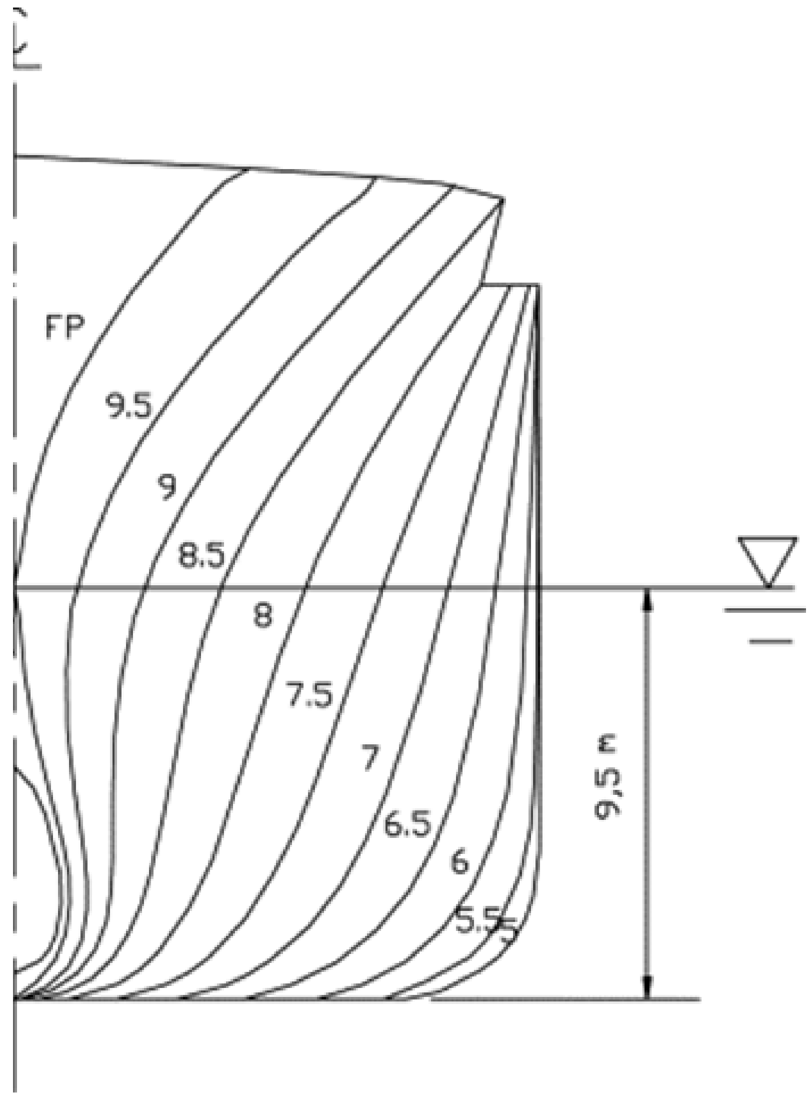
Project: The Geometry of the Ship – Task 1

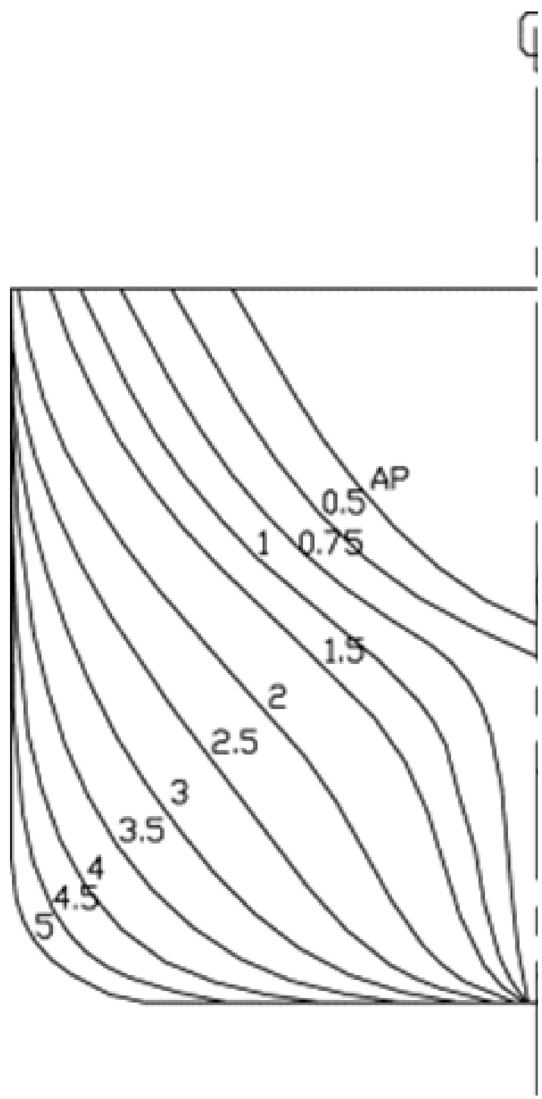
Directions: Each drawing is one half of the body plan of a naval architect's ship drawing. The first drawing depicts ½ of the ship as you would see it if you were standing directly in front of the ship looking at the bow. The second drawing depicts ½ of the ship as you would see it if you were standing directly behind the ship looking at the stern.

Your task today is to produce a precise mirror image of each drawing **of one of the three ships**, connected to the first drawing so that you have a complete view of the plan of the forward part of the ship and the aft part of the ship.

Option 1: Container Ship

	Actual Dimensions	Model Dimensions
Length Overall	965 ft.	36 in.
Maximum Beam	106 ft.	
Maximum Draft	39 ft. 6 in.	

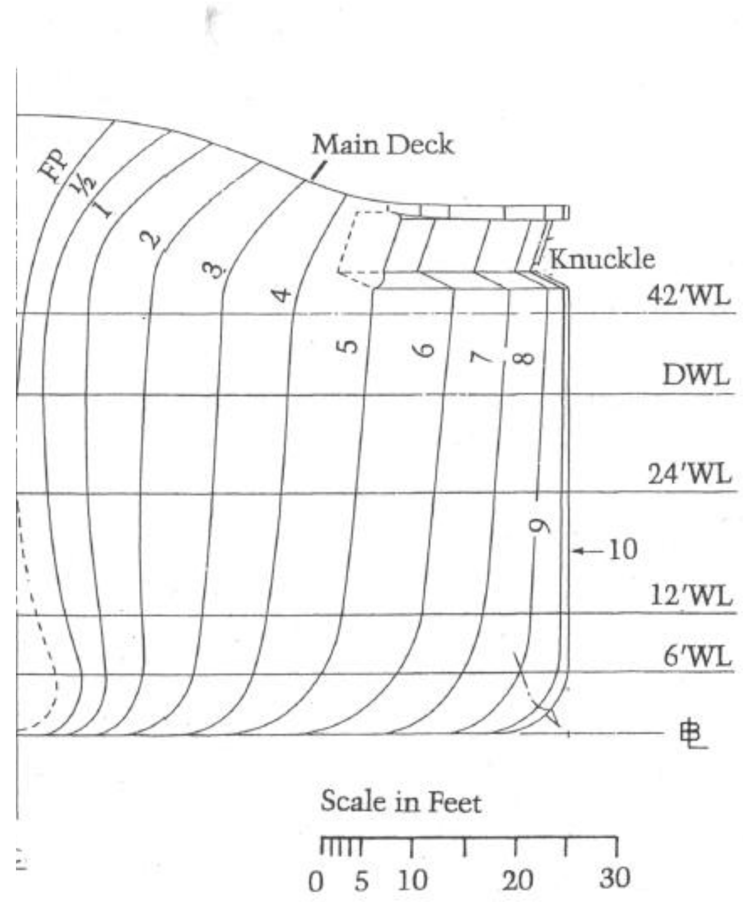


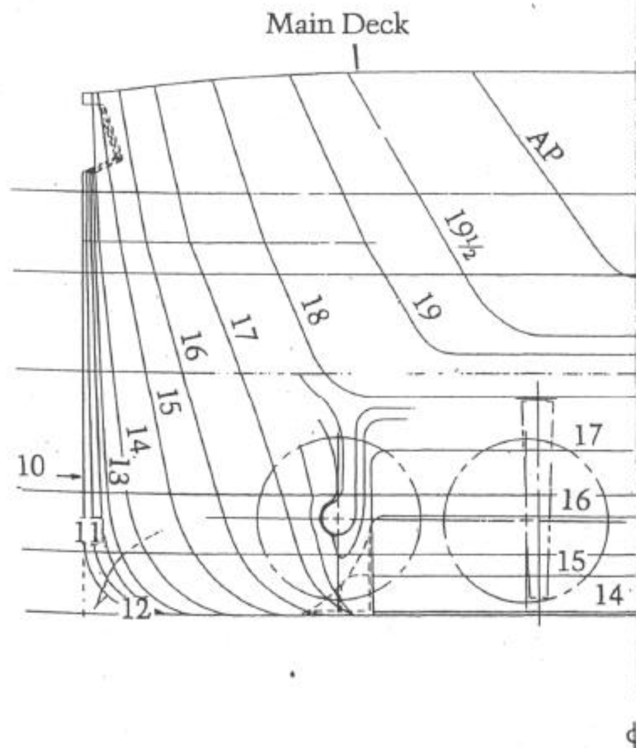


AP
0.5
1
0.75
1.5
2
2.5
3
3.5
4
4.5
5

Option 2: U.S.S. *Massachussets* (Battleship)

	Actual Dimensions	Model Dimensions
Length Overall	680 ft. 9.813 in.	36 in.
Maximum Beam	108 ft., 2.250 in.	
Maximum Draft	36 ft., 9.000 in.	




Main Deck
AP
19½
19
18
17
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15
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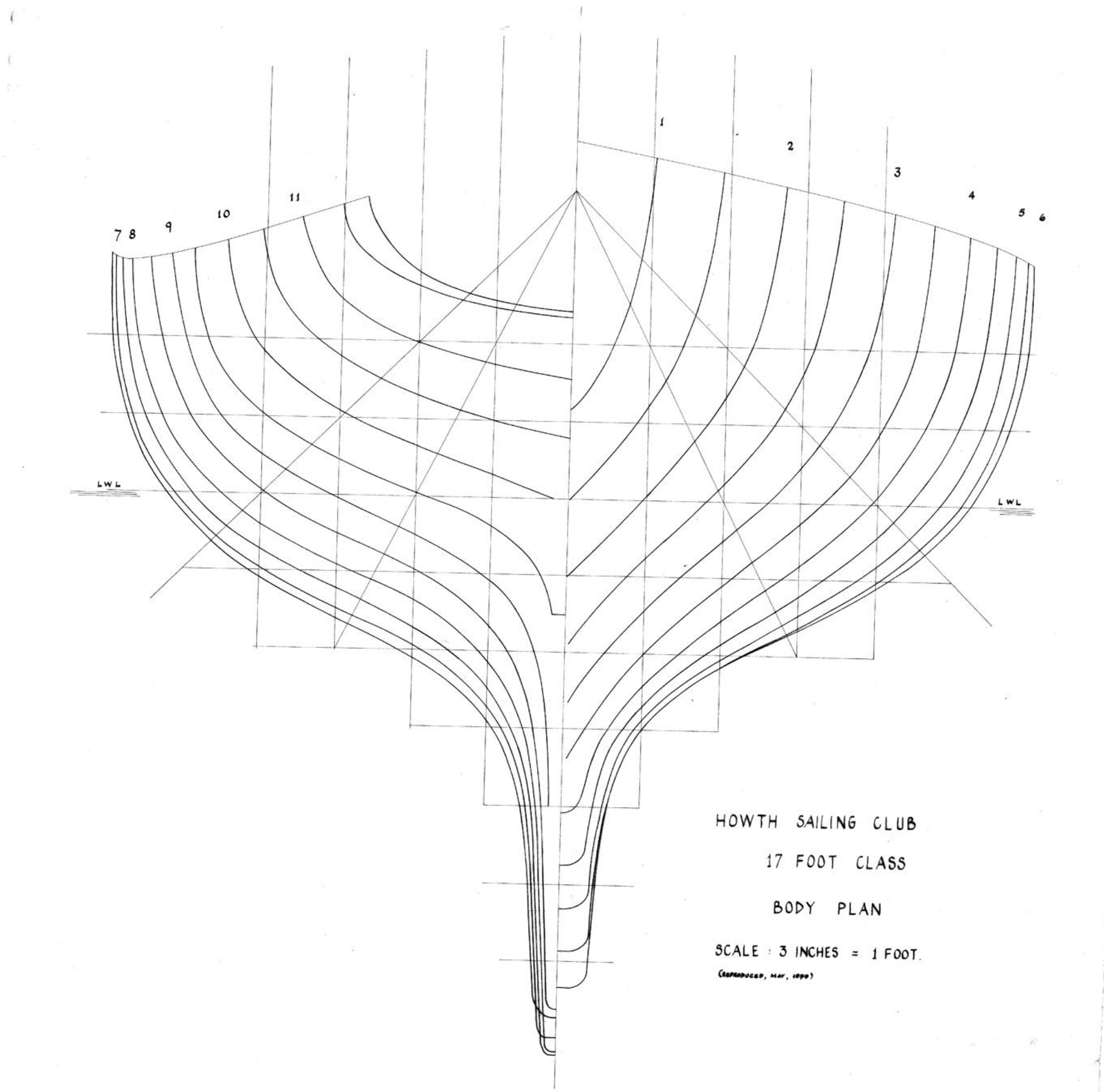
13
10
11
12
17
16
15
14

Option 3: 17 foot Sailing Yacht – Model Dimensions: 36 inches in length



Project: The Geometry of the Ship – Task 2

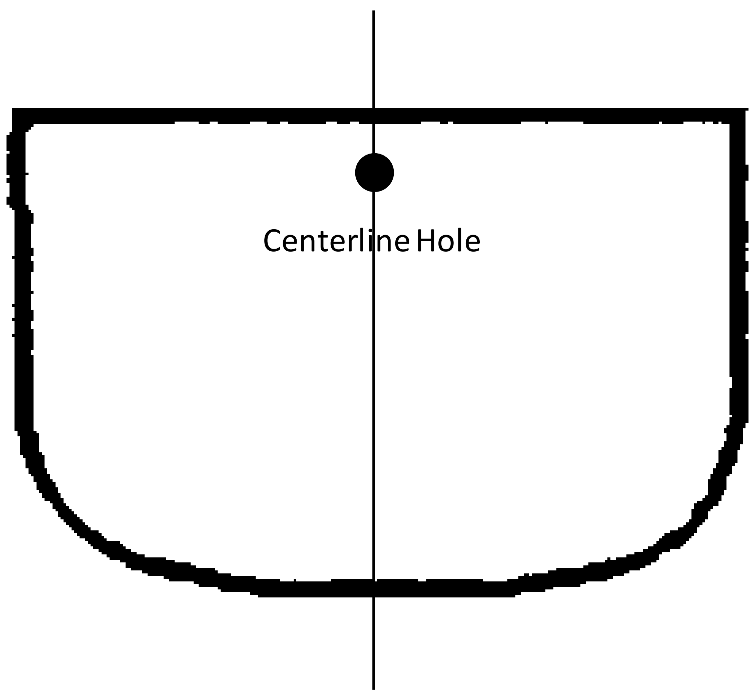
Directions: You are going to construct a builder's model ship from your drawings. When your drawings from task 1 are approved by the instructor:

1. Trace the shape of each contour curve on poster board. ***Be as precise as possible***.

2. Carefully cut each shape from the poster board.

3. Find a point on each piece that is along the centerline, equidistant from the top edge. ***Be as precise as possible.***

    a. Using one of the kabob skewers and / or a scissors, punch a hole in each shape.
    b. You should be able to slide each piece onto the skewer.



    c. Use additional skewers as necessary through different holes to make the ship model sturdy.

    d. Measure and cut a piece of poster board to be glued on top of the model (on the flat side) to act as a top deck.

    e. Cover the model with papier mache, paper, or cardboard and paint it.

Project: The Geometry of the Ship – Task 3

Background: To complete our shipbuilder's models we will construct propellers and rudders for them. This will complete our consideration of the relationship between two-dimensional and three-dimensional shapes which covered the following standards:

1. *Visualize relationships between two-dimensional and three-dimensional objects.*

2. *Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.).*

3. *Demonstrate conceptual understanding of spatial reasoning and visualization by sketching or using dynamic geometric software to generate three-dimensional objects from two-dimensional perspectives, or to generate two-dimensional perspectives from three- dimensional objects, or by solving related problems.*

A ship's propeller(s) (called a screw propeller in nautical terminology) and her rudder(s) are what propel and steer the ship through the water. Just as a window fan moves air into a room, a screw propeller moves water causing the ship to move forward. The water moving across the rudder (which is astern of the screw propeller) makes it possible for the rudder to turn the ship.

Several photos are depicted below.

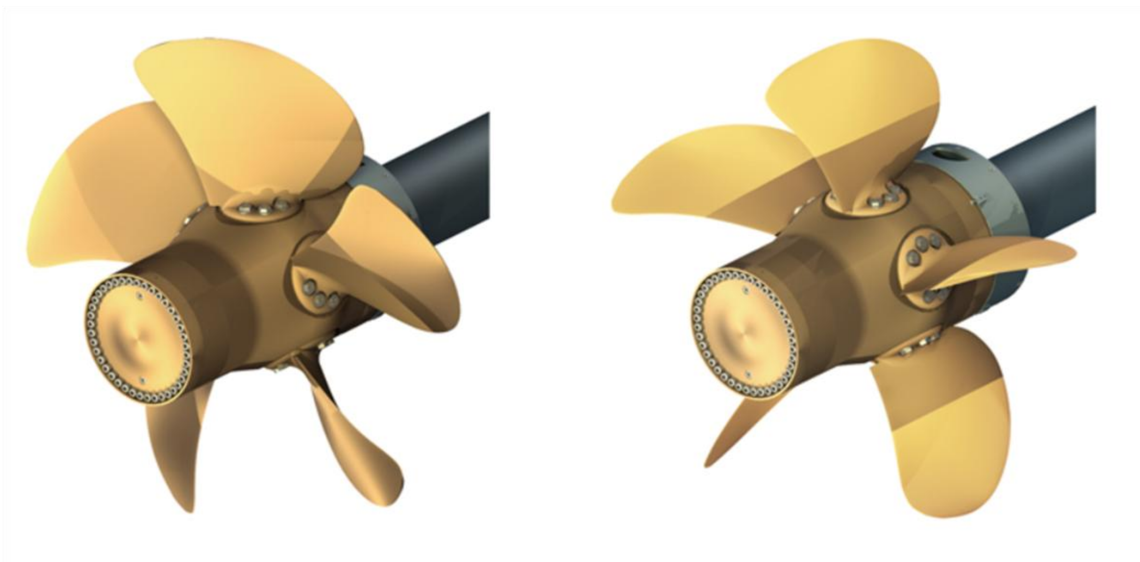
Directions: You will construct a propeller shaft, propeller, and rudder for your ship model.

1. The shaft can be made out of the kabob sticks we have been using.

2. The hub of the shaft should be a cone.

3. The propeller blades should be mounted on the hub 45º from the axis of the shaft as depicted below. There should be four blades, positioned equidistant around the side of the cone.

4. The length of the rudder should be at least the diameter of the circle inscribed by the propeller blades.



Blades 45 deg offset from axis of shaft

Blades 0 deg offset from axis of shaft

<table>
<tr><td>Grade / Content Area</td><td>8th / 9th Grade Geometry</td></tr>
<tr><td>Lesson Title</td><td>“Why Does a Ship Float?” (1 day)</td></tr>
<tr><td>Guiding Question</td><td>“Why does a ship float and what do we need to consider when loading it so it won’t sink?”</td></tr>
<tr><td rowspan="3">Content Standards</td><td>State Content Standards:

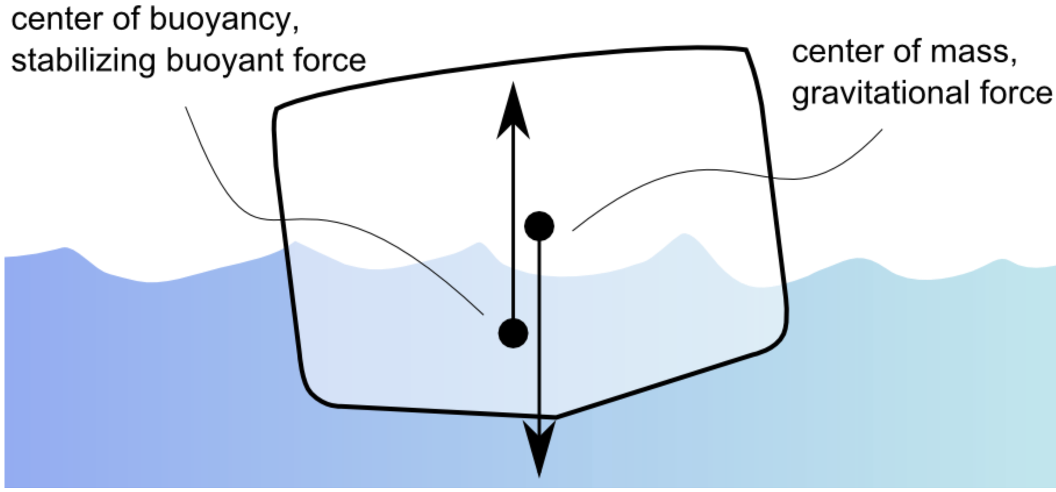
I. M(G&M)–8–5: Applies concepts of similarity to determine the impact of scaling on the volume or surface area of three-dimensional figures when linear dimensions are multiplied by a constant factor; to determine the length of sides of similar triangles, or to solve problems involving growth and rate.

II. M(G&M)–8–6: Demonstrates conceptual understanding of surface area or volume by solving problems involving surface area and volume of rectangular prisms, triangular prisms, cylinders, pyramids, or cones. Expresses all measures using appropriate units.

III. M(G&M)-10-2: Makes and defends conjectures, constructs geometric arguments, uses geometric properties or uses theorems to solve problems involving angles, lines, polygons, circles, or right triangle ratios (sine, cosine, tangent) within mathematics or across disciplines or contexts (e.g. Pythagorean Theorem, Triangle Inequality Theorem).</td></tr>
<tr><td>NCTM Standards: Middle and high school students should:
I. Analyze characteristics: precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties; understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects; create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.

II. Use visualization: draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.</td></tr>
<tr><td>Common Core Standards: None for this lesson.</td></tr>
<tr><td>Preparation</td><td>I. Classroom Organization. Students will work in groups of four in a room with sufficient sinks to provide one for each group.</td></tr>
</table>

	II. *Differentiation.* There are many things in this lesson that will appeal to the multiple intelligences of the students: specifically new terminology and hands-on exercises. This lesson also does not involve group work which may be appealing to more introverted students. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials*: A. 4 – 5 sinks filled with water. B. 5 – 7 wooden boards approximately 12" × 4" × 2" (from Home Depot). Each board should have a very long nail hammered into the center to measure the angle of heel. C. *Algeblocks* to represent container boxes and other items stowed on a ship's deck. D. Protractors to measure angle of heel.
Student Learning Objectives	I. Students will apply prior learning about volume and liquid measures to the problem of loading a ship in such a way as not to impair its stability. II. Students will understand that the stability problem involves a geometric relationship and the application of what they have learned in this unit.
Instruction and Engagement	I. *Warm-up* (**10 minutes**): II. *Opening* (**10 minutes**). I will begin with the question, *"Why does a ship float?"* After discussing that for a few minutes, I will introduce some vocabulary before conducting experiments. A. **Center of mass (a.k.a. center of gravity):** The point through which the force due to gravity, that is the weight of the body, acts. B. **Buoyancy:** The upward force acting on a floating or submerged body due to the water pressures on its boundary. C. **Center of buoyancy.** That point through which the buoyancy force acts. I will then show this diagram on a slide and ask the guiding question again. This will lead us into our experiment.



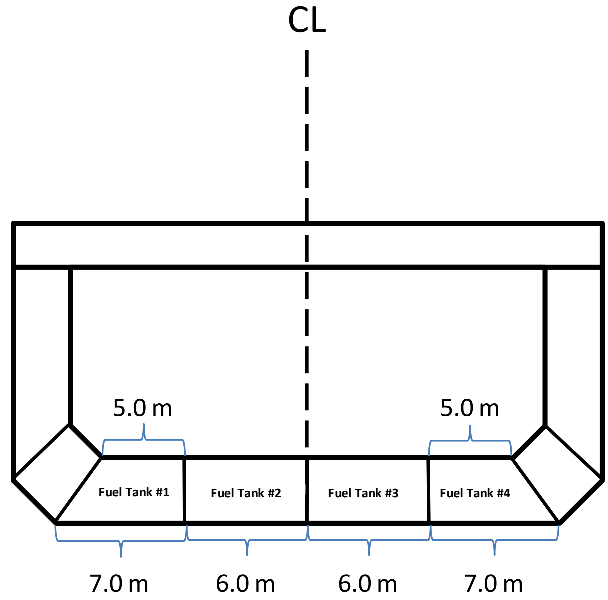
III. *Engagement:* We will perform some experiments in class and see if we can form any hypotheses from them.

A. Each group will work at a sink, equipped with the materials set forth above. They will identify the center of gravity (where the nail is hammered into the board. They will then place *Algeblocks* to determine:

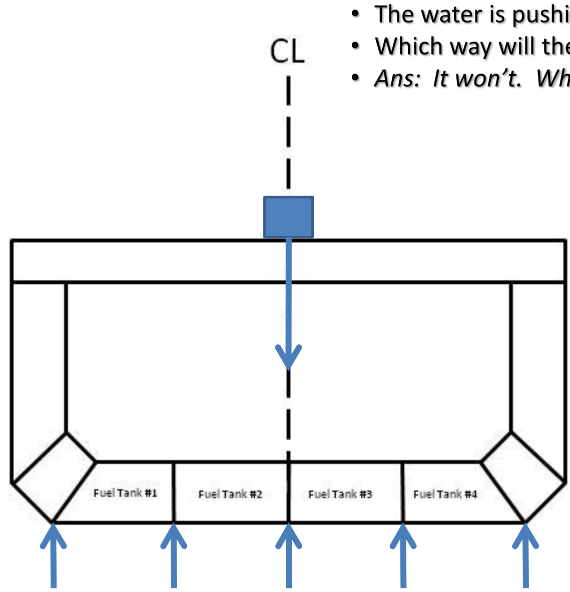
1. How many are necessary to sink the board.

2. How many and where they are placed to capsize the board. Students will measure the angle the board "heels" (using the nail and a protractor), finding the angle at which the board capsizes.

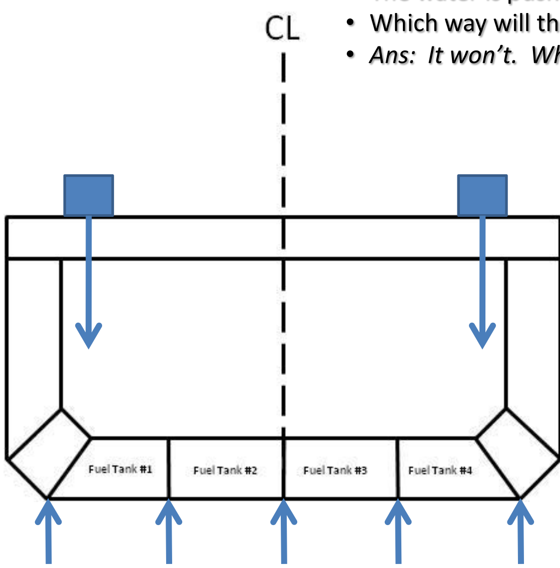
B. I will then ask students to consider the following diagrams (which we saw before in the last lesson) and answer the questions on them:

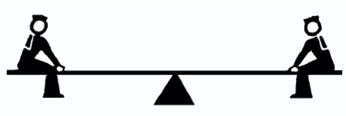


- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*

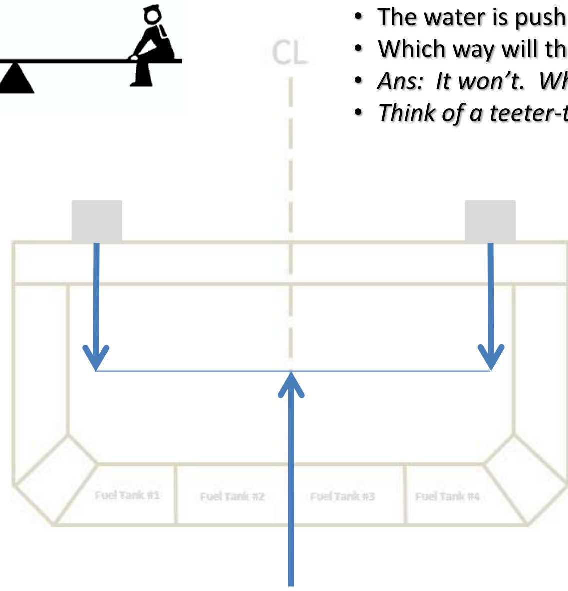


- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*

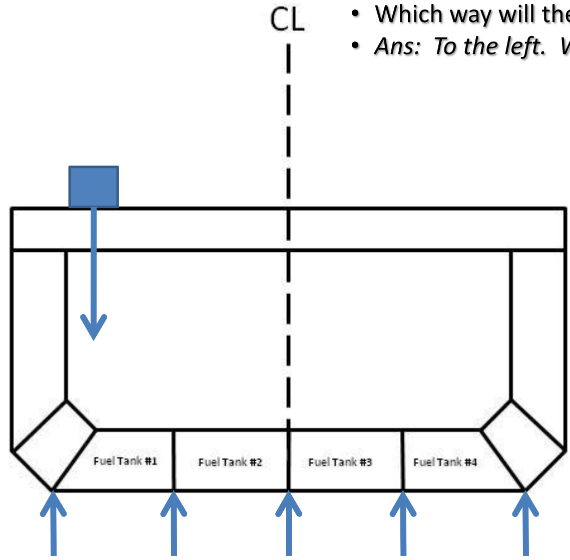


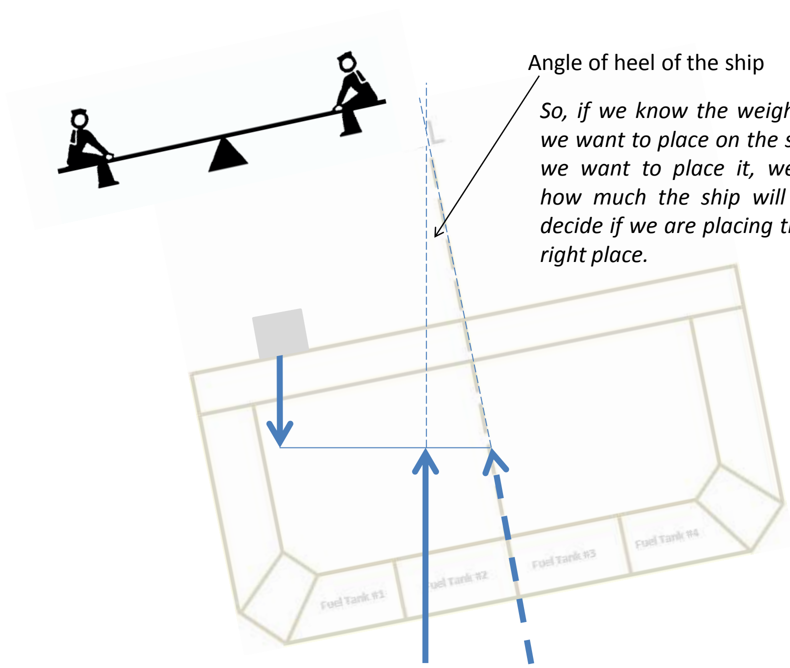


- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: It won't. Why?*
- *Think of a teeter-totter.*



- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: To the left. Why?*





So, if we know the weight of the object we want to place on the ship, and where we want to place it, we can calculate how much the ship will heel over and decide if we are placing the object in the right place.

IV. *Closing.* I will close with the conclusion that what we have is a geometric relationship among the various forces acting on the ship. I will ask the question: *"How can we use what we have learned thus far to help us determine how best to load our ship?"* The answer is that if we can compute the volumes and weights of the objects we put into our packing crates and then into our containers and then onto our ship, we can determine how best to load

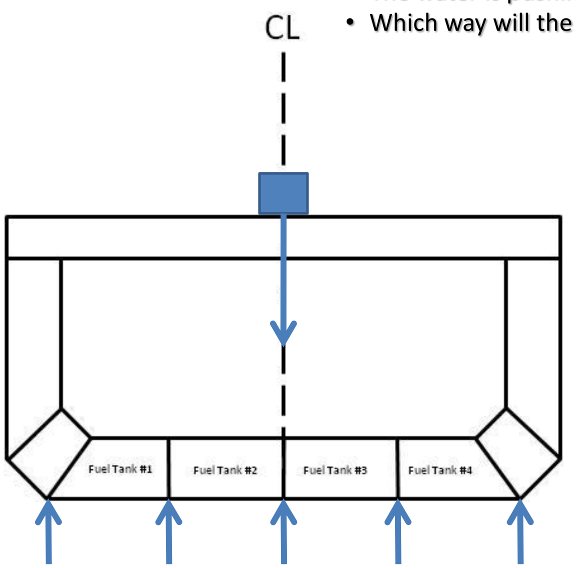
	the ship. *"That is what shippers and sea officers do every day with the hundreds of ships loaded every day."*
Assessment	While this lessons objectives and content are, to some degree, outside of the objectives of this unit, students will be required to draw on their prior learning in this unit to engage in the discussion. During the discussion, I will assess their mastery of the required concepts. I will only assess students on the material relevant to this unit in the summative assessment.

Name: ____________________
Date: ____________________
Class: ____________________

"How Does a Ship Float?"

Directions: Answer the question next to each diagram on the lines to the right of the diagram.

- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?



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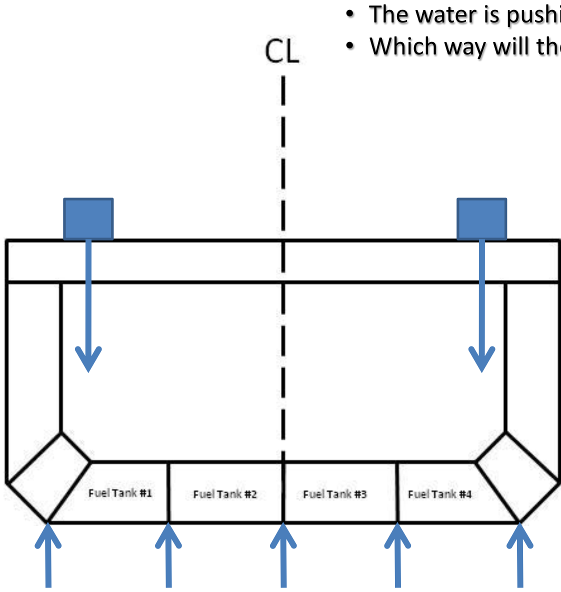
- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?



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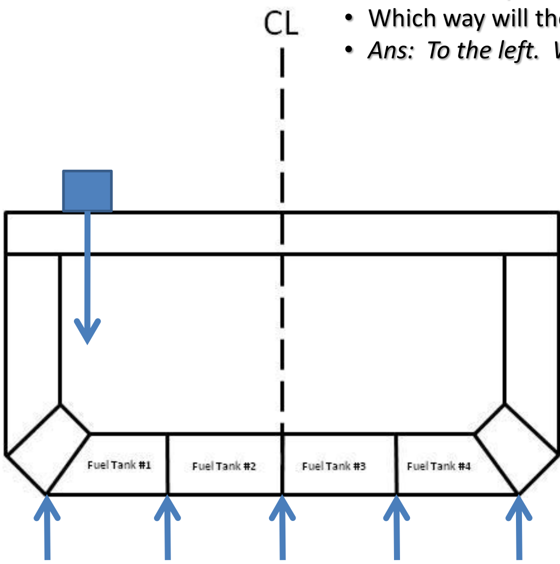

- The container is pushing down.
- The water is pushing up.
- Which way will the ship heel?
- *Ans: To the left. Why?*

____

____

____

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____

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2. ft^3. etc.)

Name:____________________
Date:_____________________
Class:_____________________

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

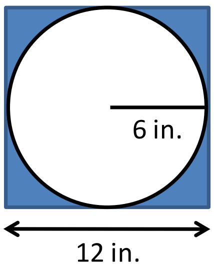
1. **Definitions:** Place the letter of the term next to the best matching definition.

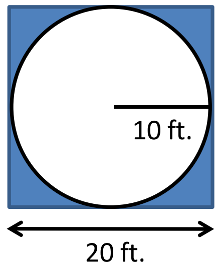
a. Circumference:	_____	Line segment whose endpoints lie on the circle.
b. Radius:	_____	The sum of the areas of the surfaces of a three-dimensional object.
c. Diameter:	_____	The distance around a closed curve such as a circle.
d. Chord:	_____	Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
e. Volume:	_____	Any line segment from the center of a circle to its perimeter.
f. Surface Area:	_____	The amount of space, inside and out, a solid body occupies.

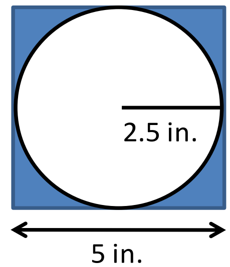
2. What is the formula for the **circumference** of a circle? ________________
3. What is the formula for the **area** of a circle ________________
4. What is the formula for the **surface area** of a rectangular prism? ________________
5. What is the formula for the **volume** of a rectangular prism? ________________
6. What is the formula for the **surface area** of a cylinder? ________________
7. What is the formula for the **volume** of a cylinder? _________________
8. What is the general formula for the **volume** of a regular prism or cylinder? _________________
9. What is the general formula for the **surface area** of a regular prism or cylinder?

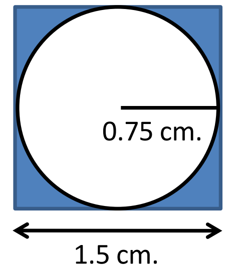
____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)









a Circle Area: ________ d Circle Area: ________ g Circle Area: ________ j Circle Area: ________

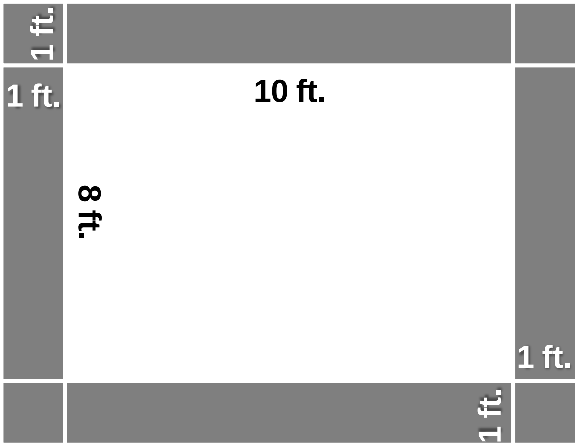
b Square Area: ________ e Square Area:________ h Square Area:________ k Square Area: _______

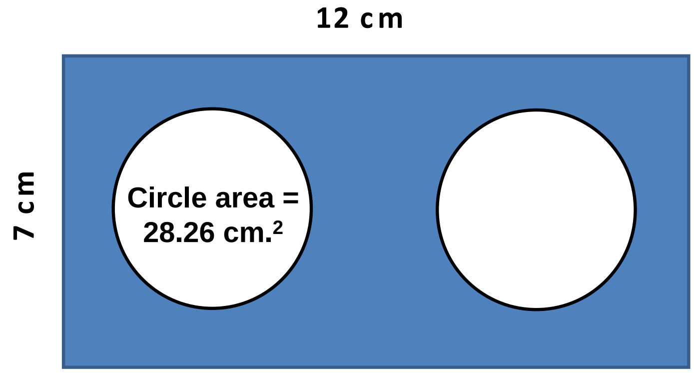
c Ratio: ________ f Ratio: ________ i Ratio: ________ l Ratio: ________

m Are c, f, i, and l the same? **Yes** **No** (circle one)

n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



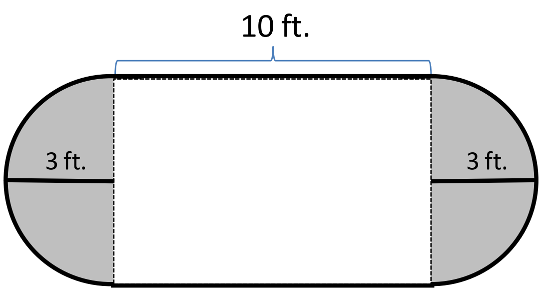


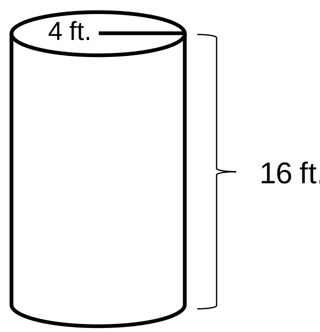
a Total Surface Area: ________

b Area of Shaded Region: ________

c Area of Shaded Region: ________

d Radius of each Circle: ________



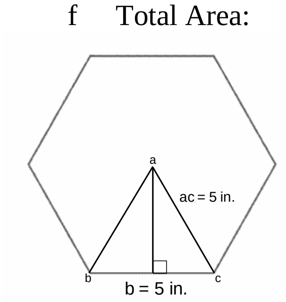


e Area of Shaded Region: ________

f Total Area: ________

g Surface Area: ________

h Volume: ________



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

i Surface Area: ________

j Volume: ________

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? ________________

b. What is the volume of a Coca-Cola can? ________________

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 $in.^3$? ***Think! Remember fitting shapes into the containers. Draw a picture.***

____

d. What is the total volume of all these Coca-Cola cans?

____

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

____

f. ***Extra Credit:*** *Is this the answer you expected? Why or why not?*

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2. ft^3. etc.)

Differentiated Version

Name:____________________

Date:____________________

Class:____________________

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions:** Place the letter of the term next to its definition.

   a. Circumference: _____ Line segment whose endpoints lie on the circle.

   b. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: _____ The distance around a closed curve such as a circle.

   d. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   e. Volume: _____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? _______________

3. What is the formula for the **area** of a circle _______________

4. What is the formula for the **surface area** of a rectangular prism? _______________

5. What is the formula for the **volume** of a rectangular prism? _______________

6. What is the formula for the **surface area** of a cylinder? _______________

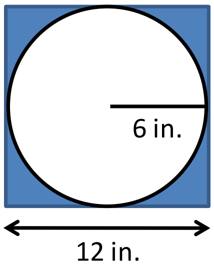
7. What is the formula for the **volume** of a cylinder? _______________

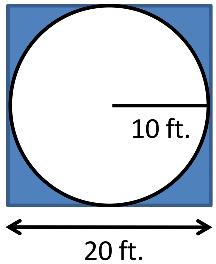
8. What is the general formula for the **volume** of a regular prism or cylinder? _______________

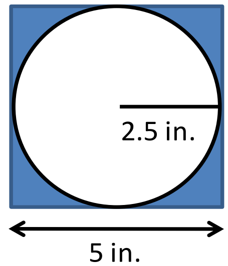
9. What is the general formula for the **surface area** of a regular prism or cylinder?

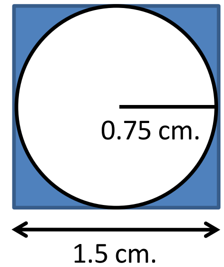
____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)









a Circle Area: ________ d Circle Area: ________ g Circle Area: ________ j Circle Area: ________

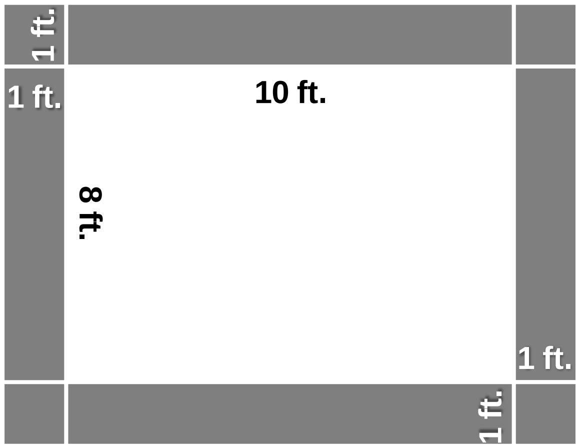
b Square Area: ________ e Square Area:________ h Square Area:________ k Square Area: _______

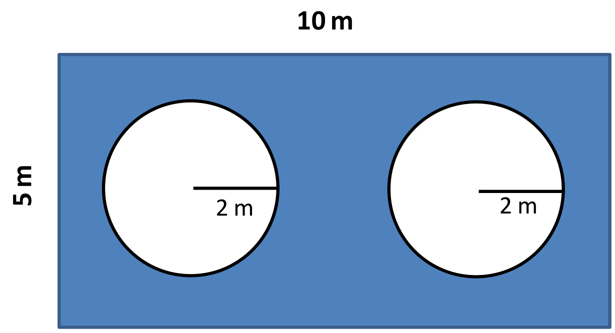
c Ratio: ________ f Ratio: ________ i Ratio: ________ l Ratio: ________

m Are c, f, i, and l the same? **Yes** **No** (circle one)

n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



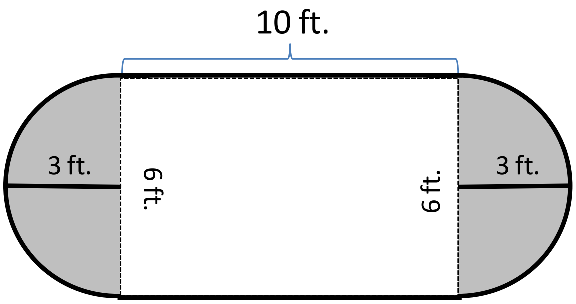


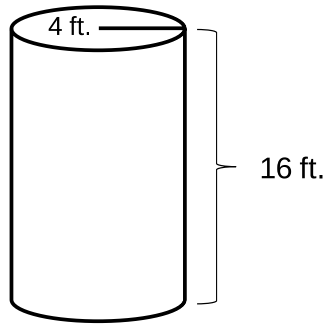
a Total Surface Area: ________

b Area of Shaded Region: ________

c Area of Each Circle: ________

d Area of Shaded Region ________



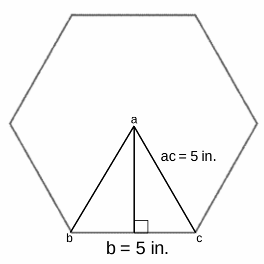


e Area of Shaded Region: ________

f Total Area: ________

g Surface Area: ________

h Volume: ________



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

i Surface Area: _______

j Volume: _______

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? ________________

b. What is the volume of a Coca-Cola can? ________________

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 $in.^3$? ***Think! Remember fitting boxes into the containers. Draw a picture.***

____

d. What is the total volume of all these Coca-Cola cans?

____

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

____

f. ***Extra Credit:*** Is this the answer you expected? Why or why not?

Answer Key

Assessment Review: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. Definitions.

   a. Circumference: **d**_____ Line segment whose endpoints lie on the circle.

   b. Radius: **f**_____ The sum of the areas of the surfaces of a three-dimensional object.

   c. Diameter: **a**_____ The distance around a closed curve such as a circle.

   d. Chord: **c**_____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   e. Volume: **b**_____ Any line segment from the center of a circle to its perimeter.

   f. Surface Area: **e**_____ The amount of space, inside and out, a solid body occupies.

2. What is the formula for the **circumference** of a circle? _________________

3. What is the formula for the **area** of a circle _________________

4. What is the formula for the **surface area** of a rectangular prism? _________________

5. What is the formula for the **volume** of a rectangular prism? _________________

6. What is the formula for the **surface area** of a cylinder? _________________

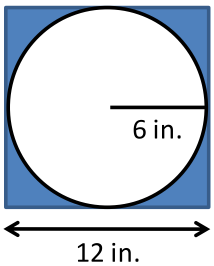
7. What is the formula for the **volume** of a cylinder? __________________

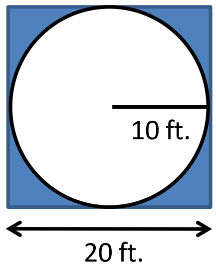
8. What is the general formula for the **volume** of a regular prism or cylinder? __________________

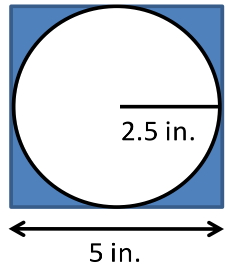
9. What is the general formula for the **surface area** of a regular prism or cylinder?

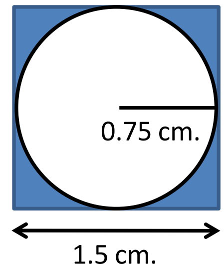
____

10. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)







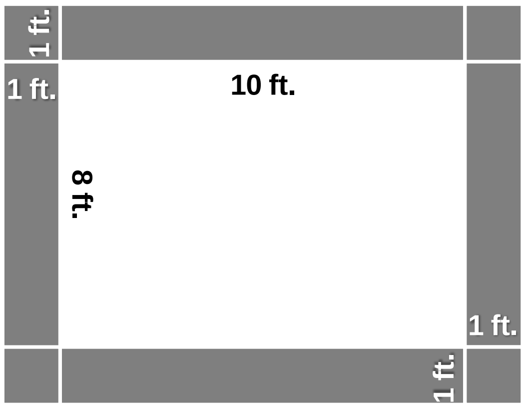


a	Circle Area:	**113.04 in^2**	d	Circle Area:	**314 ft.^{2}**	g	Circle Area:	**19.625 in.^{2}**	j	Circle Area:	**1.77 cm.^{2}**
b	Square Area:	**144 in^2**	e	Square Area:	**400 ft^2**	h	Square Area:	**25 ft.^{2}**	k	Square Area:	**2.25 cm.^{2}**
c	Ratio:	**0.785**	f	Ratio:	**0.785**	i	Ratio:	**0.785**	l	Ratio:	**0.785**

m Are c, f, i, and l the same? **Yes** **No** (circle one)

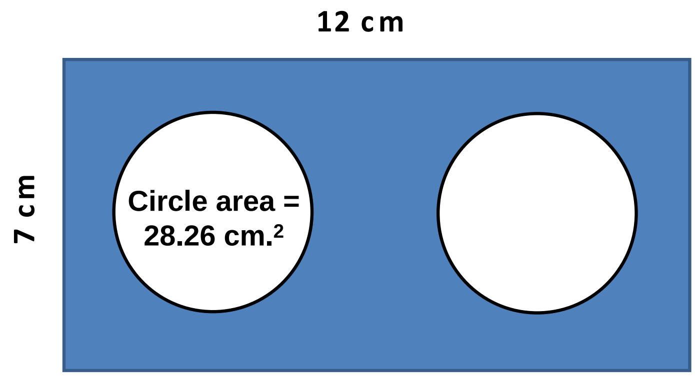
n In your opinion, why do you think the ratios are the same (or different)?

11. Find the area and volume (if asked for) of the following shapes:



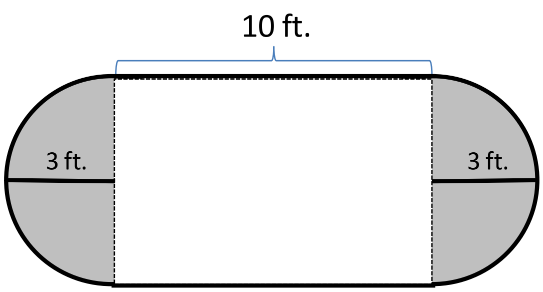
a Total Surface Area: **120 ft.^{2}**

b Area of Shaded Region: **40 ft.^{2}**



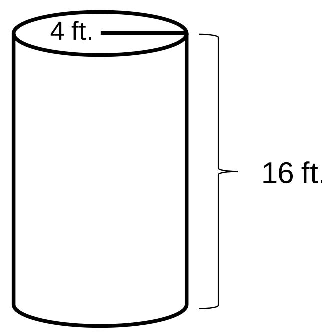
c Area of Shaded Region: **26.8 cm.^{2}**

d Radius of Each Circle **3 cm.**



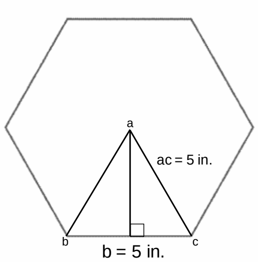
e Area of Shaded Region: **28.26 ft.^{2}**

f Total Area: **88.26 ft.^{2}**



g Surface Area: **502.4 ft.^{2}**

h Volume: **803.84 ft.^{3}**



Regular hexagonal prism with this hexagon as its base and a height of 40 in.

9.i Surface Area:

9.j Volume:

12. A Coca-Cola can has the following dimensions:

Diameter = 2.25 in.
Height = 5 in.

a. What is the surface area of a Coca-Cola can? **43.27 in.^{2}**

b. What is the volume of a Coca-Cola can? **19.87 in.^{2}**

c. How many Coca-Cola cans fit into a shipping crate that is 27 in. x 18 in. x 10 in. and has a volume of 4860 in.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

192

d. What is the total volume of all these Coca-Cola cans?

3815.04 in.^{3}

e. What is the ratio of the volume of all the Coca-Cola cans to the volume of the container?

0.785

g. ***Extra Credit:*** Is this the answer you expected? Why or why not?

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Name:____________________
Date:_____________________
Advisor:__________________

Assessment: Surface Area and Volume of 3D Shapes

SHOW ALL WORK ON A SEPARATE SHEET OF PAPER
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

* 2 – level Problems *

1. **Definitions (Matching).**

   a. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

   b. Diameter: _____ The distance around a closed curve such as a circle.

   c. Volume: _____ Any line segment from the center of a circle to its perimeter.

   d. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

   e. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

   f. Circumference: _____ Line segment whose endpoints lie on the circle.

2. **Formulas.**

   a. What is the formula for the **circumference** of a circle? _________________

   b. What is the formula for the **area** of a circle? _________________

   c. What is the formula for the **surface area** of a rectangular prism? _________________

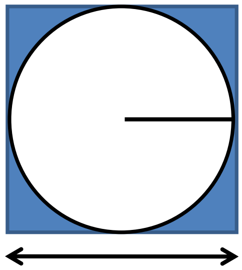
   d. What is the formula for the **volume** of a rectangular prism? _________________

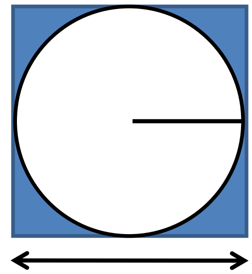
   e. What is the formula for the **surface area** of a cylinder? _________________

   f. What is the general formula for the **surface area** of a regular prism? _________________

Round all answers to the nearest hundredth

3. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)





3.a Circle Area: _______

3.b Square Area: _______

3.c Ratio: _______

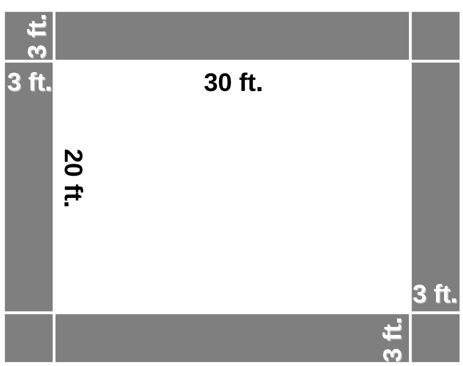
3.d Circle Area: _______

3.e Square Area: _______

3.f Ratio: _______

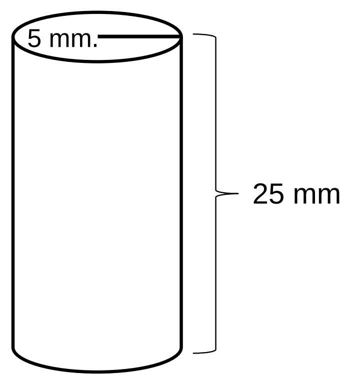
* 3 – level Problems *

4. Find the area and volume (if asked for) of the following shapes:



4.a Total Area: _______

4.b Area of Shaded Region: _______



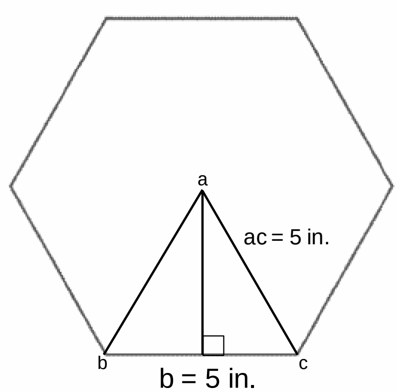
4.c Surface Area: _______

4.d Volume: _______

Regular hexagonal prism with this hexagon as its base and a height of 40 in.

4.e Surface Area: _______

4.f Volume: _______



Round all answers to the nearest hundredth

* 4 – level Problems *

5. A tennis ball can has the following dimensions:

Diameter = 6.6 cm.
Height = 20 cm.

a. What is the surface area of a tennis ball can? ________________

b. What is the volume of a tennis ball can? ________________

c. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 $cm.^3$?

Draw a picture before you try to figure it out.

____

d. What is the total volume of all these tennis ball cans?

____

e. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

____

f. Is this the answer you expected? Why or why not?

1. List all formulas.
2. Show all work on a separate sheet.
3. Ensure you include units of measure (in^2, ft^3, etc.)

Differentiated Version

Name:____________________
Date:______________________
Advisor:___________________

Test: Circle Area and Circumference / Surface Area and Volume of Cylindrical Prisms
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions.**

a. Chord: _____ Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.

b. Diameter: _____ The distance around a closed curve such as a circle.

c. Volume: _____ Any line segment from the center of a circle to its perimeter.

d. Radius: _____ The sum of the areas of the surfaces of a three-dimensional object.

e. Surface Area: _____ The amount of space, inside and out, a solid body occupies.

f. Circumference: _____ Line segment whose endpoints lie on the circle.

2. **Formulas.**

a. What is the formula for the **circumference** of a circle? _________________

b. What is the formula for the **area** of a circle? _________________

c. What is the formula for the **surface area** of a rectangular prism? _________________

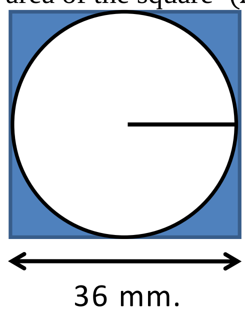
d. What is the formula for the **volume** of a rectangular prism? _________________

e. What is the formula for the **surface area** of a cylinder? _________________

f. What is the general formula for the **surface area** of a regular prism? _________________

Round all answers to the nearest hundredth

6. Find the area of the square and the inscribed circle. What is the **ratio** of the area of the circle to the area of the square $(A_{\text{circle}} \div A_{\text{square}} = ?)$

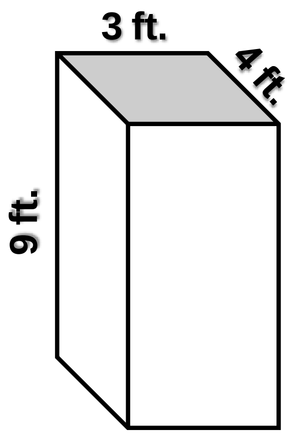


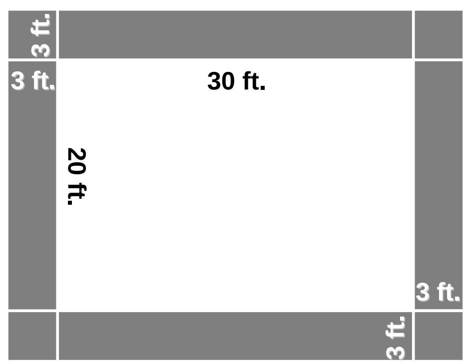
3.a Circle Area: _______

3.b Square Area: _______

3.c Ratio: **100%** **75%** **50%** **(circle one)**

7. Find the area and volume (if asked for) of the following shapes:





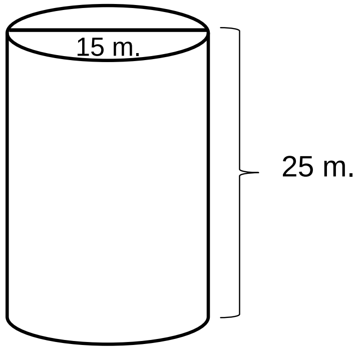
4.a Surface Area: _______

4.b Volume: _______

4.c Total Surface Area: _______

4.d Area of Shaded Region: _______

Round all answers to the nearest hundredth



4.e Surface Are: _______

4.f Volume: _______

Round all answers to the nearest hundredth

8. A tennis ball can has the following dimensions:

   Diameter = 6.6 cm.
   Height = 20 cm.

   f. What is the surface area of a tennis ball can? ________________

   g. What is the volume of a tennis ball can? ________________

   ************* Do c, d, e, and f for extra credit ***********

   h. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 $cm.^3$? ***Think! Remember fitting boxes into the containers. Draw a picture.***

   ____

   i. What is the total volume of all these tennis ball cans?

   ____

   j. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

   ____

   g. Is this the answer you expected? Why or why not?

Round all answers to the nearest hundredth

Assessment: Circle Area and Circumference / Surface Area and Volume of Cylindrical Prisms
PLACE YOUR ANSWERS IN THE SPACES ON THIS SHEET

1. **Definitions.**

a.	Chord:	**b**_____	Any straight line segment that passes through the center of the circle and whose endpoints are on the circle.
b.	Diameter:	**f**_____	The distance around a closed curve such as a circle.
c.	Volume:	**d**_____	Any line segment from the center of a circle to its perimeter.
d.	Radius:	**e**_____	The sum of the areas of the surfaces of a three-dimensional object.
e.	Surface Area:	**c**_____	The amount of space, inside and out, a solid body occupies.
f.	Circumference:	**a**_____	Line segment whose endpoints lie on the circle.

2. **Formulas.**

a. What are the two formulas for the **circumference** of a circle? $\mathbf{2 \times \pi \times r}$ or $\mathbf{\pi \times d}$

b. What is the formula for the **area** of a circle? $\mathbf{\pi \times r^2}$

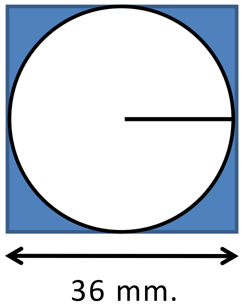
c. What are the two formulas for the **volume** of a rectangular prism? $\mathbf{l \times w \times h}$ or $\mathbf{B \times h}$ where $\mathbf{B = l \times w}$

d. What is the formula for the **surface area** of a cylinder? $\mathbf{(2 \times \pi \times r^2) + (2 \times \pi \times h)}$

e. What are the two formulas for the **volume** of a cylinder? $\mathbf{B \times h}$ **or** $\mathbf{\pi \times r^2 \times h}$

Round all answers to the nearest hundredth

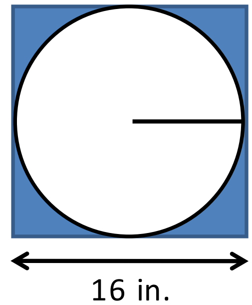
3. Find the area of the square and the inscribed circle. What is the **ratio** of the area of each circle to the area of each square ($A_{\text{circle}} \div A_{\text{square}} = ?$)



3.a Circle Area: **1017.36 mm^2**

3.b Square Area: **1296 mm^2**

3.c Ratio: **0.79**

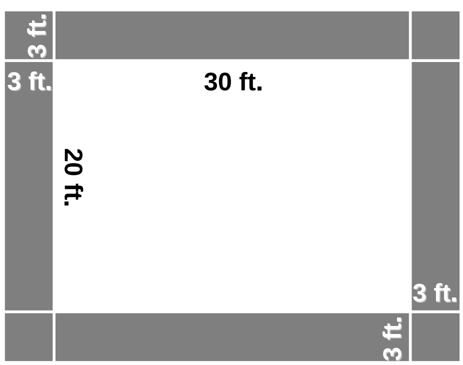


3.d Circle Area: **200.96 in^2**

3.e Square Area: **256 in^2**

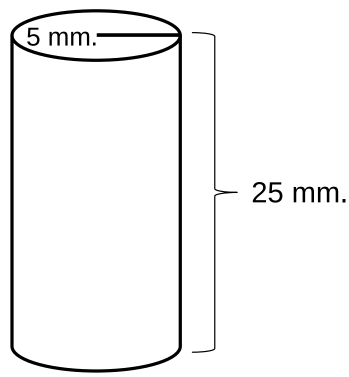
3.f Ratio: **0.79**

4. Find the area and volume (if asked for) of the following shapes:



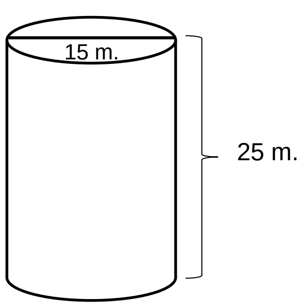
4.a Total Surface Area: **936 ft.^{2}**

4.b Area of Shaded Region: **336 ft.^{2}**



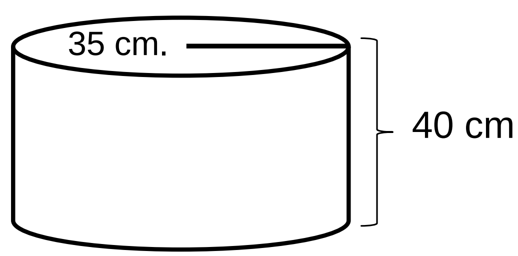
4.c Surface Area: **942 mm.^{2}**

4.d Volume: **1962.5 mm.^{3}**



4.e Surface Area: **1530.75 m.^{2}**

4.f Volume: **4415.6 m^3**



4.g Surface Area: **16,485 cm.^{2}**

4.h Volume: **153,860 cm.^{3}**

5. A tennis ball can has the following dimensions:

Diameter = 6.6 cm.
Height = 20 cm.

a. What is the surface area of a tennis ball can? **482.87 cm.^{2}**

b. What is the volume of a tennis ball can? **683.89 cm.^{3}**

c. How many tennis ball cans fit into a shipping box that is 26.4 cm. x 13.2 cm. x 40 cm. and has a volume of 13,939.2 cm.^{3}? ***Think! Remember fitting boxes into the containers. Draw a picture.***

16 cans

d. What is the total volume of all these tennis ball cans?

10,942.3 cm.^{3}

e. What is the ratio of the volume of all the tennis ball cans to the volume of the shipping box?

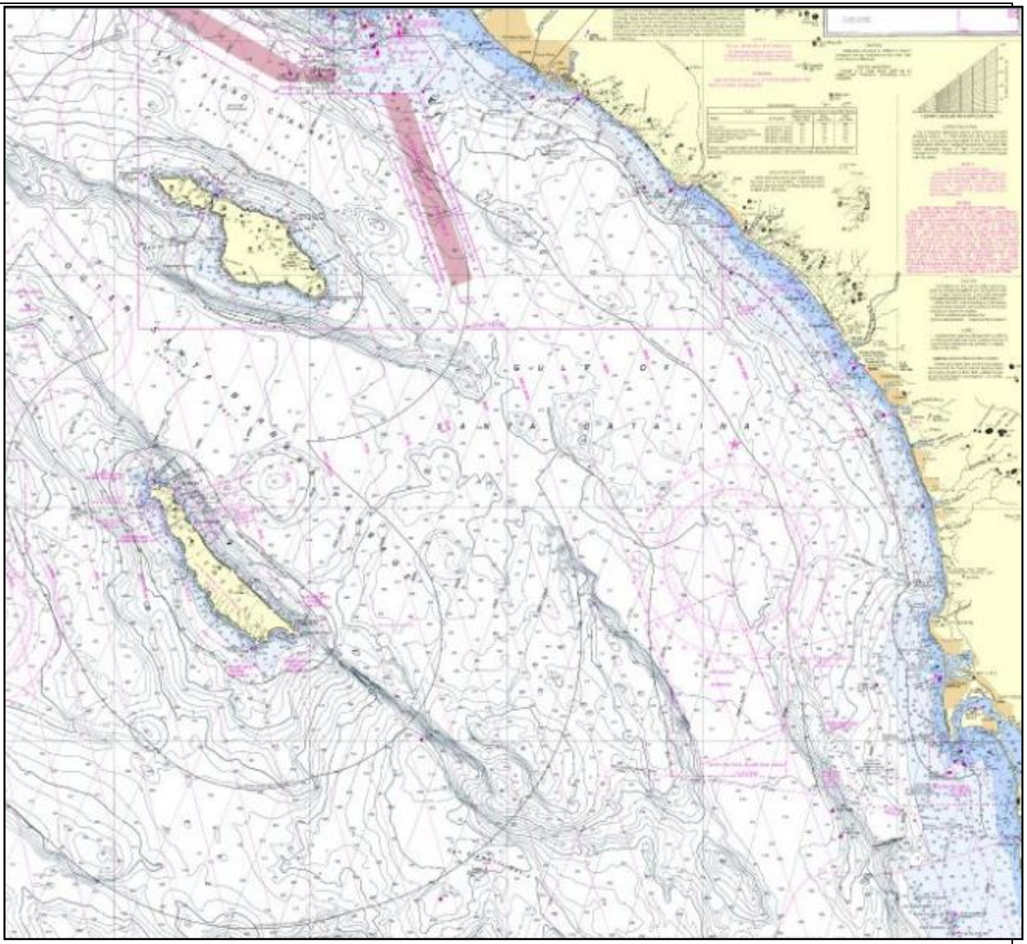
0.79

h. ***Extra Credit:*** Is this the answer you expected? Why or why not?

Grade / Content Area	**8th / 9th Grade Geometry**
Lesson Title	**Navigation and a Rescue at Sea (3 days)**
Guiding Question	*"How is Geometry used to assist mariners in navigating at sea?"*
Content Standards	*State Content Standards*: I. **M(N&O)–8–4**: Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) II. **M(G&M)–8–2:** Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local) III. **M(F&A)–8–3**: Demonstrates conceptual understanding of algebraic expressions by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when x = 4 given). (Local) IV. **M(F&A)–8–4:** Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., d = rt; d/r = t); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context. (Local)
	NCTM Standards: In middle and high school students should: I. **Specify locations:** use coordinate geometry to represent and examine the properties of geometric shapes; use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides. II. **Use visualization:** draw geometric objects with specified properties, such

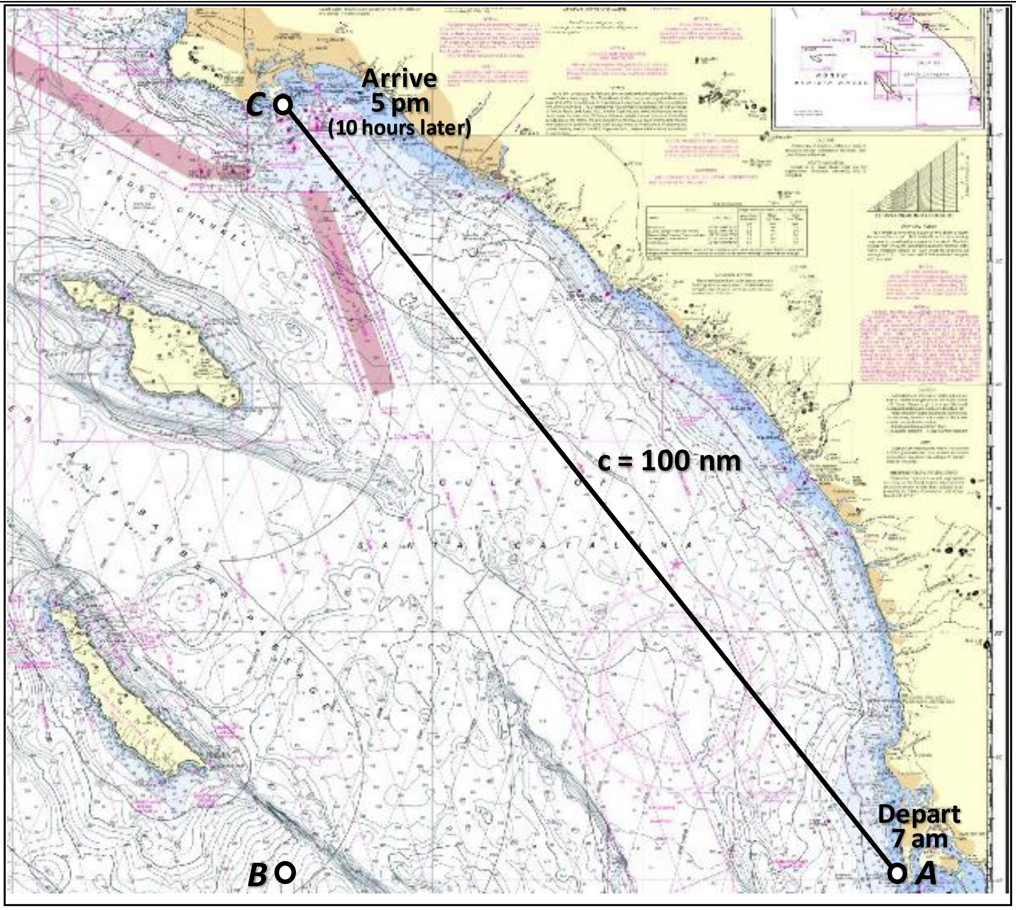
	as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Preparation	I. *Classroom Organization*: Students will work in groups of three or four. Desks will be rearranged to permit four students to work together with a common writing surface. II. *Differentiation*. As before, there are many things in this lesson that will appeal to the multiple intelligences of the students: including new terminology and hands-on exercises. Further working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. The culminating exercise is particularly challenging. Mindful of my special needs students, I may (depending on the class and the grouping) prepare worksheets that have some supports (partially filled out answers or hints) for these students. I will only use these if I and the collaborating special needs teacher feel it is appropriate. As always, I will be mindful of those students who have difficulty with comprehension. For these students, I will employ communication strategies such as checking for understanding, rephrasing questions, and communication both verbally, visually, and in writing. III. *Materials.* A. **Day 1:** For each group of students: 1. Four congruent right triangles cut from construction paper, two red, two orange. The measures of the sides should be 6 inches, 6 inches, and 8.5 inches. 2. One right triangle with sides measuring 5 inches, 12 inches, 13 inches. 3. Calculator. B. **Day 2:** For each group of students:

	1. Pythagorean Review worksheet (one per student). 2. Calculator (one per student). 3. Computer with internet access. C. **Day 3:** For each student: 1. Problem sheet "Navigating a Ship at Sea." 2. Calculator. 3. Ruler.
Student Learning Objectives	I. Given a track laid out on a navigational chart, time and distance, students will determine the velocity a ship needs to achieve to meet a scheduled port arrival. II. Given the original destination and a new intermediate point, students will apply the Pythagorean Theorem to determine the revised distance and required velocity to meet the same scheduled port arrival.
Instructional Procedures	I. *Warm-up* **Day 1** (**10 minutes**): II. *Launch.* **Day 1:** I will begin with the question, *"So, now our ship is at sea enroute to Long Beach. It's a long voyage. How do you think ships navigate at sea?"* a. To answer, I will show them this picture of a nautical cart of the coastal waters of Southern California, describing it as analogous to a road map:



b. *"Navigators plot the paths of their ships (called "tracks") on these maps (called "charts"). For example, let's say we are part of the crew of U.S.S.* Rentz*, a Navy ship stationed in San Diego California.* Rentz *is scheduled to get underway from San Diego at 7 AM and travel 100 nautical miles to the Port of Long Beach. The navigator's track would look something like this* (I will show the chart pictured below with the track laid out).

c. My next question will be, "Rentz *is scheduled to arrive at Long Beach at 5 PM, ten hours later. How fast must she travel?"* The answer is 10 nautical miles per hour. This is a fairly straightforward time – distance problem, similar to the rate problems the students solved during the linear relationships unit in the fall semester. This question serves to review some of the concepts learned in this unit.

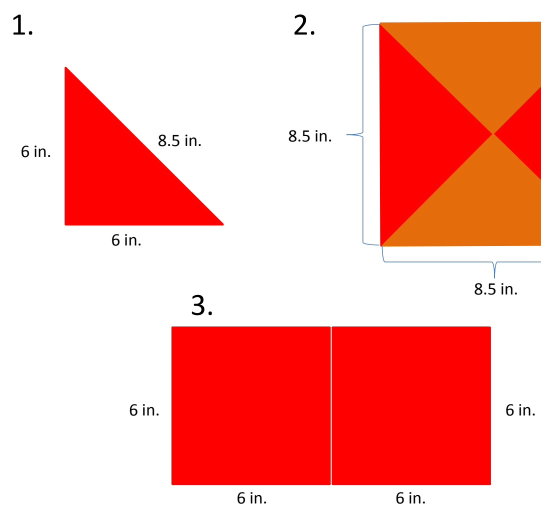
	 *d.* Now I will propose the problem. *"Our container ship is approaching Long Beach when one of our crew becomes ill. We need to get him off as soon as possible. We call the Coast Guard and they divert* Rentz*, leaving San Diego, to intercept us.* Rentz *will intercept us at Point B, 50 nautical miles west of San Diego, pick up the sick man and take him to Long Beach.* *e.* Now comes the question we will answer at the conclusion of the unit, *"If we travel to Point B, stop for one hour, and then turn toward Point C, how fast must the cutter travel to get there by 5 PM? How do we solve this problem?"* f. Of course, we could simply measure the distances and do the time-distance problem as before but we will explore the triangular relationship to determine an easier way to solve the problem. At this point, I will draw the right triangle ΔCBA and ask what kind of triangle is this. After we identify this as a right triangle, I will ask, *"Does anyone know of a relationship between the sides? Let's explore this relationship."*

III. *Engagement.* **Day 1:** First, we will examine the sides of such a triangle and speculate on the relationship among the sides. We will conduct an activity that is derived from the following interactive activity found on the NCTM *Illuminations* site: http://illuminations.nctm.org/ActivityDetail.aspx?id=30.

A. Each group of students will have four congruent triangles cut from construction paper, two red and two yellow. They will first measure and record the sides of the triangles: 6 inches, 6 inches, and 5 inches.

B. Given the measurements, students will be asked to speculate on the relationship between the sides. At this point, I would not expect that they will discover the relationship: $6^2 + 6^2 = 8.5^2$.

C. Students will now conduct a group activity. Students will be asked to arrange their triangles as in the following illustration and measure the sides of the new shapes. I will ask prompting questions as I walk around monitoring each group.



1. Question: *"What is the area of the second shape?* Students should be able to determine that it is approximately 72 square inches.

2. Question: *"What is the area of the third shape?* Students should be able to determine that it is also 72 square inches (36 sq. in. + 36 sq. in.).

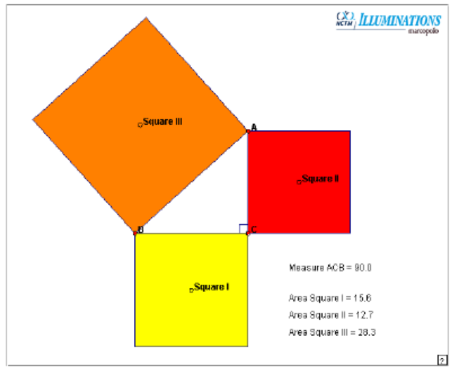
3. Question: *"So, is it correct to say that:*

	$$36 + 36 = 72$$ or $$6^2 + 6^2 = 8.5^2$$ 4. Question: *"So, what is the relationship among the sides of the triangle?* At this point, one or more students will likely get that the some of the squares of the sides equals the square of the hypotenuse or: $$a^2 + b^2 = c^2$$ IV. *Closing.* **Day 1:** We will conclude by going back to the *Illuminations* activity page (http://illuminations.nctm.org/ActivityDetail.aspx?id=30) where I will demonstrate that the relationship holds for all right triangles regardless of the lengths of the sides. I will then leave them with the question: *"Now, let's go back to our example of the Navy ship* Rentz *and our container ship. Is this the same kind of triangle? If we know the measures of two of the sides, can we determine the third and then solve the time-distance problem? We'll look at this over the next couple of days."* V. *Warm-up:* **Day 2** (**10 minutes**): VI. *Launch:* **Day 2:** We will spend a second day on the concept of the Pythagorean Theorem. I will begin this lesson with a review of what we learned on the previous day. VII. *Engagement:* **Day 2:** After I review the basic concepts, student groups will work at computers with internet access, logging on to the NCTM *Illuminations* to the following applet: http://illuminations.nctm.org/ActivityDetail.aspx?ID=164. Using the applet, the following worksheet, and a calculator, students will refamiliarize themselves with the relationship among the sides of a right triangle.

Pythagorean Review

NAME ______________________

You know that the Pythagorean theorem relates the lengths of the sides of a right triangle. But did you also know it is a relationship about areas? On the Illuminations site, open the Pythagorean Review applet. You should see the following right triangle with a square attached to each side.



1. Use your calculator to see how the areas of Squares I and II relate to the area of Square III. You can change the size of the triangle by dragging point A or point B. Investigate the relationship, and write your observations below.

The area of Square III is ____

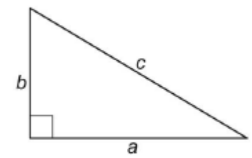
____

____

____.

The Pythagorean theorem states:

In a right triangle with legs a and b and hypotenuse c, the sum of the square of the legs is equal to the square of the hypotenuse. That is, $a^2 + b^2 = c^2$.



On the previous page, you should have noticed that the area of Square I plus the area of Square II is equal to the area of Square III. (If you did not get this result, go back and verify it with your calculator.)

2. How do a^2, b^2, and c^2 relate to the area of each square? Briefly explain your thinking.

3. What is meant by the statement, "The Pythagorean theorem is a relationship of areas?"

ILLUMINATIONS Resources for Teaching Math

VIII. *Closing:* **Day 2:** We will now share our findings in this exploratory exercise. Once we complete this activity, we will conclude the lesson with some fairly straightforward equations, solving exercises to ensure students are comfortable with manipulating the Pythagorean equation. This will set the stage for the next day's activity. We will discuss how to solve for each side of the triangle, i.e.:

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$b^2 = c^2 - a^2$$

	I will then remind students of our original problem, solving the time-distance problem for the medical rescue at sea and leaving them to consider, with their new learning, how they might solve it. IX. *Warm-up:* **Day 3** (**10 minutes**): X. *Launch:* **Day 3:** After a brief review of the concept of the Pythagorean Theorem using the *Illuminations* applet students worked with on the previous day, we will embark on the main exercise of the lesson: solving the time distance problem. XI. *Engagement:* **Day 3:** Students, working in groups, will solve the problems on the worksheet "Navigating a Ship at Sea." This is a fairly challenging task because it integrates the concepts of the Pythagorean Theorem and linear relationships (both learned this year) into a real-world problem with new terminology. This is not unlike the lesson I taught earlier in the year on linear relationships, using the space shuttle as my example. However, I will be mindful of this challenge as I walk around the classroom, observing and helping with student work. XII. *Closing:* **Day 3:** We will share our findings and review the answers. Depending on how long it takes students to solve these problems, I may allow them to finish the exercise as homework, reviewing it the following day. I will close the lesson with the following question, *"OK, so now we have seen how we can use the Pythagorean Theorem to compute the lengths of the sides of a right triangle – using that skill to solve a real-world problem...but what can we do if the triangle is NOT a right triangle?"* We will not cover this topic this year but I will discuss it with interested students.
Assessment	The completed worksheets from days 2 and 3 will serve as informal assessments. For the day 3 exercise, I will assess if the: 1. Student correctly assesses that velocity is a factor of distance and time and applies this knowledge to determine the speed necessary for the ship to achieve an on-time port arrival. I will also assess if the time-distance problem is correctly set up, arithmetic is correct resulting in a correct answer. 2. Response has no flaws in reasoning or computation. Student understands how to use the Pythagorean Theorem to get correct solutions. It contains

	solid explanations on how the student determined the length of segements "a" and "b" and used that information to determine the length of the hypotenuse "c". Response communicates student's thoughts clearly and effectively. 3. Student correctly assesses that application of the Pythagorean Theorem is appropriate to determine distances. Student demonstrates understanding of the relationship among distance, time, and velocity in a practical voyage planning problem. All work is shown, arithmetic is correct and correct answers are arrived at.

Coastal Piloting of a U. S. Navy Ship

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Name

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Class

____

Date

Coastal Piloting of a U. S. Navy Ship

I. You are the Navigator of the U. S. Navy ship USS *Rentz* (pictured below). *Rentz* is scheduled to depart the U. S. Naval Station at San Diego, California (point A on the *navigation chart* on page 3 below) at 7 AM tomorrow morning and travel to the U. S. Naval Weapons Station at Long Beach, California (point C on the navigation chart) to take on ammunition for an upcoming deployment to the Arabian Gulf. You must arrive at the Weapons Station no later than 5 PM tomorrow afternoon. Long Beach is 100 *nautical miles* from San Diego.

II. Background Information.

A. **Nautical Mile**: The most common measure of distance at sea. 1 nautical mile (nm) = 1.15 “land miles.”

B. **Knots**: A measure of speed used by ships. One knot = one nautical mile per hour.

C. **Navigation Chart**: A map used by Navigators to find where they are going and how to get there at sea.

III. At what speed (in *knots*) do you recommend *Rentz* travel to get from San Diego to Long Beach on time?

- *Ans:* $\frac{100nm}{10hrs} = \frac{x\ nm}{1hr} = 10\frac{nm}{hr} = 10\ nautical\ miles\ per\ hour = 10\ knots$

IV. At 6 AM, one hour before you sail, the Captain tells you that *Rentz* has been ordered to travel due West to point B (just south of San Clemente Island) to pick up a sick man on a containership that is traveling to Long Beach. Point B is 50 nautical miles West of San Diego.

A. Once you pick up the sick crewman at point B you turn 90 degrees to the North. How far (in nautical miles) must *Rentz* travel due North to get to Long Beach (point C)?

- *Ans: Since you traveled due West to point B and then turned 90 degrees to due North, you can use Pythagorean Theorem.*

$$a^2 + b^2 = c^2$$

$$50^2 + b^2 = 100^2$$

$$2500 + b^2 = 10000$$

$$b^2 = 10000 - 2500 = 7500$$

$$b = \sqrt{7500} = 86.6\ nm$$

B. What is the total distance (in nautical miles) *Rentz* will need to travel to pick up the sick man and then proceed to Long Beach?

- *Ans:*

$$50\ nm + 86.6\ nm = 136.6\ nm$$

C. If you leave San Diego on time (7 AM), at what speed (in knots) will *Rentz* need to travel to get from San Diego to point B and then from point B to Long Beach on time (by 5 PM) if you stop at point B for one hour to pick up the sick man?

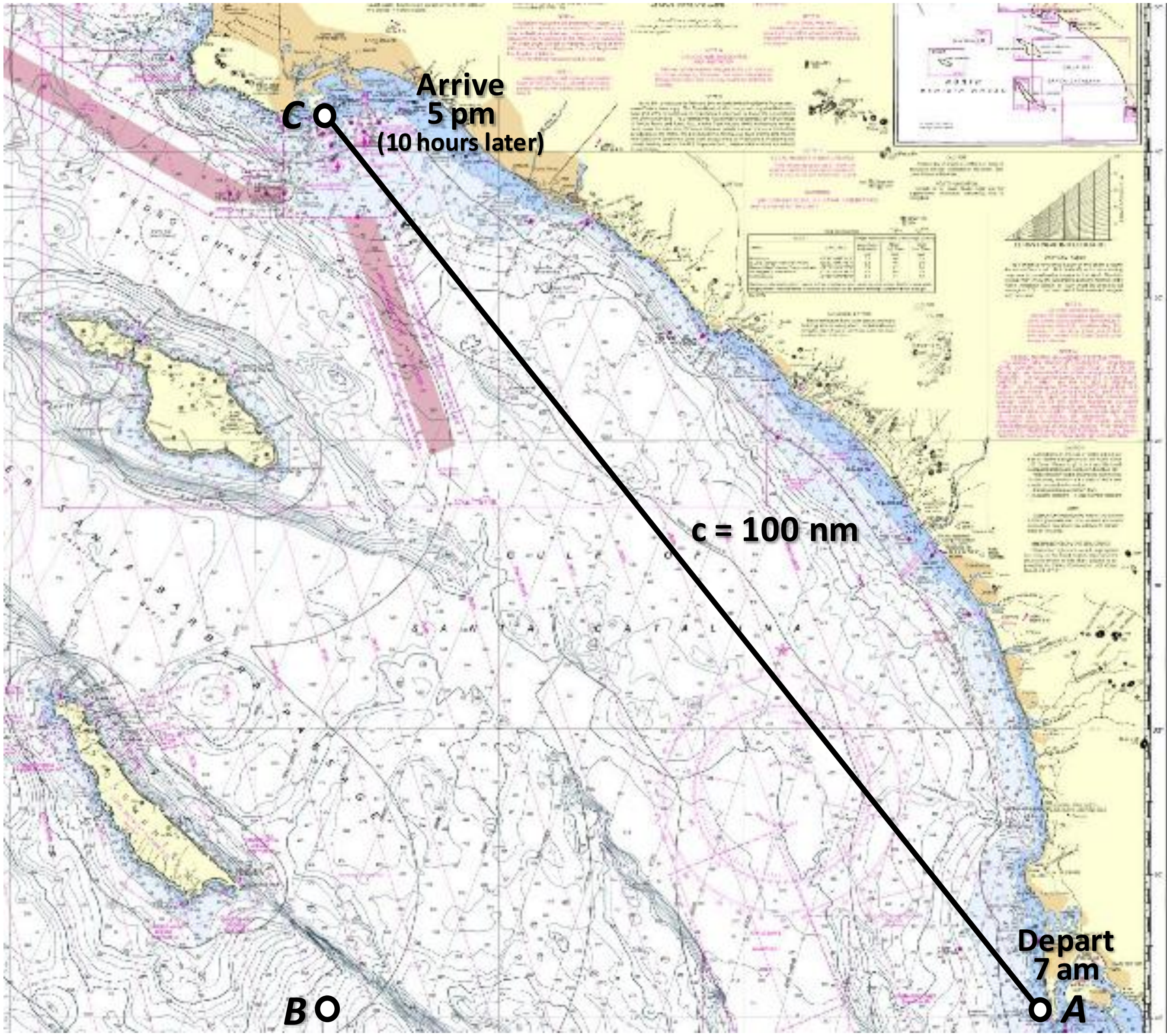
- *Ans:*

  - *First, determine how many hours you will actually be traveling.*

$$10\ hours\ (7\ AM - 5\ PM) - 1\ hour\ (to\ pick\ up\ the\ fishermen) = 9\ hours$$

- *Next, compute the speed Rentz will need to make to get to Long Beach on time.*

$$\frac{136.6\ nm}{9\ hours} = \frac{x\ nm}{1\ hour} = 15.2\ nautical\ miles\ per\ hour = 15.2\ knots$$


Arrive
5 pm
(10 hours later)
C
c = 100 nm
Depart
7 am
A
B

Grade / Content Area	# Grade 8 / 9 Algebra and Geometry
Lesson Title	**A Rescue At Sea Using the Distance Formula**
Guiding Question	*"How is Geometry used to assist mariners in navigating at sea?"*
Content Standards	*State Content Standards:* I. **M(N&O)–8–4**: Accurately solves problems involving proportional reasoning (percent increase or decrease, interest rates, markups, or rates); multiplication or division of integers; and squares, cubes, and taking square or cube roots. (Local) II. **M(G&M)–8–2:** Applies the Pythagorean Theorem to find a missing side of a right triangle, or in problem solving situations. (Local) III. **M(F&A)–8–3**: Demonstrates conceptual understanding of algebraic expressions by evaluating and simplifying algebraic expressions (including those with square roots, whole number exponents, or rational numbers); or by evaluating an expression within an equation (e.g., determine the value of y when x = 4 given). (Local) IV. **M(F&A)–8–4:** Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g., d = rt; d/r = t); by solving multi-step linear equations with integer coefficients; by showing that two expressions are or are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of linear equations in a context. (Local)
	NCTM Standards: In middle and high school students should: I. **Specify locations:** use coordinate geometry to represent and examine the properties of geometric shapes; use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.

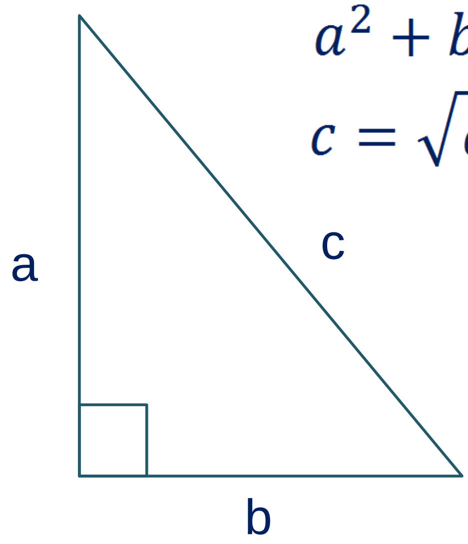
	II. **Use visualization:** draw geometric objects with specified properties, such as side lengths or angle measures; use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume; use visual tools such as networks to represent and solve problems; use geometric models to represent and explain numerical and algebraic relationships; recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.
	Common Core Standards: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
Preparation	VI. *Classroom Organization*: Students will work in groups of three to five. Desks will be rearranged to permit students to work together with a common writing surface. VII. *Differentiation*. Working in groups promotes inclusion provided students are organized such that they can leverage off each others' strengths and minimize their individual weaknesses. I will be particularly mindful when grouping students of where I place my English language learners, those with reading comprehension challenges, and those who are already having difficulty with the course material. VIII. *Materials.* One worksheet for each student.
Student Learning Objectives	V. Correctly identify all inputs to a function from a given word problem. VI. Understand that maps such as nautical charts are used to graphically portray special coordinates and distances. VII. Apply the Pythagorean Theorem to determine distances in a spatial coordinate system. VIII. Derive the Distance Formula from application of the Pythagorean Theorem in a spatial coordinate system.
Instruction and Engagement	*Launch (Opening):* I. *How do ships navigate at sea – where everything looks the same? If a ship needs help at sea, how do we find him?* I'll give students an opportunity to think about this and offer ideas. II. *Today we are going to solve a navigation problem involving a rescue at sea.* III. *Who can tell me what the Pythagorean Theorem is?* I will give students an

opportunity to offer answers. I will then ensure the following concept is clear to all students:

A. *Given a right triangle, the length of the hypotenuse is equal to the square root of the sum of the squares of the two sides.*

$$a^2 + b^2 = c^2$$

$$c = \sqrt{a^2 + b^2}$$



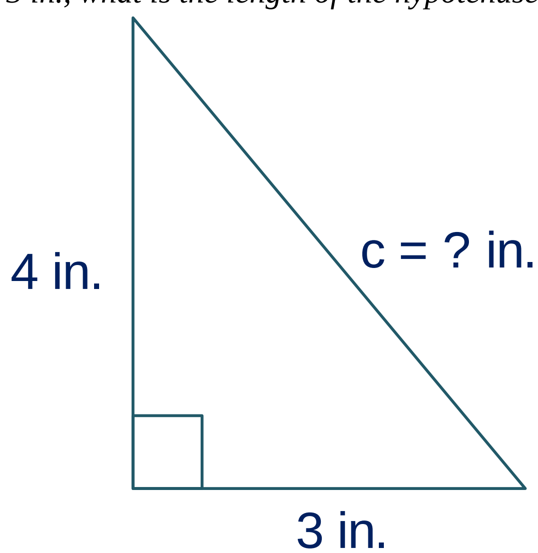
B. *Normally we write this as:*

$$a^2 + b^2 = c^2$$

C. *But it can also be written as:*

$$c = \sqrt{a^2 + b^2}$$

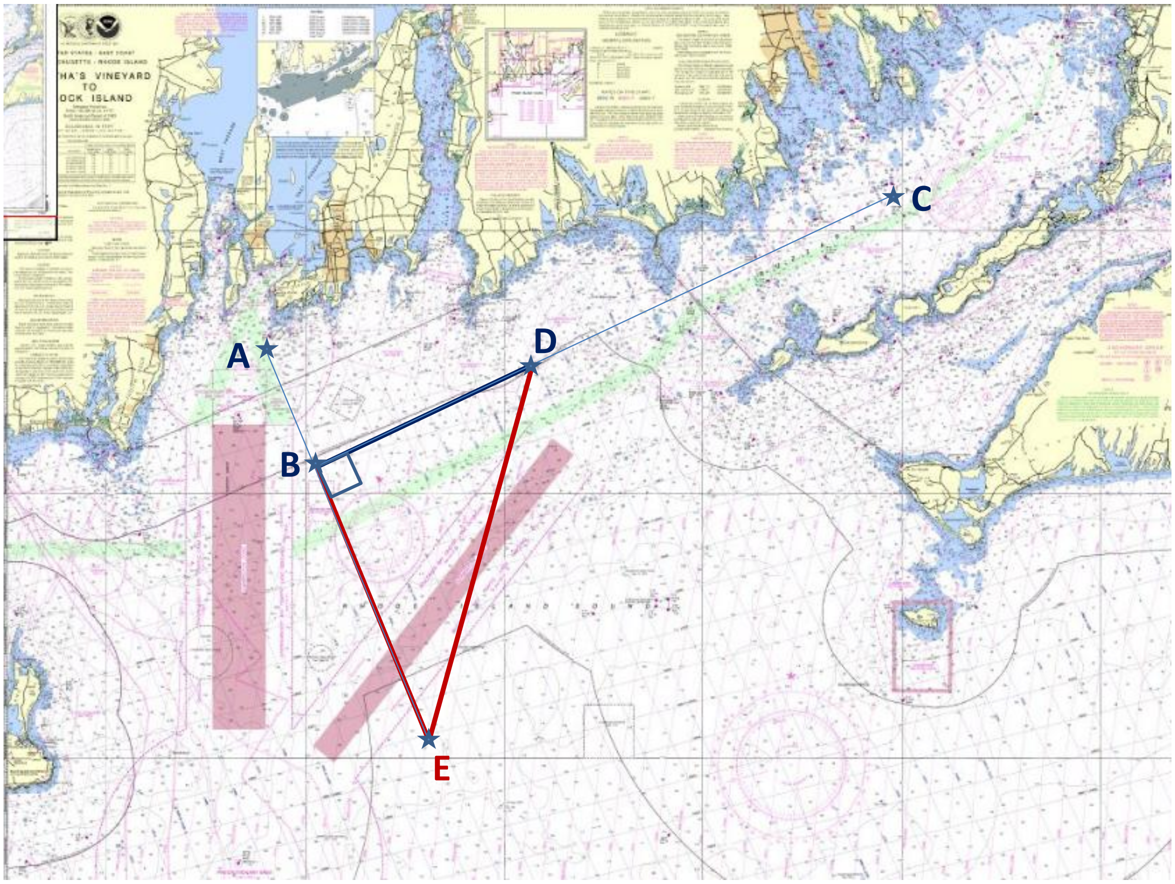
D. *Here is an example: Given a right triangle with side a = 4 in. and side b = 3 in., what is the length of the hypotenuse c?*



	$$c = \sqrt{4^2 + 3^2} = \sqrt{16 + 9} = \sqrt{25} = 5$$ IV. *Now let's look at our rescue at sea.* Pass out performance task.
	Explore (Engagement): I. I will hand out the worksheet and allow students to begin working on the problems in groups while I walk around, monitor, and ask questions such as: A. *To answer the first two questions, how would you organize the information that you do know?* B. *What information about this triangle helps you decide how to answer the third question? What is the equation we would use to solve for this distance?* C. *What is the relationship between x_2 and x_1? How about y_2 and y_1? How would you substitute x_1, x_2, y_1, y_2 into are equation to solve for the distance between Point D and Point E?*
	Summarize / Share (Closure): To conclude, we will consider the following question: *What does a Cartesian coordinate system allow us to do? Think of a circle. If we plot it in a Cartesian coordinate system, how could we express that circle?*
Assessment	V. I will assess student understanding by monitoring progress in completion

Name: ___________________________

Date: ___________________________



At 2 PM on Tuesday, June 8th, U. S. Coast Guard Southeastern New England headquarters receives a distress call from a fishing vessel located 16 nautical miles south, southeast of the entrance to Narragansett Bay (Point E). The vessel is sinking and will not remain afloat for more than four hours. The Coast Guard headquarters dispatches two cutters, USCGC *Tiger Shark* from Newport, Rhode Island and USCGC *Hammerhead* from New Bedford, Massachusetts to find and rescue the fishing boat.

At 5 PM:

A. *Tiger Shark* is at Point B which is 26 nautical miles from the entrance to New Bedford Harbor (Point C) and 5 nautical miles from the entrance to Narragansett Bay (Point A).

B. *Hammerhead* is at Point D which is 17 nautical miles from the entrance to New Bedford Harbor (Point C).

C. The fishing boat is at point E which is 16 nautical miles from the entrance to Narragansett Bay (Point A).

You must rendezvous with the fishing boat by 6 PM in order to have time to get the crew off before the boat sinks.

1. How far is *Hammerhead* (Point D) from *Tiger Shark* (Point B)?

$$26\ nm - 17\ nm = 9\ nm$$

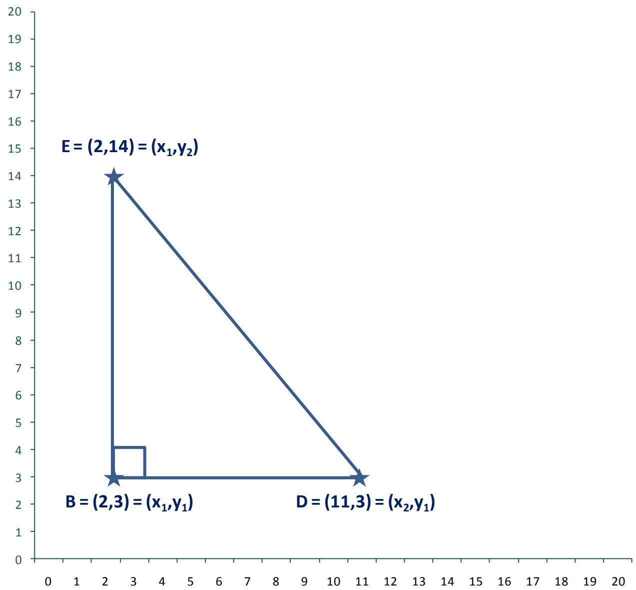
2. How far is *Tiger Shark* (Point B) from the fishing boat (Point E)?

$$16\ nm - 5\ nm = 11\ nm$$

3. How far is *Hammerhead* (Point D) from the fishing boat (Point E)?

$$c = \sqrt{9^2 + 11^2} = \sqrt{81 + 121} \cong 14.21$$

4. Now, let's put that triangle on a coordinate grid so that it looks like the picture below. Can you derive a formula for the distance between Point D and Point E expressed in terms of x_i and y_i?



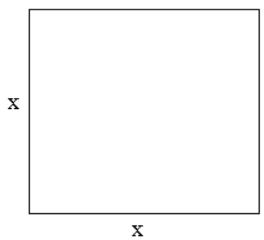
$$c = \sqrt{(x_2 - x_1) + (y_2 - y_1)}$$

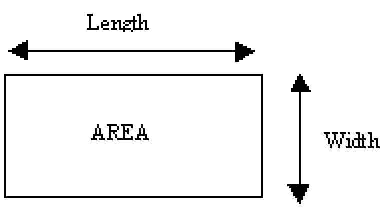
5. *Bonus:* If both *Tiger Shark* and *Hammerhead* have one hour to get to the fishing boat, how fast does each ship need to go?

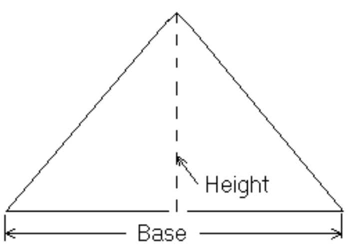
Hammerhead: $\frac{14.21nm}{1hr} = 14.21\frac{nm}{hr}$

Tiger Shark: $\frac{11nm}{1hr} = \frac{11nm}{hr}$

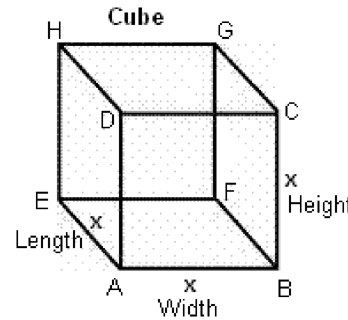
Surface Area and Volume of 3D Shapes – *Save this forever!!!!!!*

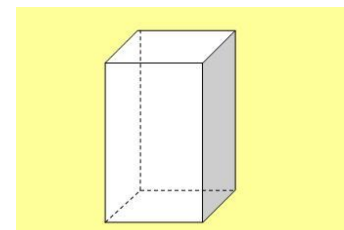


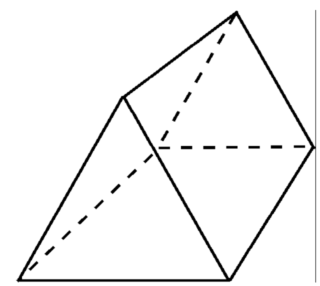


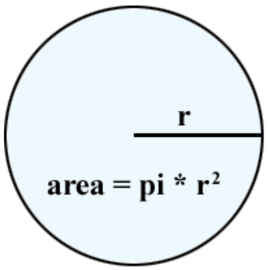


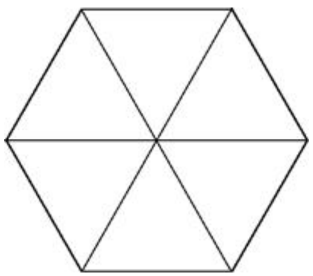
Area of a Square:	$\boldsymbol{A = s^2}$ where s = the length of a side.	***Right Square Prism:*** • $\boldsymbol{SA = 2(s^2) + 4(sH)}$ • $\boldsymbol{V = s^2 H}$ ...where s = the length of a side and H = height of the prism. ***Right Square Pyramid:*** • $\boldsymbol{SA = s^2 + 4\left(\frac{1}{2} b \times sh\right)}$ • $\boldsymbol{V = \frac{1}{3} sH}$ ...where s = the length of a side and H = height of the prism.
Area of a Rectangle:	$\boldsymbol{A = l \times w}$ where l = the length of the rectangle and w = the width.	***Right Rectangular Prism:*** • $\boldsymbol{SA = 2\big((lw) + (lH) + (wH)\big) = 2B + PH}$ • $\boldsymbol{V = l \times w \times H = PH}$ ...where l = the length, w = the width, P = perimeter of the base, H = height of the prism.
Area of a Triangle:	$\boldsymbol{A = \frac{1}{2} bh}$ where b = the base of the triangle and h = the height of the triangle.	***Right Triangular Prism:*** • $\boldsymbol{SA = 2\left(\frac{1}{2} bh\right) + (S_1 + S_2 + S_2)H}$ • $\boldsymbol{V = \left(\frac{1}{2} bh\right) H}$ ...where S_i is the length of side i, b is the length of the base of the base triangle, h is the height of the base triangle, and H is the height of the prism. ***Right Triangular Pyramid:*** • $\boldsymbol{SA = \frac{1}{2} bh + 3(\frac{1}{2} b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}\left(\frac{1}{2} bh\right) H}$ ...b is the length of the base of the base triangle, h is the height of the base triangle, sh is the height of each side triangle, and H is the height of the pyramid.



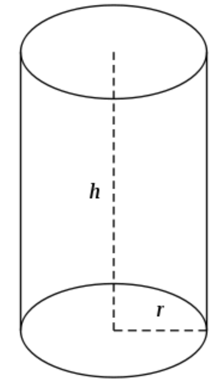


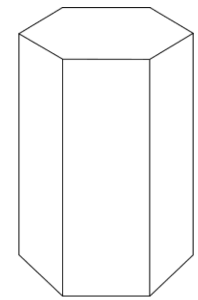


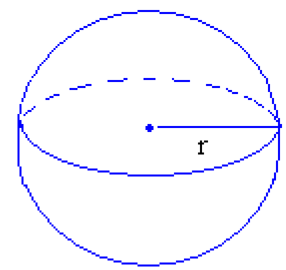


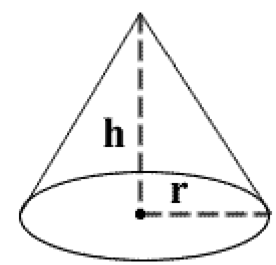


Area of a Circle:	$\boldsymbol{A = \pi r^2}$ where r = the radius of the circle.	***Right Cylinder:*** • $\boldsymbol{SA = 2\pi r^2 + 2\pi r H}$ • $\boldsymbol{V = \pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Right Cone:*** • $\boldsymbol{SA = \pi r^2 + \pi r \times \sqrt{r^2 + H^2}}$ • $\boldsymbol{V = \frac{1}{3}\pi r^2 H}$ ...where r is the radius of the base circle and H is the height of the cylinder. ***Sphere:*** • $\boldsymbol{SA = 4\pi r^2}$ • $\boldsymbol{V = \frac{4}{3}\pi r^3}$ ...where r is the radius of the sphere.
Area of a Regular Hexagon:	$\boldsymbol{A = \frac{1}{2}aP}$ where a = the apothem and P = the perimeter of the base.	***Right Regular Hexagonal Prism:*** • $\boldsymbol{SA = 2 \times (\frac{1}{2}aP) + PH}$ • $\boldsymbol{V = \frac{1}{2}aP \times H}$ ...where a = the apothem, P = the perimeter of the base, and H = the height of the prism.
General Formulas		***Prism and Cylinder:*** • $\boldsymbol{SA = 2B + PH}$ • $\boldsymbol{V = BH}$ ...where B is the area of the base, P is the perimeter of the base, and H is the height of the prism. ***Pyramid:*** • $\boldsymbol{SA = B + N(\frac{1}{2}b \times sh)}$ • $\boldsymbol{V = \frac{1}{3}BH}$ ...where B is the area of the base, N is the number of sides, b is the length of the base of a side triangle, and sh is the slant height of the side triangle, and H is the height of the pyramid.









PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth

Seeing with Sound

Equations and Their Uses

Thomas R. Beall
Captain, U. S. Navy (Ret.)

2013 – 2014 Edition

Introduction to SONAR

Many ships use SONAR (short for ***SO***und ***N***avigation and ***R***anging) to find things under water. Scientific ships use it to map the bottom of the ocean and find objects that are very deep. Warships use SONAR to find enemy submarines. Because one cannot see underwater easily, sound is used to hear things that are underwater.

Listening with SONAR is much like listening with your ears. Imagine yourself at a party where music is playing and people are talking and laughing. Your friend is standing near you talking to you. Your ability to hear him is based on many factors. Can you think of what those factors are? List as many as you can below:

____

____

____

____

Let's give each of these a name.

1. Detection Threshold (DT): ____
2. Source level (SL): ____
3. Noise level (NL): ____
4. Transmission Loss (TL): ____
5. Target Strength (TS): ____

You could put these terms together in a sentence:

The sound level at which you can just barely hear your friend equals how loudly your friend speaks minus how far away from you your friend is minus how much noise is in the background.

Now, let's rewrite this sentence, putting in the abbreviations for each term:

The sound level (DT) at which you can just barely hear your friend equals how loudly your friend speaks (SL) plus how well someone could hear him standing one foot from him (TS) minus twice how far away from you your friend is (2TL) minus how much noise is in the background (NL).

Now, let's translate this into an equation:

$$DT = SL + TS - 2TL - NL$$

This is the basic SONAR equation we will use in this unit.

We can write this equation in terms of each variable. For example we can rewrite the equation in terms of SL by solving for SL in the basic SONAR equation above:

Source level (SL) equals detection threshold (DT) plus twice the transmission loss (2TL) plus noise level (NL)

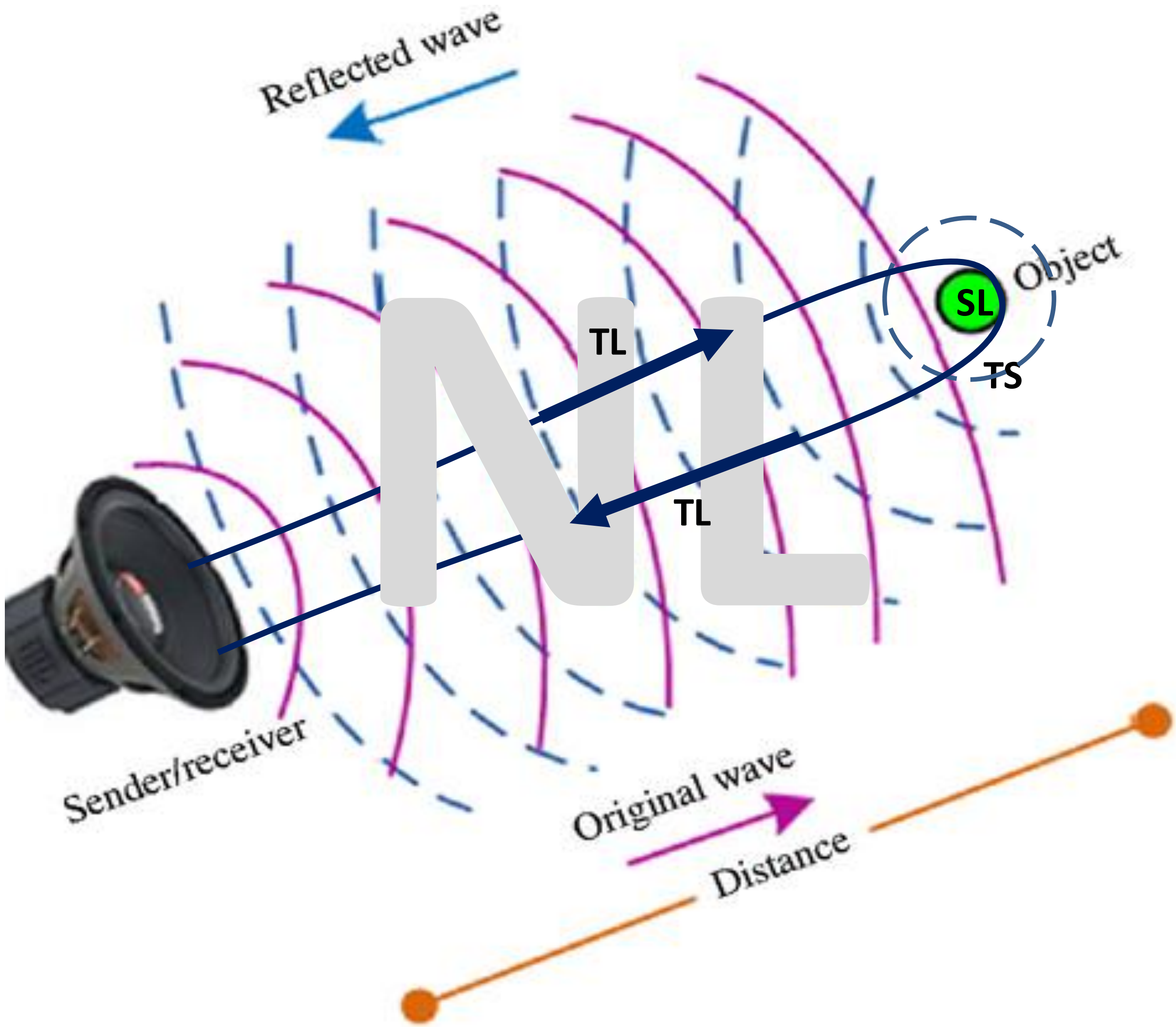
$$SL = DT + 2TL + NL - TS$$

Now try to rewrite the equation in terms of transmission loss (TL) and noise level (NL):

$TL =$ ______________________________________

$NL =$ ______________________________________

How SONAR Works



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Solving Equations of One Variable – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. **How to solve equations of one variable:**

   a. $$\begin{array}{r} 3x + 4 = 15 \\ \underline{\quad -4 \quad -4} \\ 3x + 0 = 11 \end{array}$$

   b. $\frac{3x}{3} = \frac{11}{3}$

   c. $x = \frac{11}{3}$

2. **Problem Set:**

a) $5x + 9 = 39$

b) $9p + 11 = -7$

c) $6 - 2d = 42$

d) $9 - c = -13$

e) $2m + 5 = 17$

f) $9p + 20 = -7$

g) $5x + 9 = 54$

h) $3 + 2x = 21$

i) $6 - 8d = -42$

j) $127 = 2x + 17$

j) $5.2 + 1.3x = -1.3$

k) $13 = \frac{a}{3} - 2$

Name: ____________________________
Date: ______________________________
Advisor: ___________________________

Solving Equations of One Variable – II

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = TS + SL - 2TL - NL$$

II. **Variables** (all variables express sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

III. **Problems:**

A. $10db = TS + 4db - 2(5db) - 12db$

1. Add given numbers on right-hand side together: $4db - 2(5db) - 12db =$

$$4db - 10db - 12db = -18db$$

2. Substitute the sum for the given numbers: $10db = TS - 18db$

3. Add 18db to both sides:

$$\begin{aligned} 10db &= TS - 18db \\ \underline{+18db} &\ \ \underline{\qquad + 18db} \\ 28db &= TS \ - 0db \end{aligned}$$

4. And the answer is..... $TS = 28db$

"2 Level" Problems	**"3 Level" Problems**	**"4 Level" Problems**
B. $30db = 10db - NL$	J. $30db = 5db - NL + 4db$	R. $30db = 5db - NL + SL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
C. $46db = TS - 41db$	K. $46db = TS + 65db - 41db$	S. $46db = TS + 65db - NL$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
D. $80db = 44db + SL$	L. $80db = 37db + SL - 21db$	T. $80db = TS + SL - 21db$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________
E. $12db = 87db - 2TL$	M. $12db = 35db - 2TL - 15db$	U. $12db = TS - 2TL - 15db$
$2TL =$ ______________	$2TL =$ ______________	$2TL =$ ______________
$TL =$ ______________	$TL =$ ______________	$TL =$ ______________
F. $55db = 21db - NL$	N. $55db = 81db - NL + 24db$	V. $55db = TS - NL + 24db$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
G. $8.2db = TS - 4.1db$	O. $8.2db = 15db - TS - 4.1db$	W. $8.2db = SL - TS - 4.1db$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
H. $9.7db = 6.9db - NL$	P. $9.7db = 2.3d + 6.9db - NL$	X. $9.7db = SL + 6.9db - NL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
I. $56db = 33.3db + SL$	Q. $0db = 33.3db + SL - 56db$	Y. $0db = 33.3db + SL - NL$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________

Name: ___________________________
Date: _____________________________
Advisor: ___________________________

Solving Equations of One Variable – III

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = TS + SL - 2TL - NL$$

II. **Variables** (all variables represent sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

If the detection threshold (DT) is less than 1.0db, it will be impossible to detect the target. If DT is less than 10db, it is highly unlikely that the target will be detected.

III. **Problems:**

A. A nuclear-powered submarine uses pumps to cool its nuclear reactor. These pumps make a great deal of noise. A destroyer searching for such a submarine uses a SONAR to listen for the sound of those pumps. If the source level (SL) of the submarine is 40db, the target strength (TS) of the submarine is 34db, the transmission loss (TL) is 20db and the noise level (NL) is 20db, will the destroyer detect the submarine?

B. Will the destroyer detect the submarine if the noise level (NL) increases to 44db?

C. A frigate's SONAR has a detection threshold (DT) for a submarine of 10db. If the noise level (NL) is 20db, the transmission loss (TL) is 10db, and the submarine's target strength (TS) is 28db, what is the source level (SL) of the submarine?

D. What will the new detection threshold (DT) be if the submarine's source level (SL) increases by 5db over the level that is the answer to problem "C"?

E. A National Ocean Survey ship uses SONAR to detect underwater natural gas emissions. If the SONAR has a detection threshold (DT) of 15db with a noise level (NL) of 8db, a transmission loss (TL) of 12db, and a source level (SL) of 0db, what is the target strength of the natural gas emission?

F. What would the new detection threshold (DT) be if the noise level (NL) decreased by 4db?

Name: ___________________________
Date: _____________________________
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Solving Multi-Step Equations – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. **How to solve multi-step equations:**

   a. $4x - 2 = 3x + 4$
   $\underline{-3x \qquad -3x}$
   $1x - 2 = 0 \quad + 4$

   b. $x - 2 = 4$
   $\underline{\quad +2 \ +2}$
   $x + 0 = 6$

   c. $x = 6$

2. **Problem Set:**

   a. **Basic Skills Practice.** Solve each equation:

1) $2x - 2 = 4x + 6$
2) $3x + 5 = 2x + 2$
3) $4x + 3 = 5x - 4$

4) $2x - 5 = 4x - 1$
5) $5x + 24 = 2x + 15$
6) $5y - 10 = 14 - 3y$

7) $12 - 6z = 10 - 5z$
8) $5m - 7 = -6m - 29$
9) $-10x + 3 = -3x + 12 - 4x$

10) $6p - 12 = -4p + 18$
11) $\frac{w}{2} + 7 = \frac{w}{3} + 9$
12) $6 - \frac{t}{4} = 8 + \frac{t}{2}$

Name: ___________________________
Date: _____________________________
Advisor: ___________________________

Solving Multi-Step Equations – II

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

I. **SONAR Equation.** All problems will use the following basic SONAR equation we have already discussed:

$$DT = TS + SL - 2TL - NL$$

II. **Variables** (all variables express sound levels in decibels [db]):

A. DT = Detection Threshold
B. TS = Target Strength
C. SL = Source Level
D. TL = Transmission Loss
E. NL = Noise Level

III. **Problems:**

A. $10db + 2TS = 3TS + 4db - 2(5db) - 12db$

1. Add given numbers on right-hand side together: $4db - 2(5db) - 12db =$

$$4db - 10db - 12db = -18db$$

2. Substitute the sum for the given numbers: $10db + 2TS = 3TS - 18db$

3. Add 18db to both sides:

$$\begin{array}{r} 10db + 2TS = 3TS - 18db \\ \underline{+18db \qquad\qquad\quad + 18db} \\ 28db + 2TS = 3TS \;\; - 0db \end{array}$$

4. Subtract 2TS from both sides:

$$\begin{array}{r} 28db + 2TS = 3TS \;\; - 0db \\ \underline{-2TS \;\; - 2TS \qquad\quad} \end{array}$$

5. And the answer is..... $TS = 28db$

"2 Level" Problems	"3 Level" Problems	"4 Level" Problems
B. $30db - 2NL = 10db - NL$	J. $30db - 2NL = 5db - NL + 4db$	R. $30db - 6SL = 5db - NL - 5SL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
C. $46db + 3TS = 4TS - 41db$	K. $46db + 2TS = 3TS + 65db - 41db$	S. $-2NL + 46db + 2TS = TS + 65db - NL$
$TS =$ ______________	$TS =$ ______________	$TS =$ ______________
D. $80db - 4SL = 44db - 3SL$	L. $80db - 6SL = 37db + SL - 21db - 6SL$	T. $80db - 2SL + TS = TS - SL - 21db$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________
E. $12db = 4TL + 87db - 2TL$	M. $12db - 5TL = 35db - 3TL - 15db$	U. $18.7TS + 12db - 6TS = 12.7TS - 2TL - 15db$
$2TL =$ _____________	$2TL =$ _____________	$2TL =$ _____________
$TL =$ ______________	$TL =$ ______________	$TL =$ ______________
F. $55db - 17NL = 21db - 16NL$	N. $55db + 5NL = 81db - NL + 24db + 5NL$	V. $2(TS - NL) + 55db = TS + 2NL + 24db$
$NL =$ ______________	$NL =$ ______________	$TS =$ ______________
G. $8.2db + 12TS = 13TS - 4.1db$	O. $4TS + 8.2db - TS = 15db + 4TS - 4.1db$	W. $8.2db - 4(SL + TS) = -3SL - 3TS - 4.1db$
$TS =$ ______________	$TS =$ ______________	$SL =$ ______________
H. $9.7db + 4NL = 6NL + 6.9db - NL$	P. $9.7db + 3NL = 2.3db + 5NL + 6.9db - NL$	X. $6(2SL - 3NL) + 9.7db = 2(6SL + 6.9db) - 17NL$
$NL =$ ______________	$NL =$ ______________	$NL =$ ______________
I. $56db + 12SL = 33.3db - 11SL$	Q. $0db + 12SL = 33.3db + SL - 56db + 12SL$	Y. $7(-3NL + 0db) = 33.3db + SL - 21NL$
$SL =$ ______________	$SL =$ ______________	$SL =$ ______________

Name: ___________________________
Date: _____________________________
Advisor: __________________________

Solving Equations with Distributive Property – I

Solve the following equations for the variable indicated. Show all work on separate sheets of paper.

1. Solving equations using the distributive property **(show work on separate sheet of paper)**:

$3(x-4)=48$

$3x-12=48$

$\underline{\quad +12 \; + \; 12}$

$3x+0=60$

$\dfrac{3x}{3}=\dfrac{60}{3}$

$x=20$

$4x-8(x+1)=8$

$4x-8x-8=8$

$-4x-8=8$

$\underline{\quad +8 \; +8}$

$-4x \quad =16$

$-\dfrac{4x}{-4}=\dfrac{16}{-4}$

$x=-4$

2. **Problem Set.**

   a. **Basic Skills Practice**.

1) $4n-2+7n=20$
2) $3(r-4)=9$
3) $6x-4(2x+1)=12$
4) $3y-2(3y+2)=8$
5) $3(x+1)=2x+7$
6) $3w-1-4w=4-2w$
7) $8x-5=4x+4-2x$
8) $15-3y=y+13+y$
9) $4a-4=-2a+14$
10) $4m-5=3m+7$
11) $2(x+1)=3x-3$
12) $2m-4=2(6-7m)$

Name: ___________________________
Date: ____________________________
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Solving Equations with Distributive Property – II

Calculating Fuel Consumption

1. **Background.** In order to find the sunken submarine, our ships need to remain at sea for days at a time, burning fuel to move and maintain the light, power, and air conditioning necessary to operate the ship and sustain the crew. Being very careful, a destroyer-type ship with a full load of fuel can remain at sea without refueling for approximately three weeks.

   It is the job of the ship's Chief Engineer (CHENG) and his engineers to monitor fuel use and report to the ship's Captain daily on how much has been burned, how fast it was burned and how much is left. With this information, the Captain can decide if he will keep his ship on its mission or take time out to refuel from a tanker at sea.

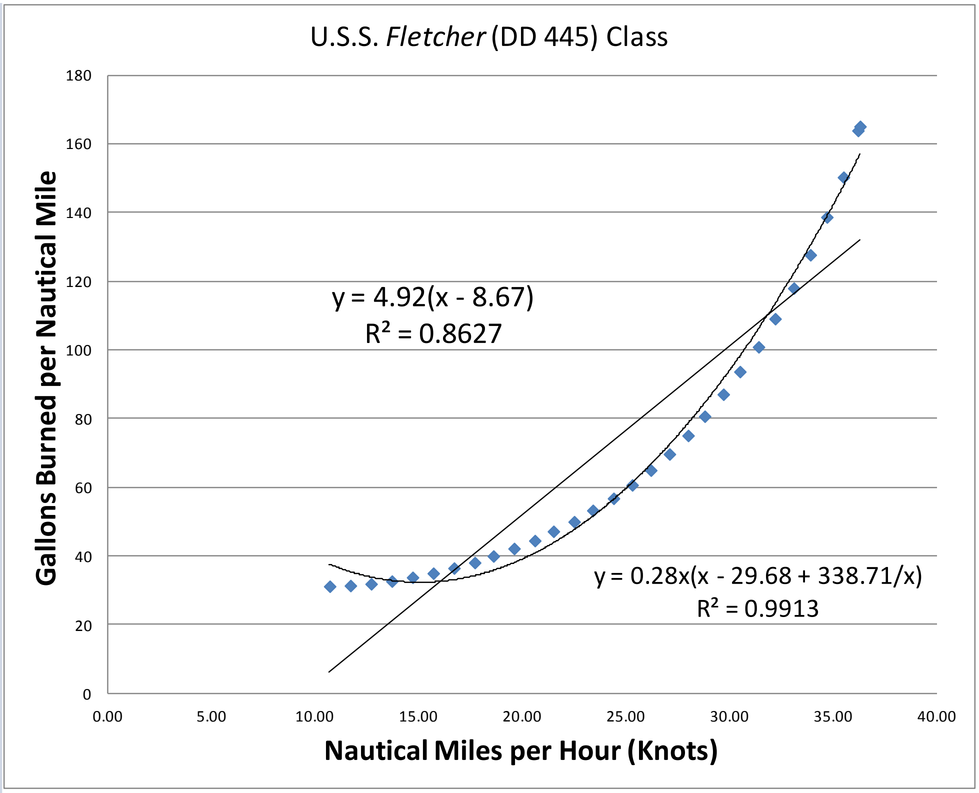
   A typical destroyer is pictured below. The

U.S.S. *Fletcher* (DD 445)

   The following table provides the data that the Chief Engineer and his engineers use to make their recommendations to the Captain.[11] The table shows that at a given speed in nautical miles per hour (knots), the ship will burn a certain number of gallons. This information is graphically depicted in the scatter plot next to the table.

[11] *From War Service Fuel Consumption of U.S. Naval Surface Vessels* (FTP 218), UNITED STATES FLEET, Headquarters of the Commander in Chief, 1945. Online at: http://www.ibiblio.org/hyperwar/USN/ref/Fuel/.

Knots	Gallons per NM
10.70	31.20
11.70	31.40
12.70	31.90
13.70	32.70
14.70	33.80
15.70	35.00
16.70	36.50
17.70	38.10
18.60	40.00
19.60	42.20
20.60	44.50
21.50	47.20
22.50	50.00
23.40	53.30
24.40	56.80
25.30	60.70
26.20	65.00
27.10	69.70
28.00	75.10
28.80	80.70
29.70	87.10
30.50	93.70
31.40	100.90
32.20	109.10
33.10	118.00
33.90	127.70
34.70	138.70
35.50	150.30
36.20	163.90
36.30	165.10



One line and one polynomial curve are fitted to the graph. Note that both have an R-squared value greater than 0.5. What this means is that either the linear equation or the curve's equation could be used to predict gallons burned for this class of destroyer. We will use both in the following exercises.

Linear Equation: $y = 4.92(x - 8.67)$

Polynomial Equation: $y = 0.28x(x - 29.68 + \frac{338.71}{x})$

2. **Problem Set.**

   a. Complete the table below by predicting the gallons burned per nautical mile for the given speeds using each equation.

Example: For $x = 10\ knots$ using the linear equation:

$$y_1 = 4.92(10 - 8.67) = 4.92(1.33) = 6.54\ gallons\ per\ nm.$$

$x = Knots$	$y_1 = \frac{Gallons}{NM} = 4.92(x - 8.67)$	$y_2 = \frac{Gallons}{NM} = 0.28x(x - 29.68 + \frac{338.71}{x})$	Difference
10			
15			
20			
25			
30			
35			
40			

b. Which equation provides a better predictor of fuel use (particularly at lower and high speeds? Why?

Name: ___________________________
Date: ____________________________
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The Bedford Incident – Questions

Place all answers on this sheet. 7 – 13 correct answers = 2.
14 – 20 correct answers = 3. 21 correct answers = 4.

1. What are the medical personnel in the ship's sick bay testing when the new medical officer arrives?

   Red cabbage.

2. Where does the item they are testing come from?

   A Russian submarine

3. What is the name of the Captain of U.S.S. *Bedford*?

   Captain Verlander

4. What does Mr. Munsford do for a living?

   Journalist

5. What did the German Commodore waiting in the Captain's Cabin do during the Second World War?

   Command a German U-boat

6. What is *Bedford* doing in the Denmark Strait?

   Hunting for a Russian submarine

7. What is the code name of the Russian submarine?

   Big Red

8. What angers the Captain about the Russian trawler they see?

   It dumps garbage in front of Bedford

9. What is the name of the Ensign whom the Captain yells at on the bridge?

   Ensign Ralston

10. What did the unidentified air contact turn out to be?

    A weather balloon

11. In what country's territorial waters does *Bedford* find the Russian submarine?

    Greenland

12. How many hours can the Russian submarine remain underwater?

    24 hours

13. What was Mr. Munsford doing when *Bedford* found the Russian submarine?

    Taking a shower

14. What does Commander, NATO, North Atlantic tell *Bedford* to do?

    Track the Russian submarine and take no other action

15. What does the second message from COMNATONORTH tell *Bedford* to do?

    Wait

16. What does the Russian submarine try to do to escape?

    Find a narrow path through the icebergs that Bedford *cannot get through*

17. What is the name of the *Bedford's* SONAR operator?

    Queffel

18. Why do the doctor and the Captain get into an argument?

    The Captain and the doctor disagree about what to do about Queffel when Queffel breaks down.

19. What does *Bedford* do when the Russian submarine finally surfaces?

    Tries to run the submarine down

20. Why does Mr. Ralston fire the rocket torpedo at the Russian submarine?

    He mistakes a casual remark from the Captain as an order to fire

21. What happens to *Bedford* at the end of the movie?

    She is destroyed by torpedoes fired from the Russian submarine

Name: ___________________________
Date: _____________________________
Advisor: __________________________

The Bedford Incident – Questions

Place all answers on this sheet. 7 – 13 correct answers = 2.
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4. What does Mr. Munsford do for a living?

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13. What was Mr. Munsford doing when *Bedford* found the Russian submarine?

14. What does Commander, NATO, North Atlantic tell *Bedford* to do?

15. What does the second message from COMNATONORTH tell *Bedford* to do?

16. What does the Russian submarine try to do to escape?

17. What is the name of the *Bedford's* SONAR operator?

18. Why do the doctor and the Captain get into an argument?

19. What does *Bedford* do when the Russian submarine finally surfaces?

20. Why does Mr. Ralston fire the rocket torpedo at the Russian submarine?

21. What happens to *Bedford* at the end of the movie?

Game Rules

I. Set-up.

A. **Game Board:** The exercise will be played on the classroom game board. Each grid square is 50 nautical miles by 50 nautical miles in area.

B. **Scenario:** A United States *Los Angeles* class nuclear – powered submarine has sunk in the Central Pacific Ocean. It's last known position (known as the DATUM) was 350 nautical miles northwest of the Hawaiian Islands. This would put it in grid square 01036 on the game board. Because of ocean currents, however, the submarine could be in any one of the surrounding grid squares.

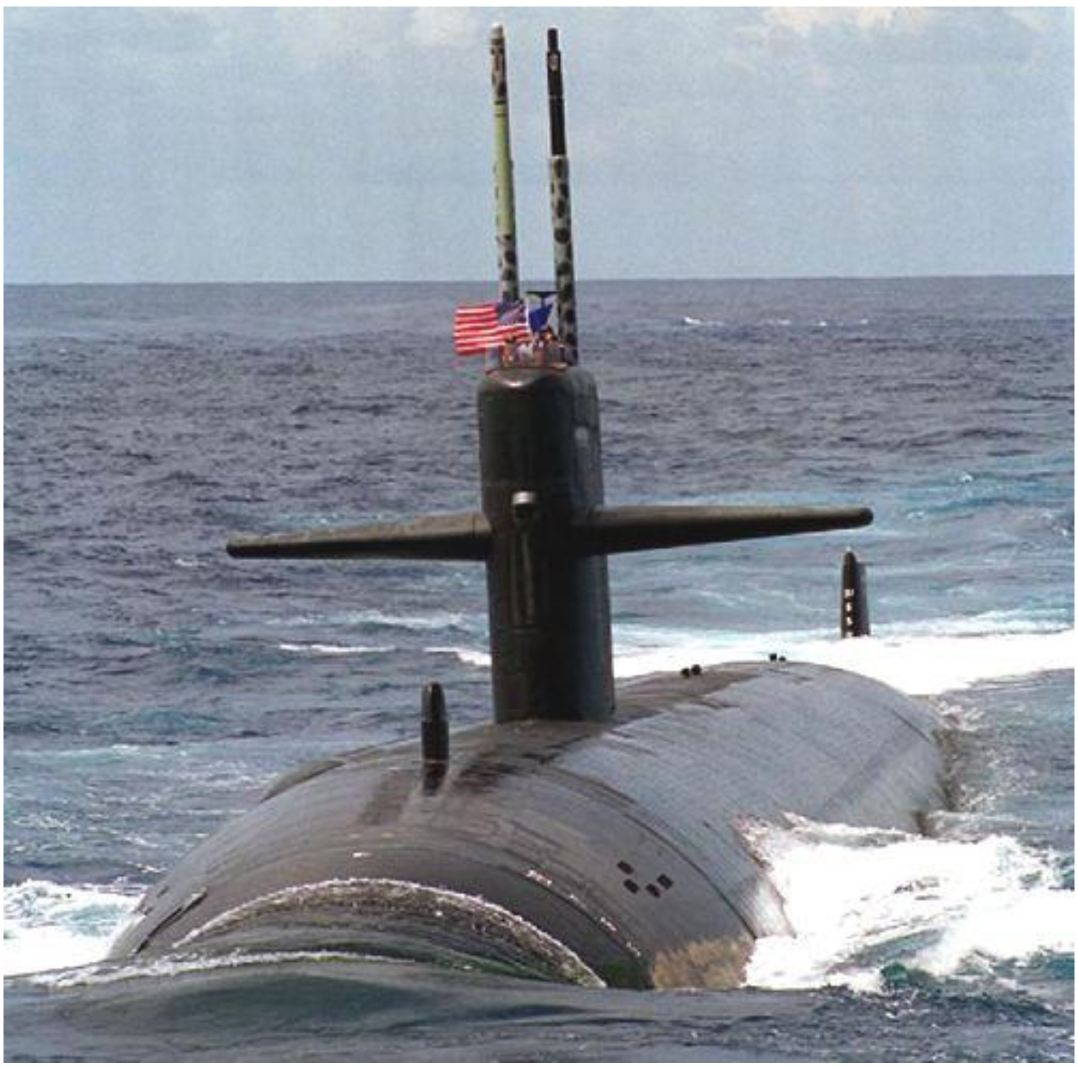


Figure 1: U.S. *Los Angeles* Class Nuclear-powered Submarine

Game Rules

Of the 141 Sailors onboard, there are 75 survivors who must be rescued within 72 hours or their power will fail and they will suffocate. The mission of your team is to find the submarine within 60 hours so that a deep sea rescue vehicle (DSRV) can descend to the submarine and rescue the survivors. **TS of the submarine is 60db and its SL is 30db.**

Figure 2: Deep Sea Rescue Vehicle (DSRV)

C. **Detection Probabilities and Ocean Environment:** The probabilities that the submarine is in each of the grid squares as well as the environmental impacts on SONAR and payoff points in each grid square are as follows:

Grid Square	Probability of Detection at 10 kts.	Time to Search at 10 kts.	Probability of Detection at 20 kts.	Time to Search at 20 kts.	Noise Level	Payoff Points
01036	1.00	20 hours	0.75	10 hours	30db	10
01037	0.50	20 hours	0.40	10 hours	30db	20
01024	0.50	20 hours	0.40	10 hours	10db	20
01048	0.50	20 hours	0.40	10 hours	10db	20
01025	0.40	20 hours	0.30	10 hours	10db	30
01049	0.40	20 hours	0.30	10 hours	10db	30
01035	0.30	20 hours	0.15	10 hours	30db	40
01023	0.20	20 hours	0.10	10 hours	10db	50
01047	0.20	20 hours	0.10	10 hours	10db	50

Figure 3: Location Probabilities and Ocean Environment

Game Rules

II. Teams.

A. **Composition.** Each class will be divided into teams of three.

B. **Team Leader.** Each team leader will be responsible for his or her teams decisions. The team leader will grade the members of his / her team. The team leader will be graded by the classroom teacher.

C. **Procedure.** Each team will be assigned a ship to conduct the search. The Transmission Loss (TL) associated with that ship's SONAR will be established based on the average of the team member's scores on the unit problem sets. The higher the (TL), lower the chances of detecting the sunken submarine. Therefore, the better the students performed on the problem sets, the better their chances of detecting the submarine.

Average Problem Set Grades	Transmission Loss (TL)
4	**10db**
3	**20db**
2	**30db**
1	**40db**

III. Game Play. The object of the game is to score the most points. Teams will be awarded points for selecting the grid square in which the submarine rests and for accurately detecting the submarine. Each team has 60 hours to search for and detect the sunken submarine.

A. Turns. A turn will be a team's search of a grid square. Each team will continue play until it runs out of hours of search.

B. Grid Selection: Each team will select a grid square to search and a search speed. Each grid square is assigned payoff points with higher points assigned to grid squares which have lower probabilities that the submarine is in that grid square. Teams will be awarded that grid square's payoff points if:

1. The team accurately detects the sunken submarine in that grid square.

2. The team completes a search of that grid square, failing to detect the submarine but it is subsequently determined that the submarine is in that grid square.

C. Speed Selection. Each team will select a search speed, either 10 knots or 20 knots. Detection probability drops at the higher speed but a team can complete a search more quickly.

D. SONAR Equation Solution. Each team will solve its SONAR equation for each grid square searched by finding its detection threshold (DT). Since DT is a measure of probability of detection, DT's will be assigned probabilities of detection, given the submarine is in the grid square, as follows:

Detection Threshold (DT)	Conditional Probability of Detection $P(DT = \_\_\_\_)$
0 – 9 db	0.20
10 – 19 db	0.50
20 – 29 db	0.75
30 db and greater	1.00

Correctly solving the SONAR equation results in the award of 10 points. For example:

1. Team 1 decides to search grid square 01025 at 10 knots. Team 1 achieved an average grade of "2" on the unit problem sets so the team's TL is 30db. The NL of 01025 is 10db.

2. The SONAR equation, therefore, is:

$$DT = TS + SL - 2TL - NL$$

$$DT = 60db + 30db - 2(30db) - 10db = 20db$$

E. Determination of Overall Detection Probability.

1. The classroom teacher, acting as umpire, will determine the team's overall detection probability. In the previous example, if the submarine is in the grid square, it would be:

$$P(overall) = P(DT = 20db) \times P(at\ speed\ of\ 20\ kts.) = 0.75 \times 0.40 = 0.30$$

2. Using a random number generator, the classroom teacher will determine the outcome of the search.

    a. If the random number is greater than $P(overall)$, then the classroom teacher will inform the team that they have not detected the submarine but that the submarine may still be in the grid square.

    b. If the random number is less than or equal to $P(overall)$, then the classroom teacher will inform the team that they have found the submarine if it is in the grid square or that it is definitely not in the grid square if, in fact, it is not. If it is not, the team will know that it need not search the square again.

F. Points Award. In the above example, points would be awarded as follows:

1. If the submarine is in grid square 01025:

$$100\ points\ (for\ finding\ the\ submarine) + 30\ points\ (payoff\ points) = 130\ points$$

2. If the submarine is in grid square 01025 but is not detected:

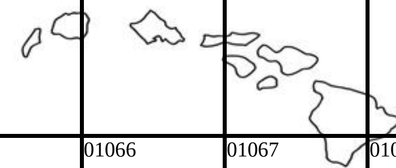
$$30\ points\ (awarded\ after\ the\ submarine\ is\ discovered\ by\ that\ team\ or\ someone\ else)$$

3. If the submarine is not in the grid square:

$$0\ points$$

Game Board

01021	01022	01023	01024	01025	01026	01027	01028	01029	01030	01031	01032
01033	01034	01035	01036 DATUM	01037	01038	01039	01040	01041	01042	01043	01044
01045	01046	01047	01048	01049	01050	01051	01052	01053	01054	01055	01056
01057	01058	01059	01060	01061	01062	01063	01064	01065	01066	01067	01068



Submarine Search Worksheet

Team: __________

Turn: __________

1. Team Grade Average: ____________________________

   Team Transmission Loss (TL): ____________________________

2. Search Speed Selection (10 or 20 kts.): ____________________________

3. Grid Square Selection:

   01036
   01037
   01024
   01048
   01025
   01049
   01035
   01023
   01047

4. Grid Square Noise Level (NL):

   a. ____________________________

   b. ____________________________

5. Sonar Equation Solution:

DT =	TS +	SL -	2TL -	NL
a. _____ b. _____	a. _____ b. _____	a. _____ b. _____	a. _____ b. _____	a. _____ b. _____

6. Overall Probability of Detection:

   a. ____________________________

   b. ____________________________

$$P(overall) = P(DT = \_\_\_\_) \times P(at\ speed\ of\ \_\_\_\ kts.)$$

7. Search Outcome: ____________________________

8. Points Awarded: ____________________________

Name: ____________________________
Date: ______________________________
Advisor: ____________________________

Algebra I Practice Assessment – Equations

1. Solve for the variable in each equation:

a. $x + 7 = 10$

b. $-9 + x = 5$

c. $k + 6.8 = -4.2$

d. $8x = 72$

e. $\frac{q}{7} = 8$

f. $-\frac{x}{8} = 44$

g. $66 = 2f - 4$

h. $-4 = \frac{x}{2} - 5$

i. $n - 6 = 2n - 14$

2. Solve for the variable in each equation:

a. $8x - 1.5 = 2x$

b. $6p - 12 = -4p + 18$

c. $-10x + 3 = -3x + 12 - 4x$

d. $5(x - 3) = 10$

e. $7x - 2(x + 6) = -2$

f. $3 - 4x = 5(x + 6)$

g. $x + \frac{5}{8} + \frac{3x}{4} = \frac{2}{3} + 5x$

h. $2x + 3 = 3x + 5$

i. $5m - 3(2m - 3) = 2(m + 3)$

Name: ____________________________
Date: ______________________________
Advisor: ____________________________

Algebra I Practice Assessment II – Equations

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet.

1. Solve for the variable in each equation:

a. $x + 9 = 15$ b. $11x = 132$ c. $k + 4.4 = -5.6$

e. $p/12 = 10$ e. $-\frac{x}{15} = 40$ f. $2n - 8 = 3n - 16$

2. Solve for the variable in each equation:

a. $10x - 4.8 = 2x$ b. $6p - 15 = -5p + 18$ c. $7(x - 4) = 14$

e. $9x - 4(x + 6) = -4$ e. $38 - 11x = 9(x + 2)$ f. $x + \frac{5}{9} + \frac{3x}{18} = \frac{2}{6} + 5x$

3. Solve each equation for the indicated variable:

a. $p + b = d, for\ b$ b. $y = zx + 12, for\ z$ c. $15t = 7r, for\ t$

d. $\frac{ma}{q} = q, for\ a$ e. $4a + 4b = 28c, for\ b$ f. $6y = 2x + 3b, for\ b$

Algebra I Assessment – Equations

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet.

1. Solve for the variable in each equation:

a. $x + 14.5 = 21.5$ b. $\frac{12x}{15} = 4$ c. $15n - 6 = 9n - 42$

d. $9(x - 2) = 27$ e. $38 - 12x = 4(2x + 2)$ f. $2x + \frac{1}{4} + \frac{4x}{6} = \frac{2}{9} + x$

g. $14x - 5.5 = 3x$ h. $37.68\ in. = 2\pi r\ in.$ i. $100°C = \frac{5}{9}(x°F - 32)$

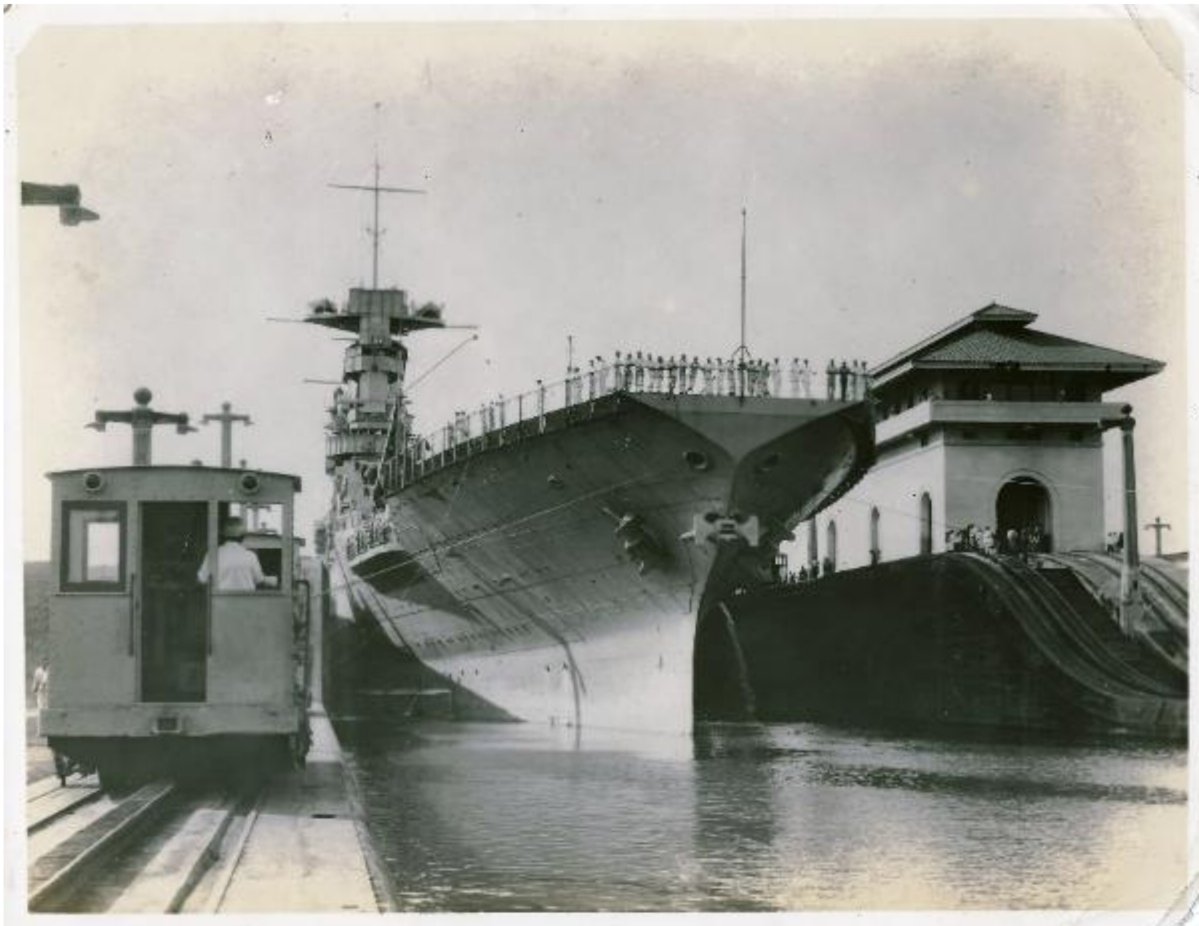
2. Solve each equation for the indicated variable:

a. $e - f = g, for\ g$ b. $6\alpha = 2\beta + 3\Phi, for\ \Phi$ c. $v = \frac{P}{m}, for\ m$

3. Use the formula $A = \frac{1}{2}h(b_1 + b_2)$ to find the length of a missing base for a trapezoid with the following dimensions. Round your answers to the nearest hundredth.

A. A = 200, b_1 = 24, h = 10

PAUL CUFFEE SCHOOL
A Maritime Charter School for Providence Youth



"A Voyage to the Panama Canal"

9th Grade Algebra and Geometry Unit Plan

Thomas R. Beall
Captain, U. S. Navy (Ret.)

Name: ______________________

Date: ______________________

Advisor: ______________________

A Voyage to the Panama Canal

1. **Background of the Panama Canal.**[12] The 50 mile – long international waterway known as the Panama Canal allows ships to pass between the Atlantic Ocean and Pacific Ocean, saving about 8000 miles (12,875 km) from a journey around the southern tip of South America, Cape Horn. Construction of the canal was completed in 1914.

   The canal makes the trip from the east coast to the west coast of the U.S. much shorter than the route taken around the tip of South America prior to 1914. Though traffic continues to increase through the canal, many oil supertankers and large warships cannot fit through the canal. There is even a class of ships known as "Panamax," those built to the maximum capacity of the Panama Canal and its locks.

   It can take anywhere from 8 to 25 hours to traverse the canal through its three sets of locks (including waiting time due to traffic). Ships passing through the canal from the Atlantic Ocean to the Pacific Ocean actually move from the northwest to the southeast, due to the east-west orientation of the Isthmus of Panama.

2. **Locks.**[13] The canal's locks are large basins which raise or lower ships to the level of the seaway the ship is entering by raising or lowering the water level under a ship. Each lock is 110 feet wide by 1000 feet long. The amount of concrete used for all locks was 3,440,488 cubic meters. The entrance and exit to each lock is through large gates are 65 feet wide and 7 feet thick. The heights vary from 47 to 82 feet depending on its position.

---

[12] Rosenberg, Matt. *Panama Canal*. Accessed online Nov. 28, 2011 at: http://geography.about.com/od/specificplacesofinterest/a/panamacanal.htm.

[13] Source: Panama Canal Authority. Accessed online Nov. 28, 2011 at: http://www.pancanal.com/eng/noticiero/canal-faqs/physical/physical-faqframeset-v2.html

Water enters the lock chamber through a system of main culverts, which are the same size as the Hudson River tubes of the Penn Central Station Railroad and are "large enough to drive a train through." From these main culverts, 10 sets of lateral culverts extend under the lock chamber from the side wall and 10 sets from the center wall. Each lateral culvert has a set of 5 openings, each measuring 4-1/2 feet in diameter. As the water is released into the main culverts, by way of a gravity flow system, by opening upper end valves and closing lower end ones, it is diverted into the 20 lateral culverts and distributed through 100 openings in the chamber floor.

An average of 52 million gallons of fresh water is used each time a lock is filled. A lock chamber takes approximately eight (8) minutes to be filled. All water used in any lock chamber comes from Gatun Lake. This lake covers 163.38 square miles and was created when Madden Dam was built. At one time, Gatun Lake was the largest artificial lake in the world.

N 8°57'11.52"
N 8°56'45.6"
N 8°56'19.68"
W 79°34'40.8"
W 79°34'14.88"
W 79°33'48.96"
W 79°33'23.04"
Calle Tabernilla
Owen St.
Calle Balboa
Calle La Boca
Calle Plank
Calle Akee
Calle Mango
Calle Empire
Calle Croton
Calle Martinida
Calle Perlas
Calle College
Av Amador
Puente de las Americas
Carretera Panama-Arraijan
Acceso a Veracruz
Calle Cedar
Calle Miller
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 12/25/2009
2004
8°57'01.72" N 79°33'46.18" W elev 75 ft

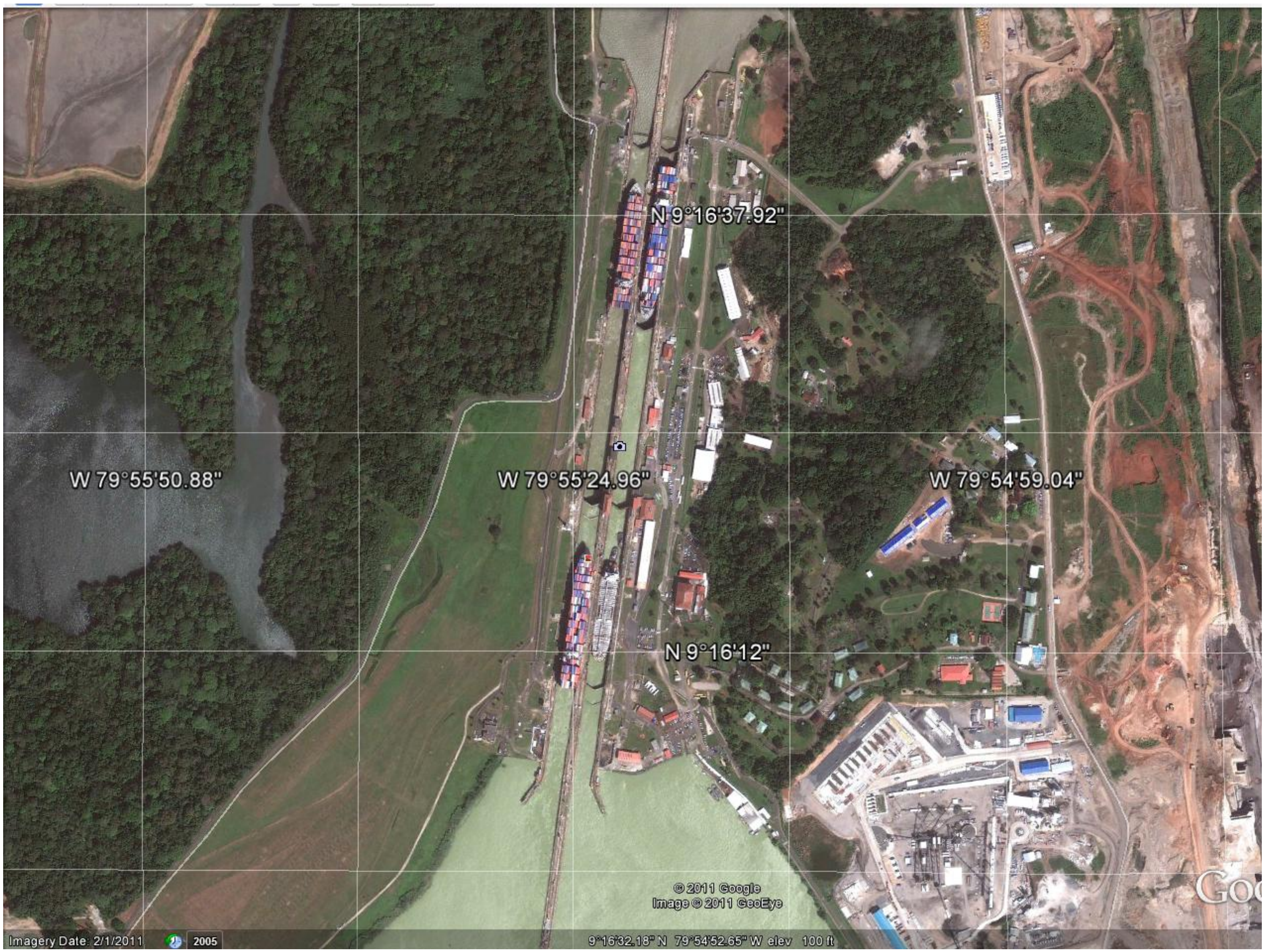
N 8°59'47.04"
56"
W 79°35'32.64"
W 79°35'6.72"
N 8°59'21.12"
Image © 2011 DigitalGlobe
© 2011 Google
Calle Morse
Cidade-do-Saber
Via Vieja De Gaillard
Calle Landrick
Calle Hamilton
Calle Salteman


N 9°01'56.64"
W 79°38'8.16"
W 79°37'42.24"
N 9°01'30.72"
Puente Centenario
Via Centenario
Calle Nata
Calle Sigri
Calle Espaba
Calle Otoque
Paraiso Rd.
Calle Jaraba
Calle Guiana
Calle Hamilton
Calle Palo Alto
Calle Chiriqui
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 12/25/2009
2002
9°01'43.42" N 79°37'37.48" W elev 138 ft


W 79°39'25.92"
N 9°03'14.4"
W 79°39'
Image © 2011 DigitalGlobe
© 2011 Google
Imagery Date: 1/11/2008
2002
9°03'04.66" N 79°38'53.27" W elev 246 ft


N 9°07'7.68"
W 79°45'2.88"
W 79°44'36.96"
W 79°44'11.04"
N 9°06'41.76"
© 2011 Google
Image © 2011 GeoEye
Google earth
2010
9°07'20.26" N 79°44'01.92" W elev 93 ft
Eye alt 6974 ft


W 79°46'20.64"
© 2011 Google
Image © 2011 GeoEye
Imagery Date 2/20/2011
2010
9°06'41.87" N 79°46'15.80" W elev 92 ft


N 9°16'37.92"
W 79°55'50.88"
W 79°55'24.96"
W 79°54'59.04"
N 9°16'12"
© 2011 Google
Image © 2011 GeoEye
Imagery Date: 2/1/2011
2005
9°16'32.18" N 79°54'52.65" W elev 100 ft


N 9°23'32.64"
W 79°55'24.96"
N 9°23'6.72"
W 79°54'59.04"
W 79°54'33.12"
© 2011 Europa Technologies
© 2011 Google
Image © 2011 GeoEye
2/1/2011
2004
9°23'12.93" N 79°54'52.86" W elev -12 ft
Google

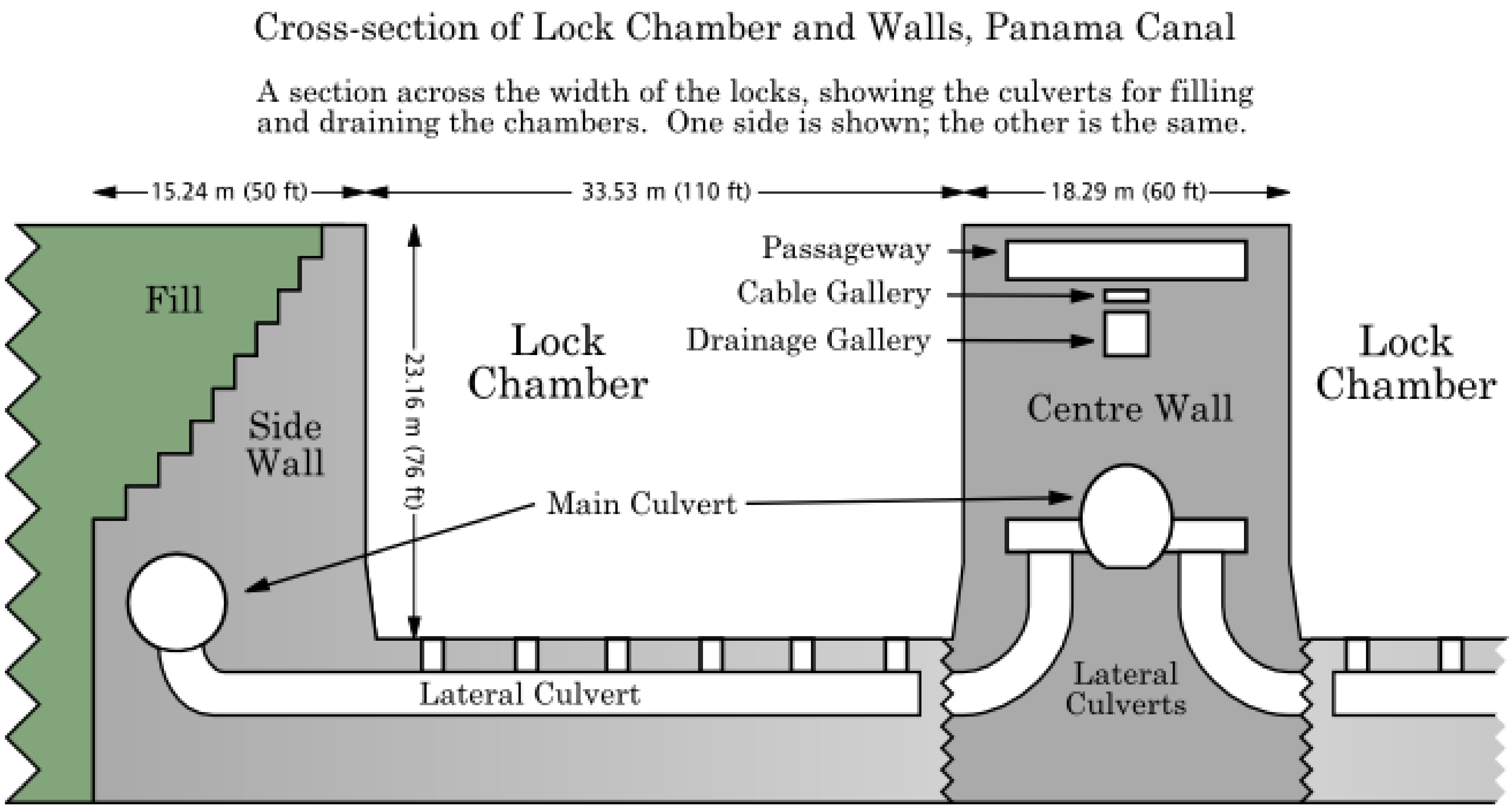


Figure 1: Lock Cross-section

3. **Transit Planning.** We will plan a voyage from the U. S. Naval Station at San Diego, California (32° 40.75’ N, 117° 07.64’ W) to the Panama Canal (9° 23’ 19” N, 79° 55’ 10” W).

Figure 2: Naval Station San Diego in the 1940’s

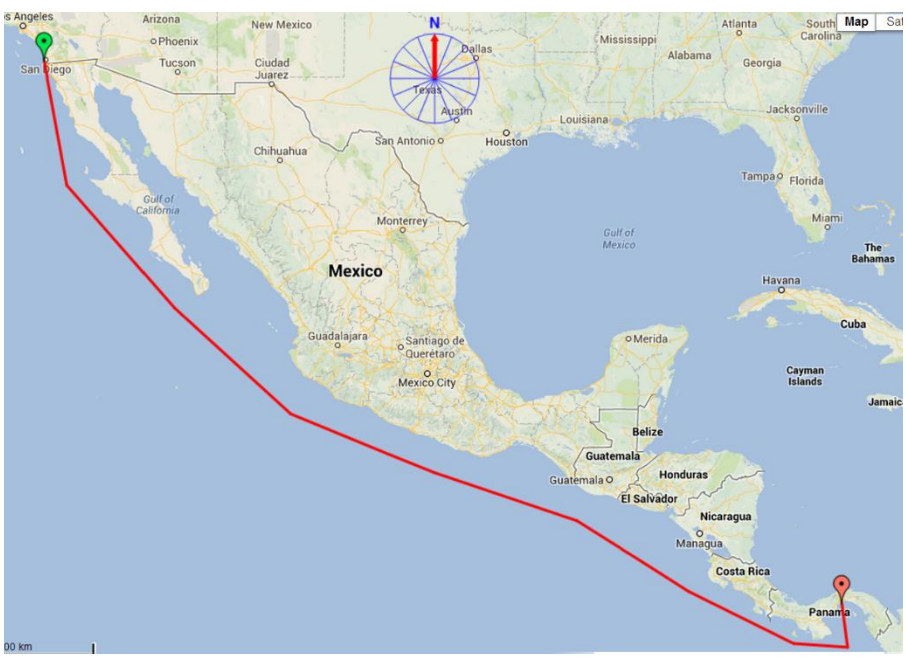
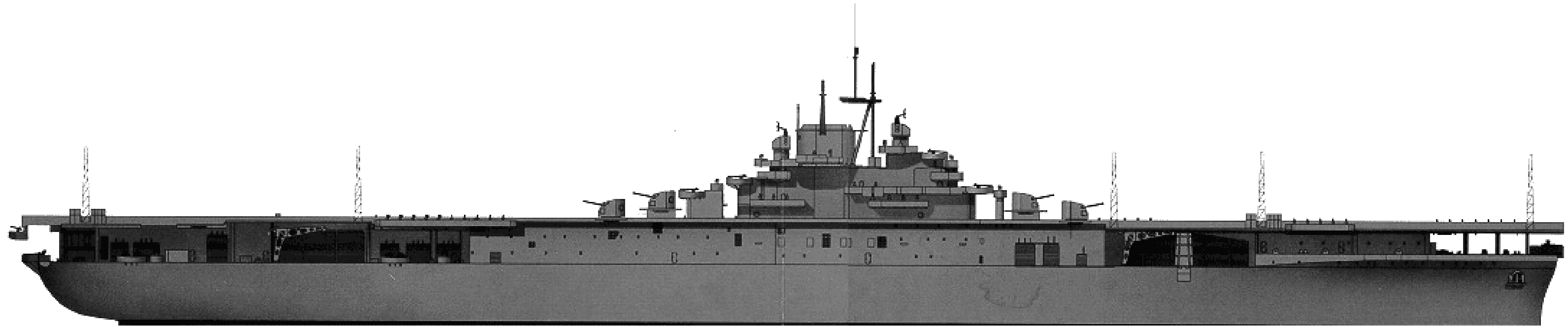
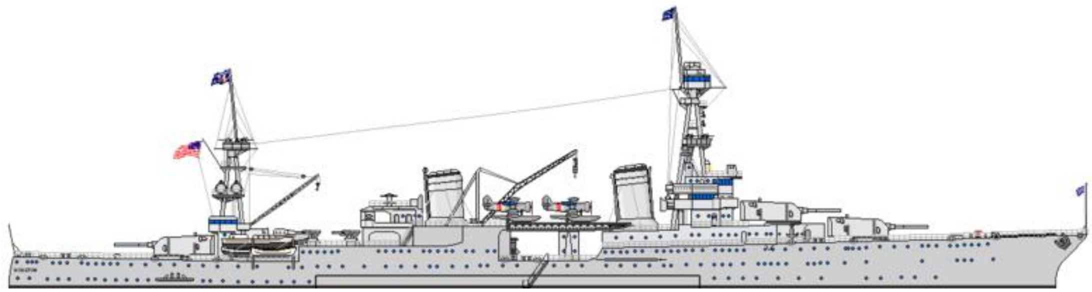


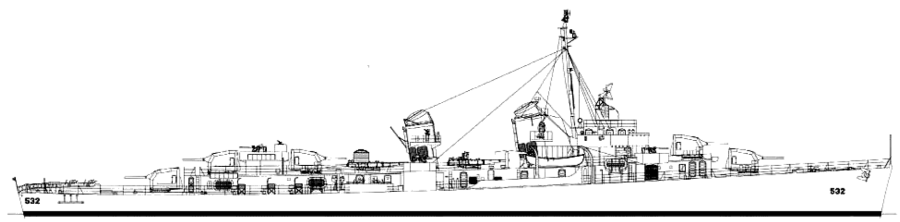
Figure 3: Transit Track



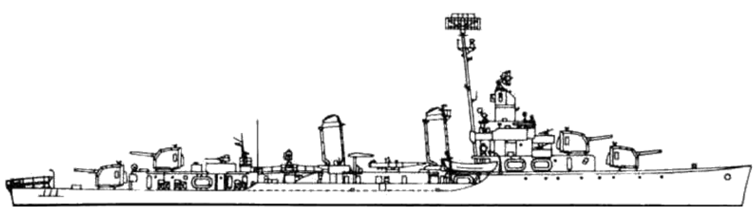
U.S.S. *Essex* (CV – 9)



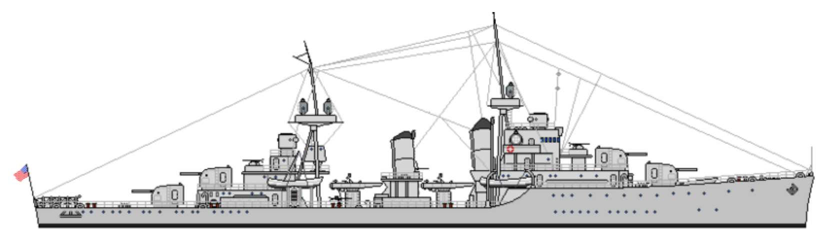
U.S.S. *Augusta* (CA – 31)



U.S.S. *Fletcher* (DD – 445)



U.S.S. *Benson* (DD – 421)



U.S.S. *Conyngham* (DD – 371)

Panama Canal Transit Planning – Task I

1. **Background.** A U. S. Navy Task Force is traveling from San Diego, CA to the Panama Canal. Before the Task Force arrives at the Canal, each ship must provide the Panama Canal Authority (ACP) with information about that ship. You will use the attached sheets, online resources, and the models of your ships provided to assemble the data. The Task Force is organized as follows:

Task Force 5
Commander (CTF FIVE): Captain T. R. Beall, USN (Ret.)

Ship	**Billet**	**Algebra I – 1**	**Algebra I – 2**	**Geometry**
U.S.S. *Essex*	Captain			
(CV 9)	Navigator			
Aircraft Carrier	Quartermaster			
	Chief Engineer			
	M.P.A.			
U.S.S. *Augusta*	Captain			
(CA 31)	Navigator			
Heavy Cruiser	Quartermaster			
	Chief Engineer			
U.S.S. *Fletcher*	Captain			
(DD 445)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			
U.S.S. *Benson*	Captain			
(DD 421)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			
U.S.S. *Conyngham*	Captain			
(DD 371)	Navigator			
Destroyer	Quartermaster			
	Chief Engineer			

Table 1: Navigation Unit Assignments

2. **Instructions.** Each person will complete the attached form, attaching the required graph and arithmetic work. The team "Captain" will approve and sign each person's submission. **No signature, no credit.**

Name: ____________________________

Date:_____________________________

Advisor:___________________________

Panama Canal Ship Data

1. Ship's Name: ________________________

2. Captain's Name: ________________________

3. Ship's Measurements (*Dictionary of American Naval Fighting Ships*[14]):

   a. Displacement (dp): ________________________

   b. Length Overall (l): ________________________

   c. Beam (b): ________________________

   d. Draft (d): ________________________

4. Blind Distances:

   a. Forward: ________________________

   b. Aft: ________________________

5. Fuel Statistics (FTP 218[15]):

   a. Full Load (gallons): ________________________

   b. Fuel Rate at 200 R.P.M. (Table 1):

      i. Gallons / hour (column 2): ________________________

      ii. Speed (column 9): ________________________

      iii. Gallons / engine mile (column 11): ________________________

[14] http://www.history.navy.mil/danfs/
[15] http://www.ibiblio.org/hyperwar/USN/ref/Fuel/

c. From Table 1, use column 1 and column 11 to construct a table and a graph of gallons burned per engine mile for each propeller RPM listed. Fit a linear model and a polynomial model to the data and display the equations (see example on next page).

i. Attach the graph to this sheet.

ii. Write the equations below:

Linear Model: $y =$ ______________________________

Polynomial Model: $y =$ ______________________________

____
Signature of Commanding Officer

Example of a graph of propeller shaft R.P.M. vs. gallons burned per engine mile – U.S.S. *Massachusetts* (BB – 59).

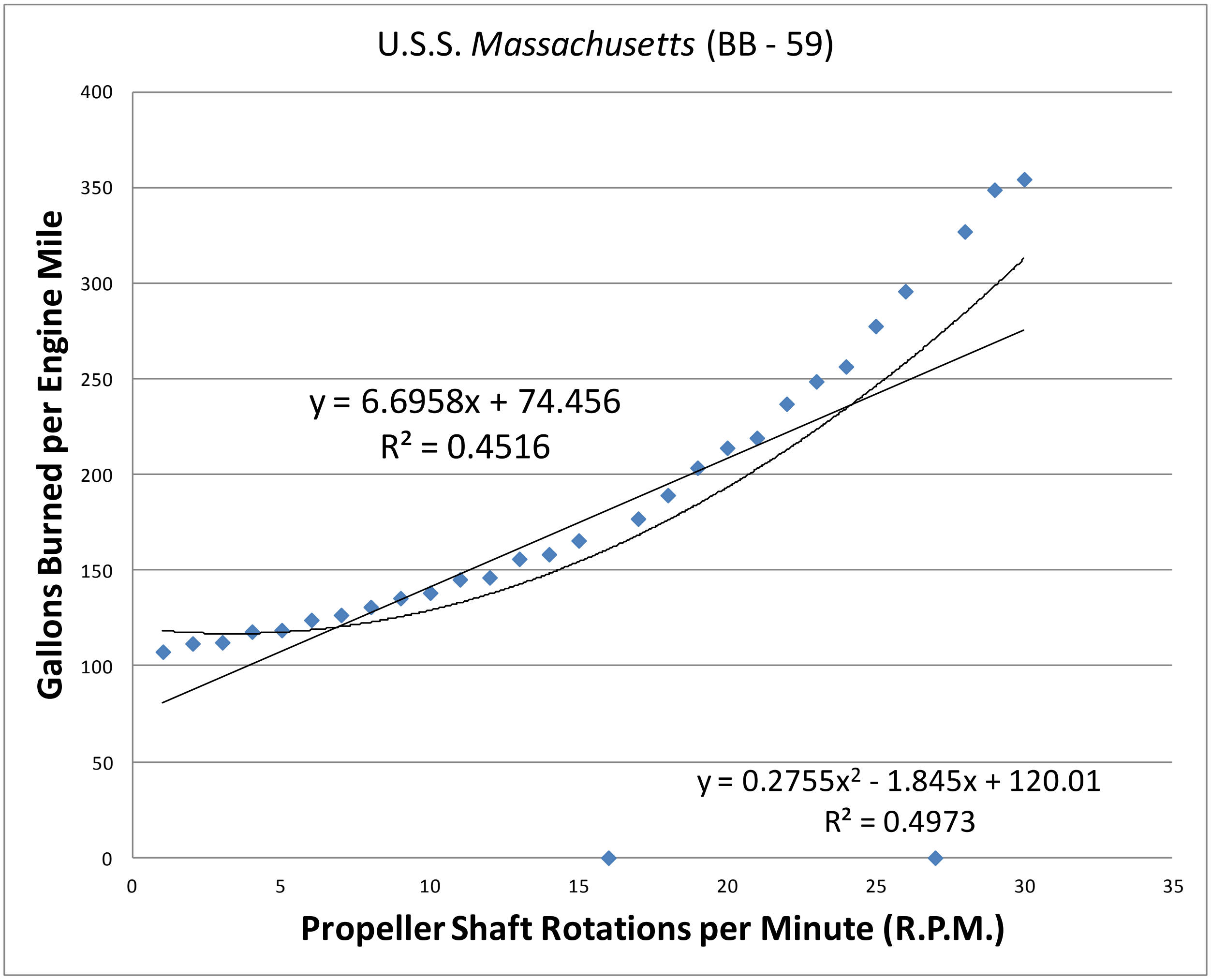


Figure 4: U.S.S. *Massachusetts* (BB – 59) R.PM. vs. Gallons Burned per Engine Mile

Finding "Blind Distance"

1. **Background.** In order to proceed through the canal, you must provide the ACP with a lot of information about your ship. One key piece of information is the ship's "blind distance". This is the distance from the front of the ship (the bow) or the back of the ship (the stern) over which you cannot see the water because it is hidden by the ship. The following diagrams depict the blind distances for an officer standing the ship's bridge (Conning Position No. 1) from the bow and the stern.

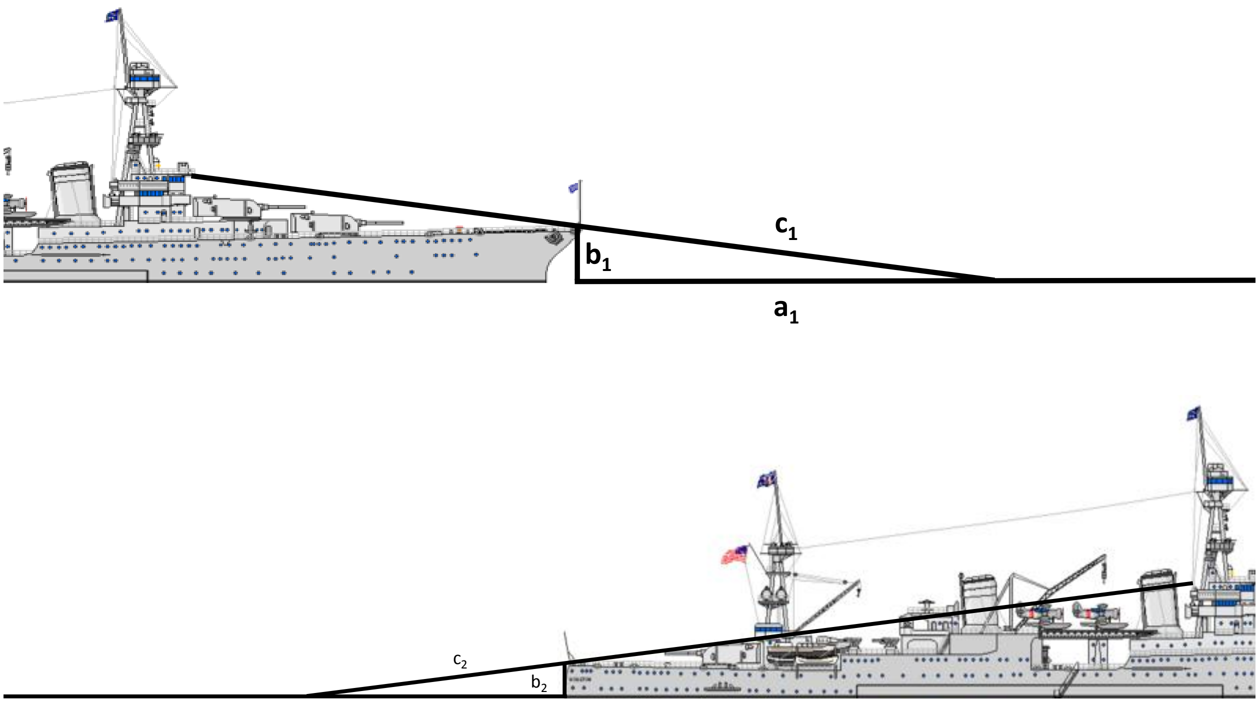
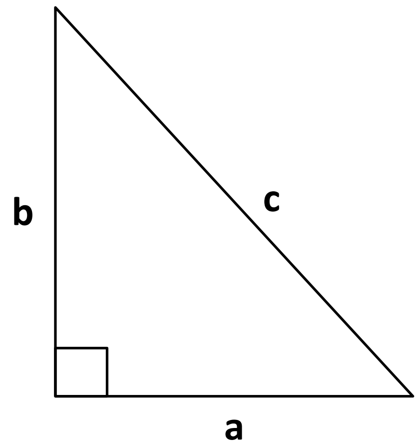


Figure 5: Blind Distances for U.S.S. *Augusta* (CA – 31)

2. To find the "blind distance" from the bow and the stern, you can use a relationship known as the Pythagorean Theorem which states that for any right triangle with sides *a* and *b* and hypotenuse *c*, the following relationship is true:



$$a^2 + b^2 = c^2$$

3. **Task.** To complete the data forms, you must determine the forward and aft "Blind Distances" for your ship. You will use a small scale model of your ship to complete the computations.

   a. Determine the scale factor to convert the model's length in inches to the actual length of the ship in inches.

   b. Measure the distance from the waterline to the deck of the model and use the scale factor to find the actual distance.

   c. Measure the distance from the bridge, through the forepeak, to the waterline of the model and use the scale factor to find the actual distance.

   d. You now have c_i and b_i. Use the Pythagorean Theorem to find a_i. Convert inches to feet and inches for the final answer.

   Ensure you show all work on separate sheets of paper, attached to the data form.

Name: ________________________

Date:_________________________

Advisor:_________________________

Panama Canal Transit Planning – Task II

1. The drawing on the next page is a cross – section view of the canal locks, showing maximum clearances. Your first task will be to determine if your ship can move safely through the locks. Reproduce the lock cross-section drawing on the next page using a scale factor of 0.6.

    a. Multiply each dimension by 0.6 to convert the measurement in meters into centimeters.

    b. Tape four pieces of graph paper together.

    c. Using a ruler and pencil and the centimeter measurements, reproduce the lock cross – section drawing.

2. Draw a cross-section of your ship hull using the same scale factor and place it in the center of the lock to determine if your ship will fit into the lock.

    a. Convert the beam and draft of your ship from feet and inches to meters.

    b. Multiply the new beam and draft measures by 0.6 to convert to centimeters.

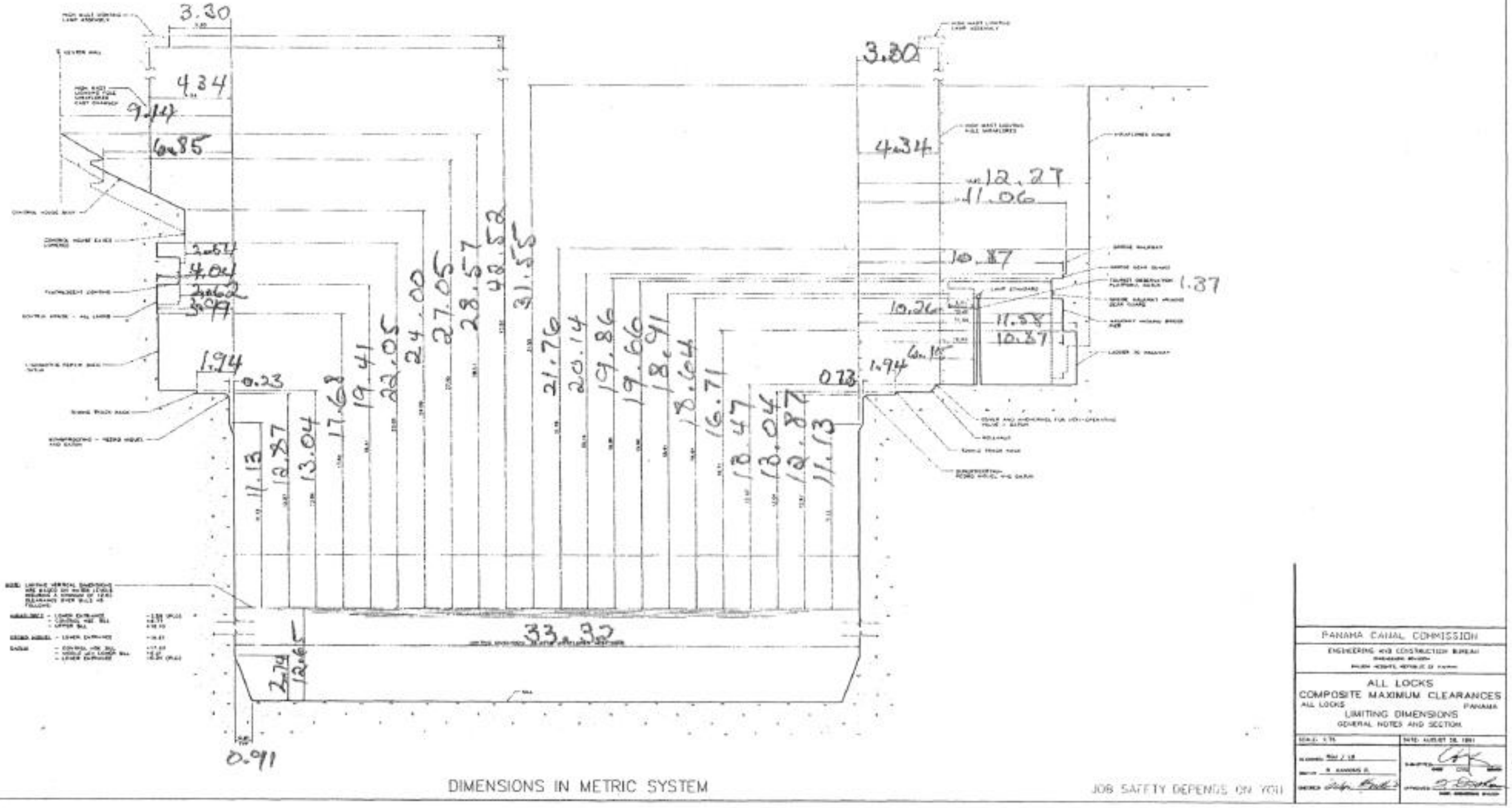
    c. Draw the cross-section of the ship into the center of the lock.

    d. Determine the distance in meters from each side of the ship to the lock wall and the bottom of the ship to the lock floor. Label those distances on your lock drawing.

    e. Your drawing will look like figure 2 on the page following the lock drawing.

ANNEX 2

All Locks Composite Maximum Clearances



SUPERSEDS SAME DWG. 5044-424, DATED 10-17-63

5044-424 SHEET 1 OF 2

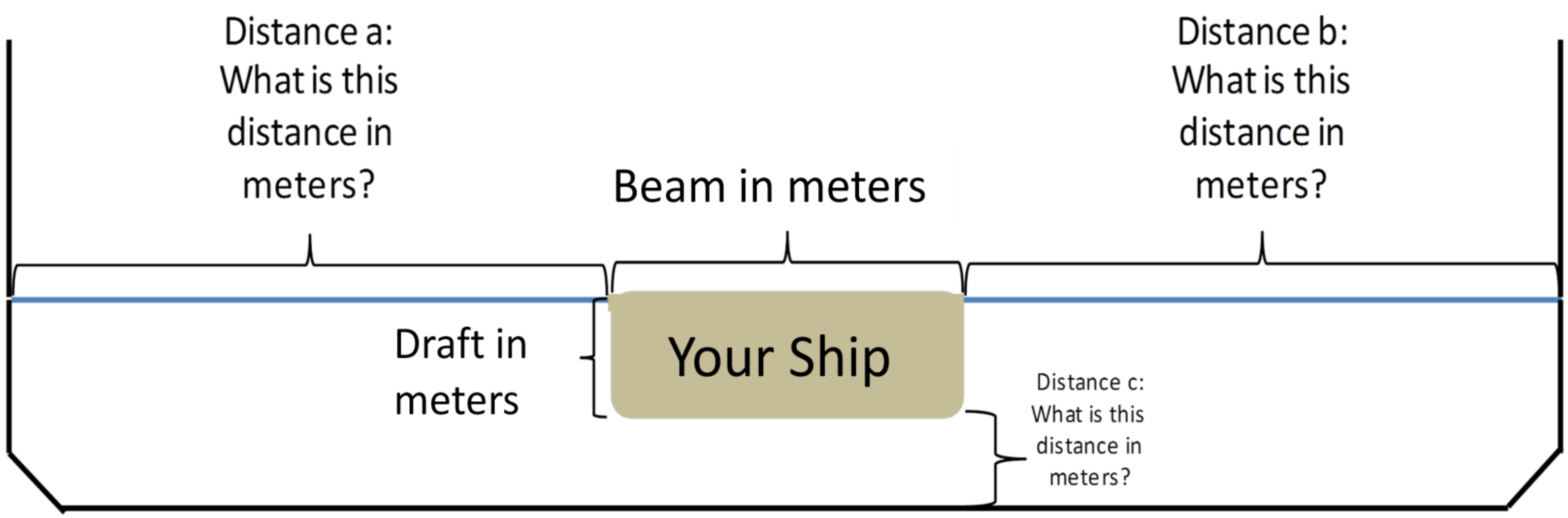


Figure 6: Cross-section of Your Ship in Your Lock Drawing

Measurement Conversions
(Use this page to make your measurement conversions)

$$\mathbf{0.6 \times m = c}$$

Where: $m = the\ actual\ measurement\ in\ meters$

$c = the\ scaled\ measurement\ in\ centimeters$

m	$\times\ scale\ factor =$	c
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
	$\times 0.6 =$	
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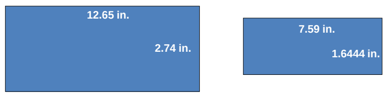
Lock Drawing Slides

Slide 1

Two Geometry Concepts

Congruence: The relationship between figures having the same shape and the same size; congruent segments are segments that match exactly, congruent angles have the same measure.

Similarity: The relationship between figures having congruent angles and proportional sides.



Are these rectangles similar?

Slide 2

Beginning Your Lock Drawing

Step 1: Number Conversions. Multiply each measurement by 0.6 and then convert to centimeters.

m	scale factor	d
12.65	0.6	7.59
2.74	0.6	1.64
0.91	0.6	0.55
11.13	0.6	6.68

Slide 3

Beginning Your Lock Drawing

Step 2: Limiting Dimension line. Find the "Limiting Dimension" line at the bottom of the drawing handed out in class. It measures 33.32 meters so, on your drawing, it will measure:

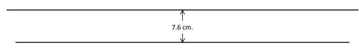
$$33.32 \times 0.6 = 20\ cm.$$

Draw this line near the bottom of your graph paper.

Slide 4

Beginning Your Lock Drawing

Step 3: Lock Sill. Next, measure 7.6 cm. below the "Limiting Dimension" line and draw a 2nd line, centered on the first, that is 1.8 cm. shorter. This line is the "sill" of the lock:

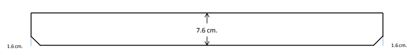


Slide 5

Beginning Your Lock Drawing

Step 4: Bottom of Lock.

- Draw 2 diagonal line segments from the sill such that the highest point of each segment is 1.6 cm. above the level of the sill and directly below the endpoint of the "Limiting Dimension" line.
- Connect the highest point of each segment to the "Limiting Dimension" line.

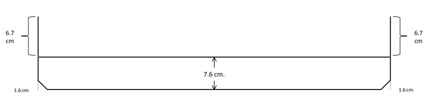


Slide 6

Beginning Your Lock Drawing

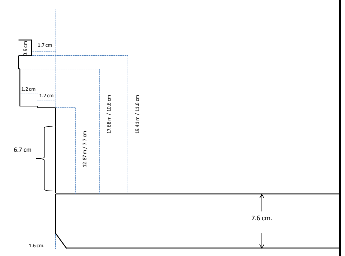
Step 5: Basin Walls. From the "Limiting Dimension" line, draw two vertical lines at each end, 6.7 cm. in length. You have now drawn the lock basin (the part that fills with water).

Now, complete the drawing.



Slide 7

Drawing the Control Tower



Slide 8

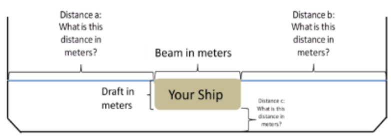
Drawing the Control Tower

If you want to shoot for a "5" or greater:

- ❑ **Use large poster board**
- ❑ **Use a scale factor of 0.8**
- ❑ **Decorate your drawing**
  - ➢ **Buildings and walls illustrated with colors**
  - ➢ **Water illustrated**
  - ➢ **Pumps and culverts per diagram near front of packet**
  - ➢ **Ship drawn / illustrated and colored**

Slide 9

Drawing your ship in the locks



Name: ____________________

Date: ____________________

Advisor: ____________________

Task III – Panama Canal Transit Fuel Consumption
Show all work on separate sheets of paper attached to this one.

3. **Background.** On her journey from San Diego to the Panama Canal, your ship will burn a lot of fuel. It will be important for you to keep track of how much your ship burns and how much is left so that you can decide if and when it is time to refuel your ship. How fast a ship is going and how much fuel is burned over a given mile is a function of how fast the propeller shaft is turning per minute (R.P.M.). The table below provides the propeller shaft R.P.M. for a given speed for all five ships.

	R.P.M.				
Knots	***Essex***	***Augusta***	***Fletcher***	***Benson***	***Conyngham***
10	77.47	96.12	94.75	81.67	189.99
15	109.36	140.24	142.55	123.14	239.84
20	146.19	189.15	193.36	164.60	289.69
25	187.97	242.85	247.17	206.06	339.55
30	234.70	301.34	303.99	247.53	389.40
35	286.36	364.62	363.82	288.99	439.25
40	342.98	432.69	426.65	330.46	489.10

Table 2: Propeller Shaft R.P.M. vs. Ship's Speed

4. **Step 1.** Using table 2, fill in the shaft R.PM. for your ship in column 2 of table 3 for each speed in column 1.

5. **Step 2.** Using the graph and equations you completed for your Panama Canal Ship Data Submission, complete the following table. Using your ship's R.P.M. data shown in the second table. Show all work on separate sheets of paper attached to this one.

$Knots$	$x = R.P.M.$	**Gallons Burned per nm using Linear Equation** $y_1 = \frac{Gallons}{nm} =$	**Distance Traveled in 24 Hours** $y_2\ nm = 24\ hours \times x \frac{nm}{hour}$	**Gallons Burned in 24 Hours**
10				
15				
20				
25				
30				
35				
40				

Table 3: Gallons Burned per Nautical Mile vs. Propeller Shaft R.P.M.

Example: If your linear equation is $4.92x - 42.6564$, then, for a speed of 10 knots:

a. R.P.M. for 10 knots is 75.15. Gallons burned per nautical mile at 75.15 R.P.M. equals:

$$1.53 \times (75.15) - 42.6564 = 114.9795 \frac{gallons}{nm}$$

b. Distance traveled in 24 hours equals:

$$24\ hours \times 10 \frac{nm}{hour} = 240\ nm$$

c. Gallons burned in 24 hours equals:

$$114.9795 \frac{gallons}{nm} \times 240\ nm = 27595.08\ gallons$$

6. **Step 3.** Next complete the following table. Show all work on separate sheets of paper attached to this one.

x = $Knots$	y_1 = Gallons Burned in 24 Hours (from previous table)	y_2 = Full Load of Fuel (from 5. a. of Panama Canal Ship Data Submission)	Gallons Remaining after 24 Hours $= y_2 - y_1$	Percent of Full Load Burned after 24 Hours $= \frac{y_1}{y_2}$
10				
15				
20				
25				
30				
35				
40				

Table 4: Fuel Remaining After 24 Hours

Example: If, at 10 knots, you burn 27595.08 gallons, and your full load of fuel is 588,625 gallons:

a. Gallons remaining after 24 hours:

$$= 588625\ gallons - 27595.08\ = 561029.92\ gallons$$

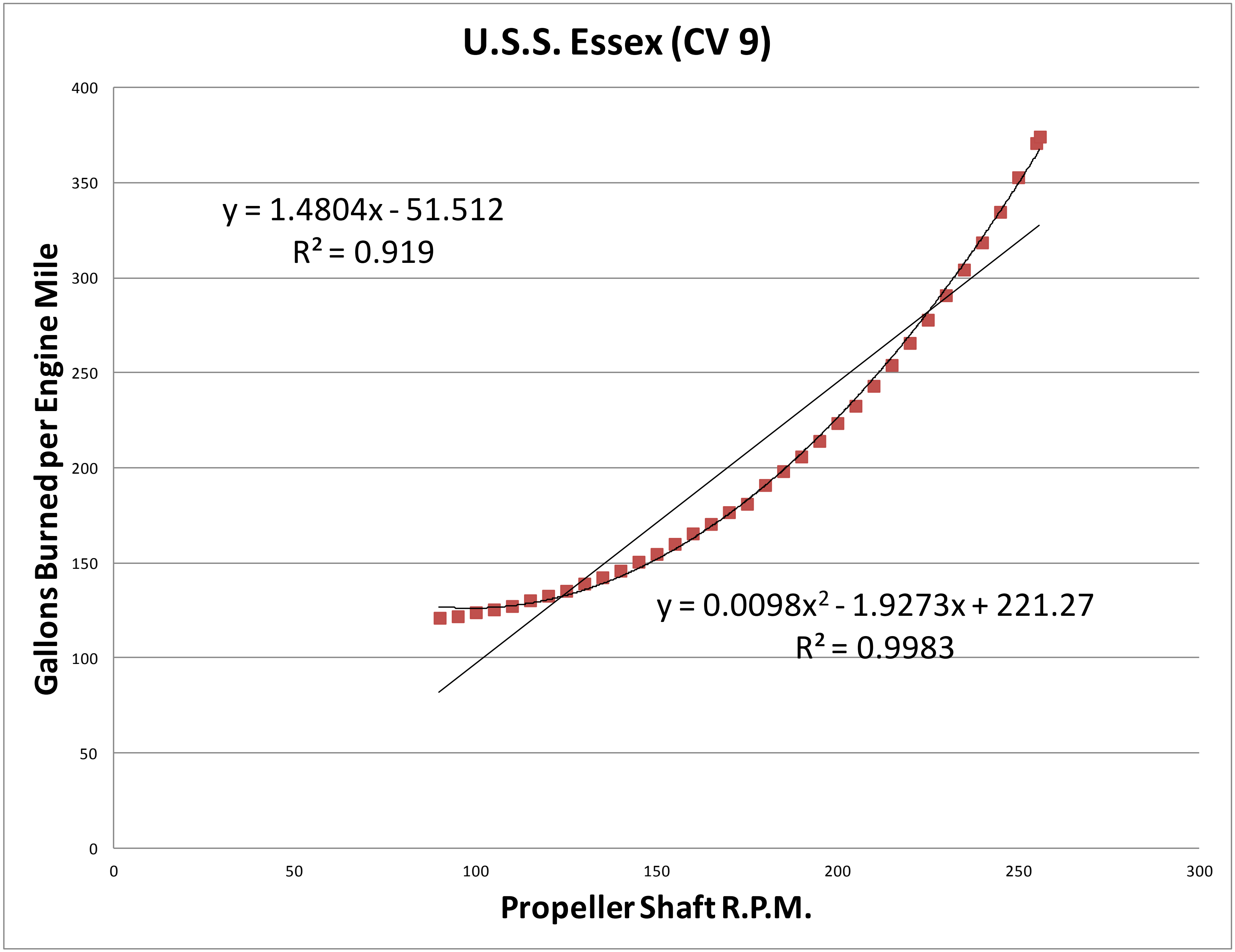
b. Percent of full load burned after 24 hours:

$$\frac{27595.08\ gallons}{588625\ gallons} = 4.69\%$$

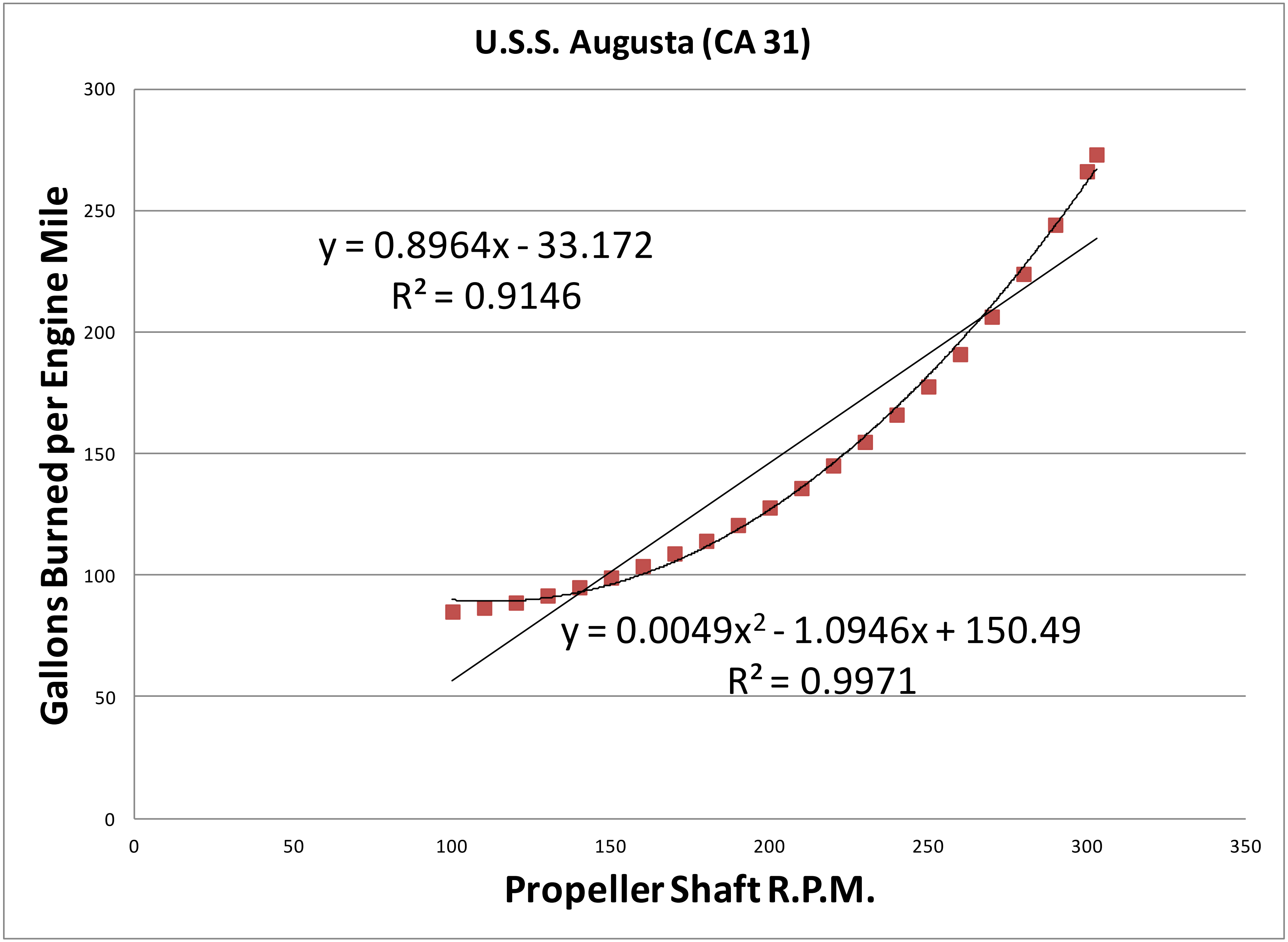
Panama Canal Ship Data Answers

	Essex	*Augusta*	*Fletcher*	*Benson*	*Conyngham*
Displacement	**27,100**	**9,050**	**2,100**	**1,620**	**1,500**
Length	**872’**	**600’ 3”**	**376’ 3”**	**348’ 2”**	**341’ 4”**
Beam	**93’**	**66’ 1”**	**39’ 8”**	**36’ 1”**	**35’**
Draft	**28’ 7”**	**16’ 4”**	**13’**	**17’ 6”**	**9’ 10”**
Blind Dist. F.	**573.03’**	**216.24’**	**127.12’**	**126.36’**	**126.35’**
Blind Dist. A.	**633.36’**	**160.63’**	**168.16’**	**193.00’**	**193.00’**
Full Load Fuel	**1,753,410**	**588,625**	**142,655**	**129,082**	**146,256**
Gal./hr. (200)	**5,837**	**2,661**	**886**	**810**	**783**
Speed (200)	**26.1±0.3**	**20.8±0.1**	**19.9±0.2**	**20.9±0.2**	**20.6±0.2**
Gal./eng. mi.	**223.6**	**127.9**	**44.5**	**38.8**	**38.0**

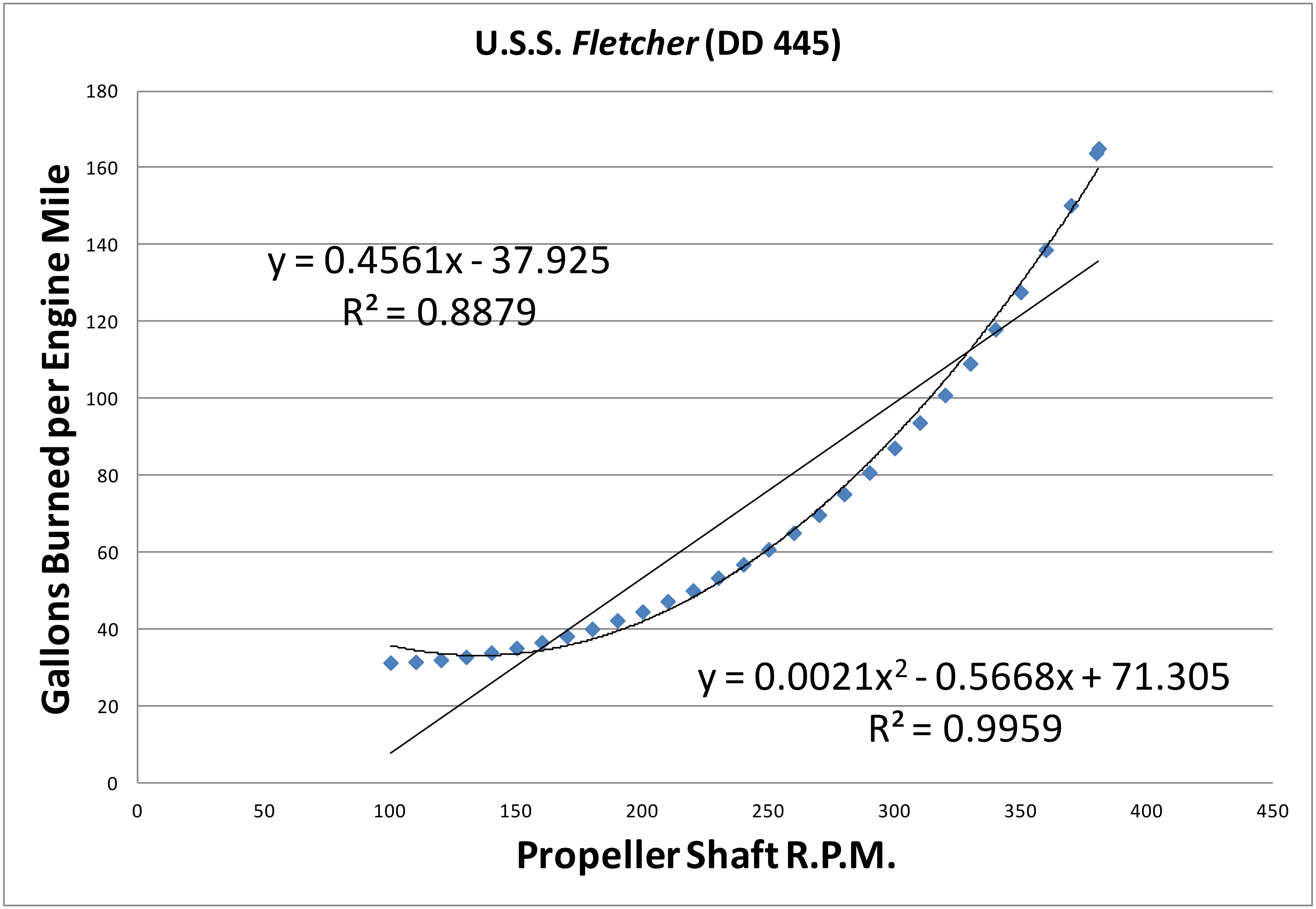
Fuel Consumption Graphs for All Ships



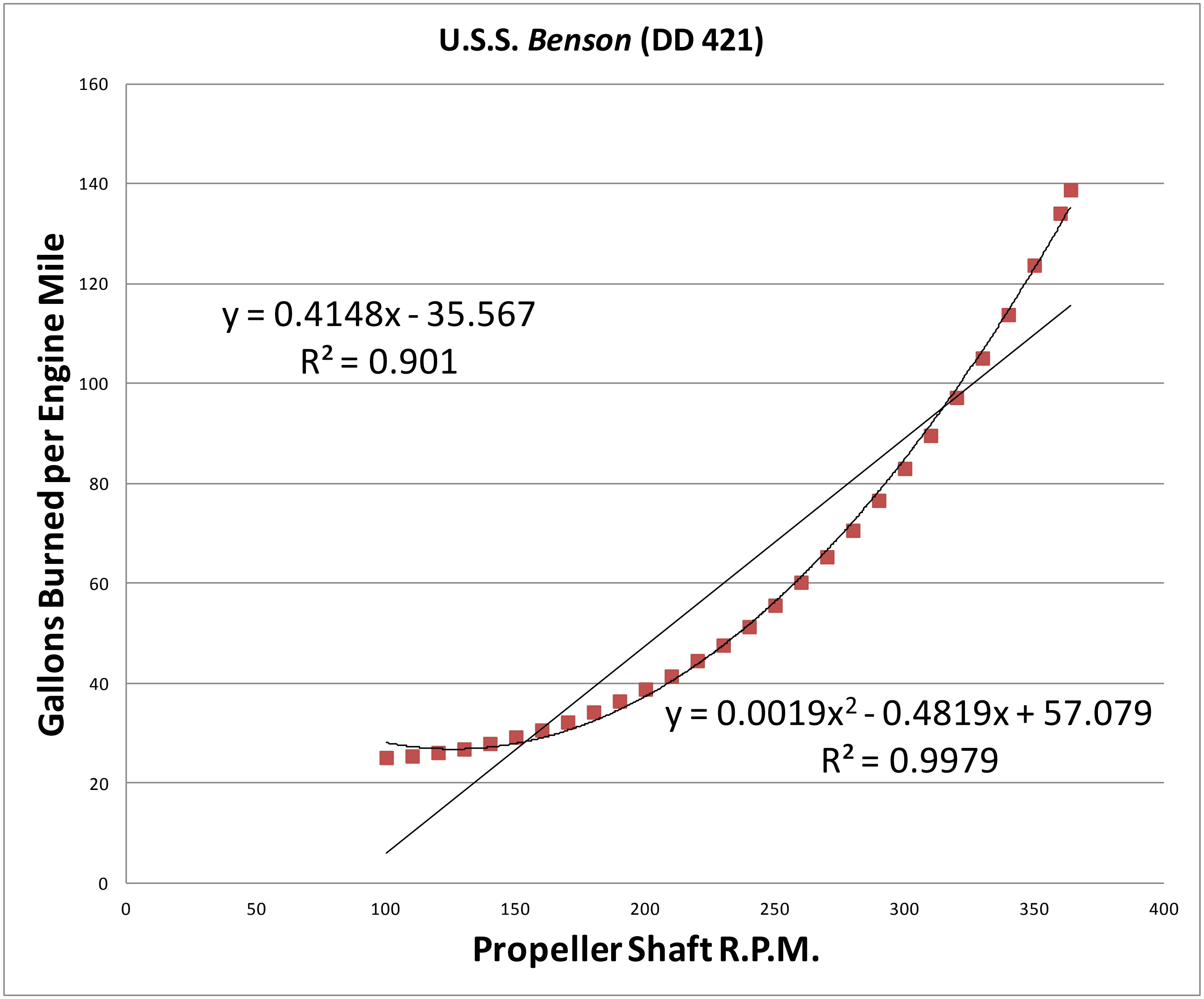
Fuel Consumption Graphs for All Ships



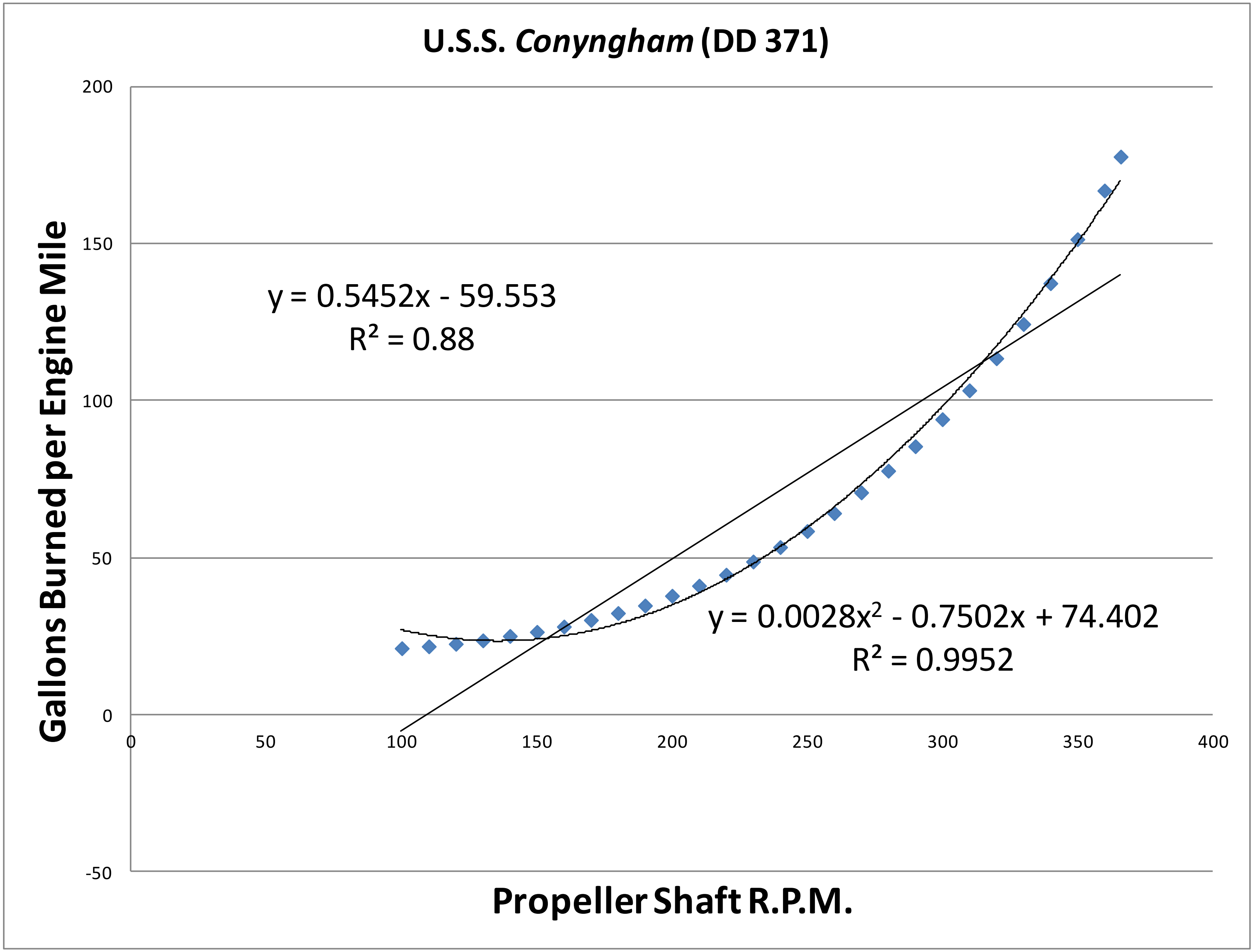
Fuel Consumption Graphs for All Ships



Fuel Consumption Graphs for All Ships



Fuel Consumption Graphs for All Ships



War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Essex* (CV - 9)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

   a. Blind distance submitted does not agree with information on file.

   b. Ship's data is incomplete or inaccurate.

   c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

   a. Captain: **4 / 3 / 2 / 1**

   b. Navigator: **4 / 3 / 2 / 1**

   c. Quartermaster: **4 / 3 / 2 / 1**

   d. Chief Engineer: **4 / 3 / 2 / 1**

   e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commandant, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Augusta* (CA - 31)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

   a. Blind distance submitted does not agree with information on file.

   b. Ship's data is incomplete or inaccurate.

   c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

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   b. Navigator: **4 / 3 / 2 / 1**

   c. Quartermaster: **4 / 3 / 2 / 1**

   d. Chief Engineer: **4 / 3 / 2 / 1**

   e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commandant, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Fletcher* (DD - 445)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commandant, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Benson* (DD - 421)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

Copy to:
Commandant, 15th Naval District
Commander, Task Force 5

War Department
Panama Canal Commission
Army Post Office Miami 34 Florida

From: Panama Canal Commission
To: Commanding Officer, U.S.S. *Conyngham* (DD - 371)

Subj: Panama Canal Transit

Ref: (a) Your project submission dtd 08 Nov

1. Your request to transit the canal northbound is **APPROVED / DISAPPROVED**

2. If **DISAPPROVED**, the reasons are:

    a. Blind distance submitted does not agree with information on file.

    b. Ship's data is incomplete or inaccurate.

    c. Fuel consumption charts are incomplete or inaccurate.

3. You must submit a corrected package no later than Monday, 18 Nov.

4. The grades for your group are as follows;

    a. Captain: **4 / 3 / 2 / 1**

    b. Navigator: **4 / 3 / 2 / 1**

    c. Quartermaster: **4 / 3 / 2 / 1**

    d. Chief Engineer: **4 / 3 / 2 / 1**

    e. M.P.A. **4 / 3 / 2 / 1**

For the Commission,

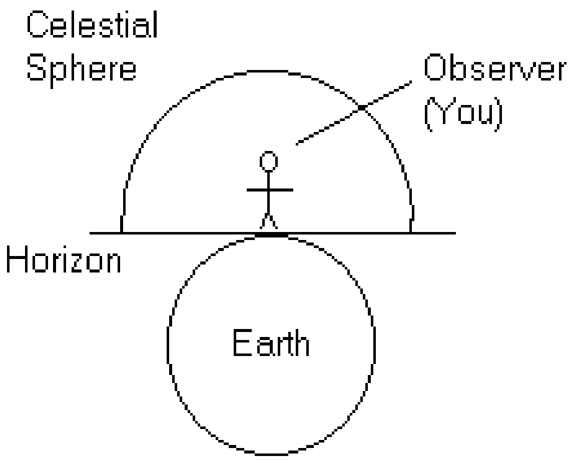
Copy to:
Commandant, 15th Naval District
Commander, Task Force 5

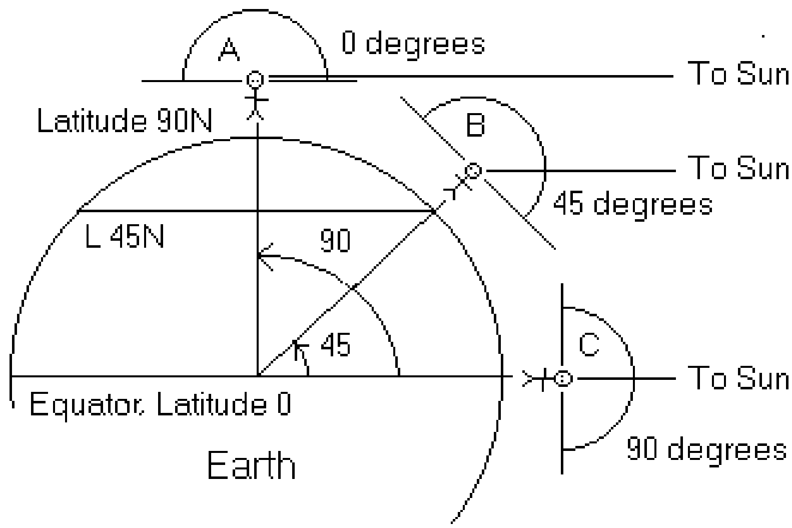
Name: ___________________________

Date:____________________________

Advisor:__________________________

Navigating by Latitude in the Northern Hemisphere[16]





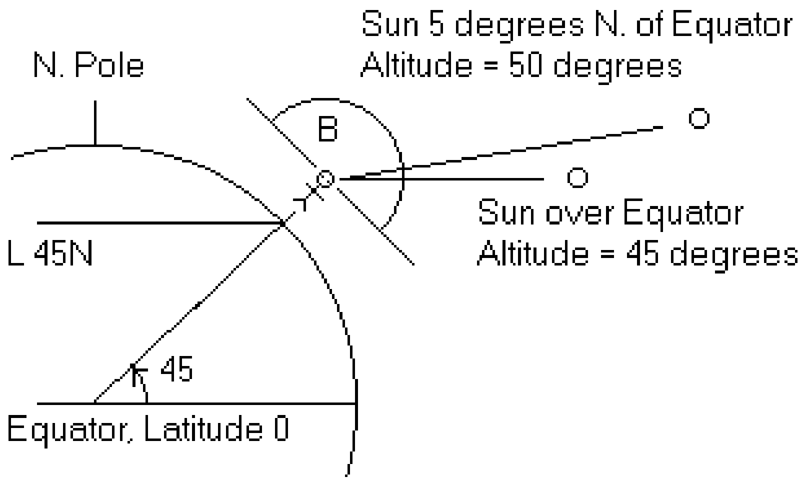
1. If the Sun is directly over the Equator, you can determine your latitude by observing the angular elevation of the sun over the horizon and then using the following formula:

$$Latitude = 90° - Sun's\ altitude$$

Find your latitude if the Sun's altitude is as follows:

Sun's Altitude	Your Latitude
15°	
27°	
45°	
70°	
90°	

[16] From Sammons, James I.: Navigating Around the World by Observing the Sun, accessed online at http://www.pbs.org/wgbh/nova/teachers/ideas/sammons/packet.html.



2. If you know how high above or below the Equator the Sun is on a given day (called the Sun's declination), you can find your latitude using the following formula:

$Latitude = 90° - Sun's\ altitude + Sun's\ declination$ (Sun in Northern Hemisphere)

$Latitude = 90° - Sun's\ altitude - Sun's\ declination$ (Sun in Southern Hemisphere)

Find your latitude if the Sun's altitude and declination are as follows:

Sun's Declination	Sun's Altitude	Latitude
15° N	42° N	
17° S	17° N	
23° N	68° N	

3. In the Northern Hemisphere, you can determine your latitude at night by observing the altitude of Polaris (the North Star)"

$$Latitude = Altitude\ of\ Polaris$$

Telling time at sea.

After 12 noon was determined by the Sun’s altitude,

1. A sailor would watch and turn a one-half-hour glass.
2. At each turning, he would ring the ship’s bell:

1 bell	12:30 AM / PM	4:30 AM / PM	8:30 AM / PM
2 bells	1:00	5:00	9:00
3 bells	1:30	5:30	9:30
4 bells	2:00	6:00	10:00
5 bells	2:30	6:30	10:30
6 bells	3:00	7:00	11:00
7 bells	3:30	7:30	11:30
8 bells	4:00	8:00	12:00

Name: ____________________________

Date: ____________________________

Advisor: ____________________________

Navigating by Latitude in the Northern Hemisphere

1. You are travelling at sea in the Northern Hemisphere. At 12 noon:

   a. You observe the Sun at an altitude of 49° above the horizon.
   b. You know that the Sun's declination today is 9° N.

   Therefore, your latitude is:

$$90° - 49° + 9° = 50°N$$

2. At midnight, you observe Polaris at 50° 21' above the horizon. How many miles north of your noon latitude have you moved?

$$21' = 21\ nm$$

3. At 12 noon the next day:

   a. You observe the Sun at an altitude of 48° above the horizon.
   b. You know the Sun's declination for today is 8° N.

   Therefore, your latitude is:

$$90° - 48° + 8° = 50°N$$

   If one degree of latitude equals 60 nautical miles, how many nautical miles north or south of yesterday's noon latitude are you today?

$$50° - 50° = 0°$$

Name: ________________________

Date:_________________________

Advisor:_________________________

Navigating by Latitude in the Northern Hemisphere

1. You are travelling at sea in the Northern Hemisphere. At 12 noon:

   a. You observe the Sun at an altitude of 49° above the horizon.
   b. You know that the Sun's declination today is 9° N.

   Therefore, your latitude is: ____

2. At midnight, you observe Polaris at 50° 21' above the horizon. If one minute of latitude equals one nautical mile, how many miles north of your noon latitude have you moved?

   ____

3. At 12 noon the next day:

   a. You observe the Sun at an altitude of 48° above the horizon.
   b. You know the Sun's declination for today is 8°.

   Therefore, your latitude is: ____

   If one degree of latitude equals 60 nautical miles, how many nautical miles north or south of yesterday's noon latitude are you today?

   ____

Name: ________________________

Date:__________________________

Advisor:_________________________

Navigating by Latitude in the Northern Hemisphere II

Introduction: You are the navigator of His Britannic Majesty's Ship *Centurion*, a 60 gun sailing battle ship sent to the Pacific Ocean in the year 1740 to seize a great treasure ship that sails each year from Manila in the Philippines to Acapulco in Mexico. You have begun to sail west from Acapulco to Manila.

1. **Basic Skills:**

   a. Determine your ship's noon latitude and record it in the log table below:

Date	Sun's Altitude	Sun's Declination	Latitude
Nov 05	55°55' N	17°15' S	16°50' N
Nov 06	55°43' N	17°32' S	16°45' N
Nov 07	55°32' N	17°48' S	16°40' N
Nov 08	55°21' N	18°04' S	16°35' N
Nov 09	55°10' N	18°20' S	16°30' N
Nov 10	55°00' N	18°35' S	16°25' N
Nov 11	54°50' N	18°50' S	16°20' N

   b. Why are the declinations of the Sun "south" at this time of year?

   Because it is after the autumnal equinox so the Sun is south of the Equator.

   c. Has your latitude increased or decreased between November 11th and November 17th? Does this mean *Centurion* is moving north or south?

d. Your log indicates the following distances travelled over a 30 – second interval at the following times during November 16th. Compute your ship's average speed for each watch using the logged data in the table below. **Show all work on separate sheets of paper.**

Time	**Noon – 4 pm**	**4 – 8 pm**	**8 pm – 12 am**	**12 – 4 am**	**4 – 8 am**	**8 am - Noon**
1 bell	69 yards	99 yards	**83.3 yards**	84.2 yards	62.7 yards	68 yards
2 bells	71.2 yards	94.5 yards	**116 yards**	100 yards	62 yards	69.2 yards
3 bells	73 yards	94.5 yards	**83.3 yards**	85.3 yards	64.3 yards	69 yards
4 bells	75.3 yards	90.2 yards	**66.67 yards**	68 yards	61 yards	67.8 yards
5 bells	**80 yards**	90 yards	**66.67 yards**	64.3 yards	61.8 yards	65.4 yards
6 bells	88.2 yards	89.4 yards	**83.3 yards**	83.3 yards	61.2 yards	68 yards
7 bells	95 yards	86 yards	**83.3 yards**	83.3 yards	66 yards	66.2 yards
8 bells	94.3 yards	84.2 yards	**66.67 yards**	60 yards	70 yards	68 yards
Average Speed						

Example: For the 8 pm to 12 am watch, average speed (A) can be computed as follows:

$$A = \frac{\left(\frac{83.3 + 116 + 83.3 + 66.67 + 66.67 + 83.3 + 83.3 + 66.67}{8}\right) \times 2 \times 60}{2000} =$$

e. Notice that your average speed increases on the Noon – 4 pm watch and decreases again on the 8 pm – 12 am watch. What do you think accounts for this?

Wind speed often increases in the afternoon when air temperature rises.

Name: ___________________________

Date:____________________________

Advisor:_________________________

Navigating by Latitude in the Northern Hemisphere II

Introduction: You are the navigator of His Britannic Majesty's Ship *Centurion*, a 60 gun sailing battle ship sent to the Pacific Ocean in the year 1740 to seize a great treasure ship that sails each year from Manila in the Philippines to Acapulco in Mexico. You have begun to sail west from Acapulco to Manila.

2. **Basic Skills:**

   a. Determine your ship's noon latitude and record it in the log table below:

Date	Sun's Altitude	Sun's Declination	Latitude
Nov 05	55°55' N	17°15' S	
Nov 06	55°43' N	17°32' S	
Nov 07	55°32' N	17°48' S	
Nov 08	55°21' N	18°04' S	
Nov 09	55°10' N	18°20' S	
Nov 10	55°00' N	18°35' S	
Nov 11	54°50' N	18°50' S	

   b. Why are the declinations of the Sun "south" at this time of year?

   c. Has your latitude increased or decreased between November 11th and November 17th? Does this mean *Centurion* is moving north or south?

d. Your log indicates the following distances travelled over a 30 – second interval at the following times during November 16th. Compute your ship's average speed for each watch using the logged data in the table below. **Show all work on separate sheets of paper.**

Time	**Noon – 4 pm**	**4 – 8 pm**	**8 pm – 12 am**	**12 – 4 am**	**4 – 8 am**	**8 am - Noon**
1 bell	69 yards	99 yards	**83.3 yards**	84.2 yards	62.7 yards	68 yards
2 bells	71.2 yards	94.5 yards	**116 yards**	100 yards	62 yards	69.2 yards
3 bells	73 yards	94.5 yards	**83.3 yards**	85.3 yards	64.3 yards	69 yards
4 bells	75.3 yards	90.2 yards	**66.67 yards**	68 yards	61 yards	67.8 yards
5 bells	**80 yards**	90 yards	**66.67 yards**	64.3 yards	61.8 yards	65.4 yards
6 bells	88.2 yards	89.4 yards	**83.3 yards**	83.3 yards	61.2 yards	68 yards
7 bells	95 yards	86 yards	**83.3 yards**	83.3 yards	66 yards	66.2 yards
8 bells	94.3 yards	84.2 yards	**66.67 yards**	60 yards	70 yards	68 yards
Average Speed						

Example: For the 8 pm to 12 am watch, average speed (A) can be computed as follows:

$$A = \frac{\left(\frac{83.3 + 116 + 83.3 + 66.67 + 66.67 + 83.3 + 83.3 + 66.67}{8}\right) \times 2 \times 60}{2000} =$$

e. Notice that your average speed increases on the Noon – 4 pm watch and decreases again on the 8 pm – 12 am watch. What do you think accounts for this?

Name: ________________________

Date:__________________________

Advisor:_______________________

Navigating by Latitude and Longitude[17]

1. To find your **latitude at sea**, all you need is to measure how far the Sun is above the horizon (*Sun's altitude*) and know what degree of latitude the Sun is over at this time of year (*Sun's declination*). We then apply the formulas we have already learned:

$$Latitude = 90° - Sun's\ altitude + Sun's\ declination$$

(Sun over Northern Hemisphere)

$$Latitude = 90° - Sun's\ altitude - Sun's\ declination$$

(Sun over Southern Hemisphere)

2. To find your **longitude at sea,** you need to determine your local time. To do this, you need an accurate time piece and you need to know how far east or west of the Prime Meridian you are.

   a. The Earth rotates on its axis and completes one revolution (on average) every 24 hours. Since the Earth is a sphere, it rotates 360° (on average) every 24 hours. Therefore, it rotates 15° every hour because:

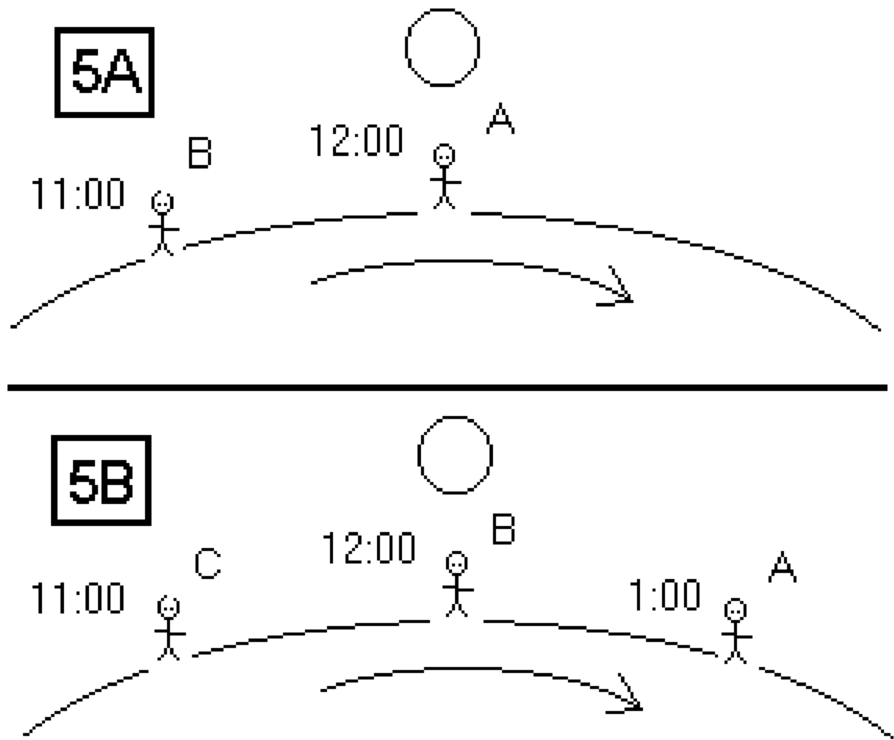
$$360° \div 24\ hours = 15°\ per\ hour = \frac{15°}{hr}.$$

   b. Taking this one step further, we can say that the Earth rotates 1° in four minutes because:

$$1\ hour \div 15° = 60\ minutes \div 15° = 4\ minutes\ per\ degree = \frac{4'}{1°}.$$

[17] From Sammons, James I.: Navigating Around the World by Observing the Sun, accessed online at http://www.pbs.org/wgbh/nova/teachers/ideas/sammons/packet.html.

c. Finding your local time is easy, just look at your watch. To determine your longitude, however, you have to compare that time with the time at the Prime Meridian (called Greenwich Mean Time or GMT). To do that, you have to consider how time differs across the Earth. Look at the following diagram:



i. In picture 5A, two people are standing on the Earth. Person B is 15° west of Person A. The Sun is directly above Person A so he is observing his Local Apparent Noon (12:00 PM). Person B, 15° west, will not observe his Local Apparent Noon for one hour (because it will take the Earth one hour to turn the 15° necessary to put Person B directly under the Sun). Person B's local time, therefore, is 11:00 AM.

ii. In picture 5B, as the Earth turns, Person A moves away from the Sun (so that the Sun appears to be moving toward the western horizon) and Person B moves closer to the Sun (so that the Sun appears to be moving directly overhead). When the Sun is directly overhead Person B, it is his Local Apparent Noon (12:00 PM). Person A, 15° east of person B, is at a local time of 1:00 PM, while Person C, 15° east of Person B, is at a local

time of 11:00 AM. This is where the idea of "time zones" comes from. Persons A, B, and C are each in a different time zone.

iii. When we compare the times of Persons A, B, and C, we find that Person C has the earliest time because he is furthest west. Person A has the latest time because he is furthest east. This leads to the following rule:

Local time earlier, position is westward.

Local time later, position is eastward.

d. Once an accurate time piece that could be taken to sea was invented, sailors determined their longitude by setting their time pieces (called chronometers) to Greenwich Mean Time (GMT – the time at the Prime Meridian) and then, each day, comparing the time of Local Apparent Noon to GMT.

(1) $$12{:}00 - 8{:}24 = 3{:}36$$

(2) $$3\ hours \times \frac{15°}{hour} = 45°$$

(3) $$36' \div \frac{4'}{1°} = 9°$$

(4) $$45° + 9° = 54°$$

Since LAN is later than GMT, we use the rule above and determine "local time later, position is eastward" so our longitude is 54° E.

Name: ___________________________

Date:_____________________________

Advisor:_____________________________

Navigating by Latitude and Longitude

Show all work on a separate sheet of paper.

1. You are aboard a ship navigating the Pacific Ocean. Each day at LAN (12:00 PM local time) you observe the Sun to determine your latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 21st	20:00	40° 13'	19° 47' S		
November 22nd	20:04	41° 50'	20° 00' S		
November 23rd	20:08	41° 27'	20° 13' S		
November 24th	20:12	41° 04'	20° 26' S		

Name: ___________________________

Date:____________________________

Advisor:____________________________

Navigating by Latitude and Longitude

Show all work on a separate sheet of paper.

2. You are aboard a ship navigating the Pacific Ocean. Each day at LAN (12:00 PM local time) you observe the Sun to determine your latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 21st	20:00	40° 13'	19° 47' S	30° 00'N	120° 00' W
November 22nd	20:04	41° 50'	20° 00' S	28° 10' N	121° 00' W
November 23rd	20:08	41° 27'	20° 13' S	28° 20' N	122° 00' W
November 24th	20:12	41° 04'	20° 26' S	28° 30' N	123° 00' W

Name: ____________________________

Date:______________________________

Advisor:______________________________

Voyage to the Panama Canal – Review Nr. 1

Show all work on separate sheets of paper.

In this unit so far we have explored a number of Algebra and Geometry concepts in the context of planning a voyage to the Panama Canal. Specifically, we have:

1. **Learned the following equations for determining a ship's latitude at sea:**

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude + Sun's\ declination$$

(Sun over Northern Hemisphere)

$$Latitude\ (at\ noon) = 90° - Sun's\ altitude - Sun's\ declination$$

(Sun over Southern Hemisphere)

$$Latitude\ (at\ night\ in\ Northern\ Hemisphere) = Altitude\ of\ Polaris$$

Review problems: Compute ship's latitude at noon for the following:

Date	Sun's Altitude	Sun's Declination	Latitude
Dec 8	55°55' N	22°46' S	
Dec 9	55°43' N	22°52' S	
Dec 10	55°32' N	22°57' S	
Dec 12	55°21' N	23°02' S	

2. **Learned how to tell time at sea:**

Telling time at sea.

After 12 noon was determined by the Sun's altitude,

1. A sailor would watch and turn a one-half-hour glass.
2. At each turning, he would ring the ship's bell:

1 bell	12:30 AM / PM	4:30 AM / PM	8:30 AM / PM
2 bells	1:00	5:00	9:00
3 bells	1:30	5:30	9:30
4 bells	2:00	6:00	10:00
5 bells	2:30	6:30	10:30
6 bells	3:00	7:00	11:00
7 bells	3:30	7:30	11:30
8 bells	4:00	8:00	12:00

3. **Learned an equation for determining ship's speed taken every half – hour:**

$$A = \frac{\left(\dfrac{sum\ of\ the\ half - hourly\ distances\ traveled\ in\ yards}{number\ of\ half - hourly\ distances\ measured}\right) \times 2 \times 60}{2000}$$

Review Problems: Compute the average speed traveled on each watch in nautical miles per hour.

Time	**Noon – 4 pm**	**4 – 8 pm**
1 bell	70 yards	98 yards
2 bells	72.2 yards	93.5 yards
3 bells	74 yards	93.5 yards
4 bells	76.3 yards	89.2 yards

5 bells	81 yards	91 yards
6 bells	89.2 yards	88.4 yards
7 bells	96 yards	85 yards
8 bells	95.3 yards	85.2 yards
Average Speed		

4. **Learned that every 15 degrees of longitude equals one hour of time and every degree of longitude equals 4 minutes of time.**

$$360° \div 24\ hours = 15°\ per\ hour = \frac{15°}{hour}.$$

$$1\ hour \div 15° = 60\ minutes \div 15° = 4\ minutes\ per\ degree = \frac{4'}{1°}.$$

5. **Learned equations for computing longitude at sea.**

For east longitude:

(1) $12{:}00 - Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) = h{:}mm$

(2) $h \times \frac{15°}{hour} = degrees\ east\ of\ 0°\ longitude$

(3) $mm \div \frac{4'}{1°} = additional\ degrees\ east\ of 0°\ longitude$

(4) $add\ the\ two\ degrees\ east\ together\ to\ get\ ship's\ longitude\ east$

For west longitude:

(1) $$Greenwich\ Mean\ Time\ of\ LAN\ (GMT\ of\ LAN) - 12{:}00 = h{:}mm$$

(2) $$h \times \frac{15°}{hour} = degrees\ west\ of\ 0°\ longitude$$

(3) $$mm \div \frac{4'}{1°} = additional\ degrees\ west\ of 0°\ longitude$$

(4) $$add\ the\ two\ degrees\ west\ together\ to\ get\ ship's\ longitude\ west$$

Review Problems: Compute your ship's longitude for the dates below.

Date	GMT of LAN	Longitude
December 8th	20:08	
December 9th	20:13	
December 10th	20:15	
December 11th	20:19	

6. **Learned how to construct a precise scale drawing of a Panama Canal lock and transiting ship using a scale factor equation to convert actual measures to drawing measures:**

$$centimeters = (scale\ factor) * meters$$

$$meteres = \frac{centimeters}{scale\ factor}$$

Review Problems: Make the following conversions using a scale factor of 0.26.

45 meters to centimeters	
67 meters to centimeters	
108 centimeters to meters	
244 centimeters to meters	
12.8 meters to centimeters	
0.67 centimeters to meters	
145 meters to centimeters	
0.005 meters to centimeters	

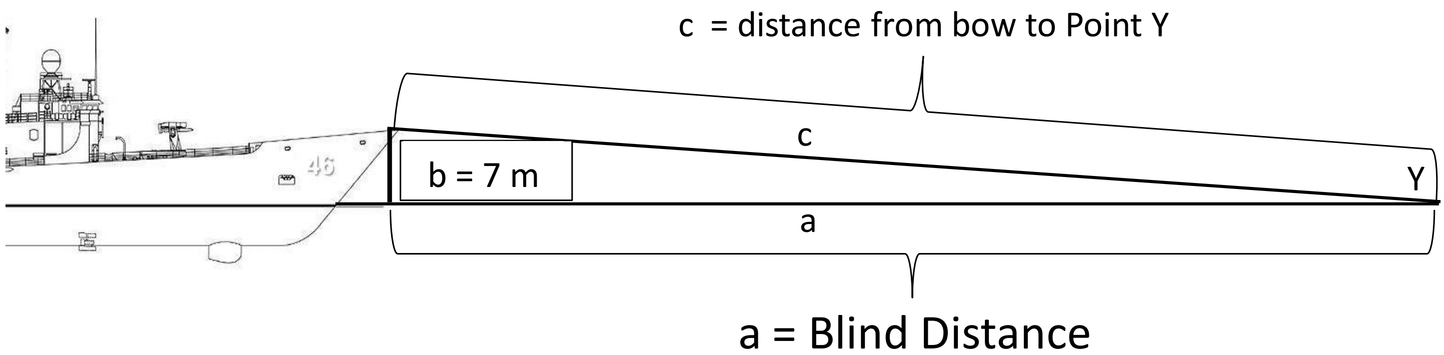
7. **Learned how to compute blind distances for your ship using the Pythagorean Theorem.**

Given any right triangle with side lengths a and b and hypotenuse length c the following relationship holds:

$$a^2 + b^2 = c^2$$

Practice Problem: Find the blind distance for the ship in the following diagram:

Blind distance in meters is: ____



8. **Learned how compute:**

a. Voyage Time and Voyage Distance:

$$voyage\ time\ in\ hours = \frac{distance\ in\ nautical\ miles}{speed\ in\ nautical\ miles\ per\ hour}$$

$$voyage\ distance = speed * transit\ time$$

Review Problem: Your ship is making a voyage of 3569 nautical miles at an average speed of 16 knots. How long in days will it take the ship to reach her destination?

b. Amount of ship's fuel used on a voyage:

$$Amt.of\ fuel\ used = (fuel\ used\ per\ nautical\ mile\ at\ ship's\ speed) * transit\ distance$$

c. Percentage of total fuel used on a voyage:

$$\%\ of\ total\ fuel\ used = \frac{amount\ of\ fuel\ used}{total\ fuel\ ship\ can\ carry}$$

Review Problem: Your ship consumes 112 gallons per nautical mile at 12 knots. Altogether she carries 306,000 gallons of fuel. How many gallons of fuel will she consume on a voyage of 2098 nautical miles and what percentage of her total fuel will she consume?

Name: ________________________

Date:_________________________

Advisor:_________________________

Task IV – Finding the Ship's Heading with a Compass

1. **Background.** We've learned how to find our ship's position at noon each day. Now we will look at how to keep our ship on a heading (a direction) that will take us to our destination. This heading is normally expressed in degrees from True North (the direction to the North Pole) on a compass and is displayed on the bridge of your ship on its compass.

Figure 7: Ship's Compass

Our ship has two types of compass. One, the gyrocompass, makes use of a gyroscope and the Earth's rotation to find True North. Because it finds True North, it is the compass most often used to find the ship's heading. The gyrocompass requires a continuous input of stable electrical power to work properly. If power is lost, the compass fails and is useless until it can be restarted and reset – normally not done at sea.

Because the gyrocompass can fail, our ship also carries a magnetic compass which does not require electrical power to operate. Magnetic compasses have a much longer history than gyrocompasses and, before the introduction of

electrical power on ships in the late nineteenth century, were the only compasses carried.

2. **How a Magnetic Compass Works.** A magnetic compass doesn't actually point to the magnetic North Pole. Instead, the compass points in the directions of the horizontal component of the Earth's magnetic field where the compass is located, and not to any single point. Knowing the magnetic declination (the angle between true north and the horizontal trace of the magnetic field) for your location allows you to correct your compass for the magnetic field in your area. A mile or two away the magnetic declination may be considerably different, requiring a different correction.[18]

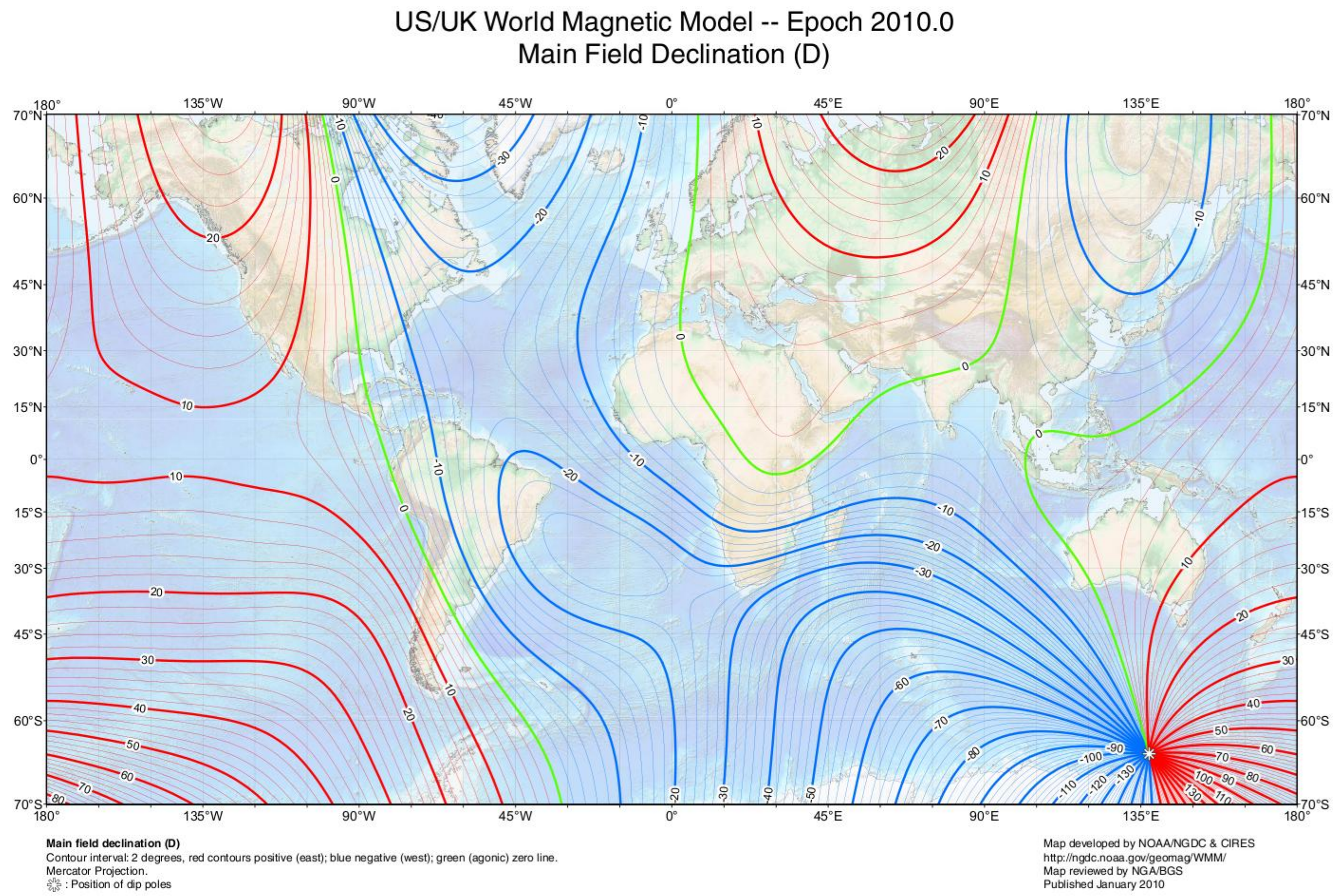
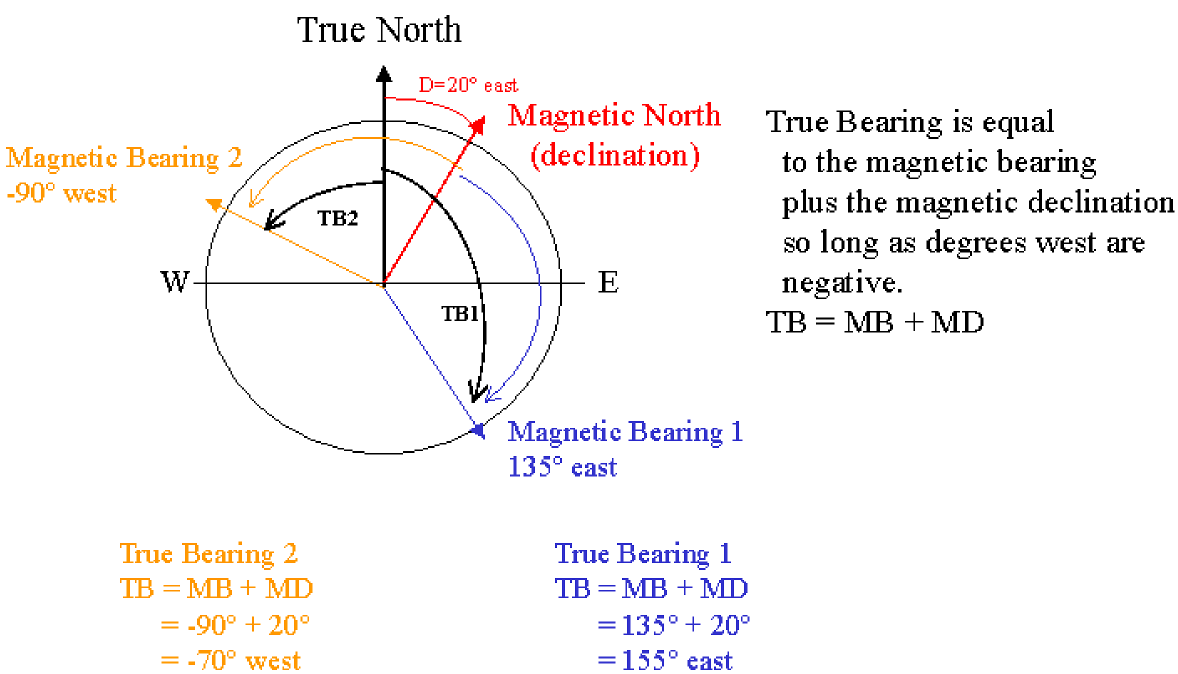


Figure 8: Magnetic Declination Worldwide

[18] National Geophysical Data Center, "Geomagnetism Frequently Asked Questions". Accessed online 09 Dec 11 at: http://www.ngdc.noaa.gov/geomag/faqgeom.shtml.

You can compute the true bearing (TB) from a magnetic bearing (MB) by adding the magnetic declination (MD) to the magnetic bearing. This works so long as you follow the convention of degrees west are negative (i.e. a magnetic declination of 10-degrees west is -10 and bearing of 45-degrees west is -45).[19] If your true bearing comes out negative, add 360°.

$$TB = MB + MD\ (+360°\ if\ result\ is\ negative)$$



Examples:

a. Magnetic bearing is 90° **West**. Magnetic declination is 20° **East**. Our true bearing is:

$$TB = -90° + 20° = -70°$$

$$-70° + 360° = 290°\,T$$

[19] Ibid.

b. Magnetic bearing is 80° **West**. Magnetic declination is 20° **West**. Our true bearing is:

$$TB = -80° - 20° = -100° + 360° = 260°\,T$$

c. Magnetic bearing is 100° **East**. Magnetic declination is 40° **West**. Our true bearing is:

$$TB = 100° - 40° = 60°\,T$$

d. Magnetic bearing is 45° **East**. Magnetic declination is 60° **East**. Our true bearing is:

$$TB = 45° + 60° = 105°\,T$$

3. **Task.** Solve the following problems. **Show all work on separate sheets of paper.**

Magnetic Bearing	**Declination**	**True Bearing**
120° W	40° E	280° T
155° E	10° E	165° T
32° W	44° W	284° T
176° E	21° E	197° T
57° W	78° W	225° T
165° W	47° E	242° T
130° E	68° W	062° T
75° W	21° E	306° T
125° W	75° W	160° T

Navigation Exercise – Voyage Plan

Waypoint	Latitude	Longitude	Bearing	Range	Time in hours at 20 knots
1 (San Diego)	32°38.2931'N	117°13.3649'W			
2	27°34.0033'N	116°8.8477'W	169°T	310 nm	15.50
3	22°18.5656'N	111°2.9883'W	138°T	421 nm	21.05
4	17°36.1283'N	105°33.3984'W	131°T	420 nm	21.00
5	14°56.6871'N	98°47.3438'W	111°T	422 nm	21.10
6	12°43.5651'N	92°1.2891'W	108°T	417 nm	20.85
7	9°26.9437'N	86°44.8828'W	122°T	368 nm	18.40
8	8°51.5000'N	85°40.0000'W	120°T	73 nm	3.65
9	8°00.0000'N	83°12.0000'W	110°T	155 nm	7.75
10	7°00.8201'N	81°46.9336'W	125°T	103 nm	5.15
11	7°09.0000'N	80°30.0000'W	085°T	77 nm	3.85
12	7°20.0000'N	79°15.8000'W	082°T	75 nm	3.75
13 (Panama Canal)	8°50.8247'N	79°29.1650'W	353°T	92 nm	4.60

Name: ________________________

Date:_________________________

Advisor:_________________________

Task V – Transit, Day 1
Show all work on separate sheets of paper attached to this one.

1. **Step 1:** Now that all planning is complete, your ship is ready to get underway for the Panama Canal. She departs Pier 4, U. S. Naval Station San Diego on the morning of November 16th, 2012 and begins her transit of San Diego Harbor to the open sea.

   As you transit the harbor, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	Declination	True Bearing
48° W	12° E	324° T
100° W	10° E	270° T
140° W	11° E	231° T

2. **Step 2.** Now that you have left San Diego harbor you will need to go alongside U.S.S. *Cimarron* (AO – 22) to refuel.

   a. Your ship can carry _______________ gallons of fuel (paragraph 5.a. from your "Panama Canal Ship Data" form. She currently has 75%. How many gallons is she carrying?

$$Full\ load\ in\ gallons \times 0.75 = Fuel\ onboard\ in\ gallons$$

   b. How much will she need to take from *Cimarron* to be full.

$$Full\ load\ in\ gallons - Fuel\ onboard\ in\ gallons = Amount\ needed$$

c. If your ship travels at **15 knots,** how far in nautical miles (nm) will she go before she burns 20% of her fuel?

$$\frac{Full\ load\ in\ gallons \times 0.20}{Gallons\ burned\ per\ nautical\ mile\ at\ 15\ kts.} = nm\ traveled$$

d. How long will it take her to burn that fuel?

$$\frac{nm\ traveled}{\frac{15\ nm}{hour}} = time\ to\ burn\ fuel$$

e. If you finish refueling by 12:00, how much fuel will you have onboard at midnight? What percentage of the total you can carry will you have onboard at midnight?

Amount onboard:

$$Amount\ burned = \left(12\ hours \times \left(15\frac{nm}{hour}\right)\right) \times gallons\ per\ nautical\ mile\ at\ 15\ knots$$

$$Amount\ onboard\ at\ close\ of\ Day\ 1 = Full\ load\ in\ gallons - Amount\ burned\ in\ gallons$$

Percentage of total onboard:

$$\frac{Amount\ onboard\ in\ gallons\ at\ close\ of\ Day\ 1}{Full\ load\ in\ gallons} = Percentage\ onboard\ at\ close\ of\ Day\ 1$$

3. **Step 3:** It is now almost 12:00 noon and time to determine the ship's position. From the following information, find the ship's latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 14th		34° 19'	23° 02' S	32° 39' N	

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information. **Your group's classroom grade for today will be based in part on how accurately you complete this message.**

O 150001Z NOV 13
FM USS ________________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT MIDNIGHT:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

A. UNDERWAY 0900T FM NAVSTA SDGO ENR PANAMA CANAL.
B. COMPLETED UNREP WITH USS CIMARRON. RECEIVED ________________________ GALLONS OF FUEL.
C. ENR PANAMA CANAL AT 15 KTS.

BT

Name: _______________________

Date:_______________________

Advisor:_______________________

Task V – Transit, Day 1
Show all work on separate sheets of paper attached to this one.

1. **Step 1:** Now that all planning is complete, your ship is ready to get underway for the Panama Canal. She departs Pier 4, U. S. Naval Station San Diego on the morning of November 16^{th}, 2012 and begins her transit of San Diego Harbor to the open sea.

   As you transit the harbor, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	Declination	True Bearing
48° W	12° E	
100° W	10° E	
140° W	11° E	

2. **Step 2.** Now that you have left San Diego harbor you will need to go alongside U.S.S. *Cimarron* (AO – 22) to refuel.

   a. Your ship can carry ______________ gallons of fuel (paragraph 5.a. from your "Panama Canal Ship Data" form. She currently has 75%. How many gallons is she carrying?

   b. How much will she need to take from *Cimarron* to be full.

c. If your ship travels at **15 knots,** how far in nautical miles (nm) will she go before she burns 20% of her fuel?

d. How long will it take her to burn that fuel?

e. If you finish refueling by 12:00, how much fuel will you have onboard at midnight? What percentage of the total you can carry will you have onboard at midnight?

Amount onboard:

Percentage of total onboard:

3. **Step 3:** It is now almost 12:00 noon and time to determine the ship's position. From the following information, find the ship's latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		34° 19'	23° 02' S		

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information. **Your group's classroom grade for today will be based in part on how accurately you complete this message.**

O 150001Z NOV 13
FM USS ________________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________ //
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT MIDNIGHT:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

D. UNDERWAY 0900T FM NAVSTA SDGO ENR PANAMA CANAL.
E. COMPLETED UNREP WITH USS CIMARRON. RECEIVED ________________________ GALLONS OF FUEL.
F. ENR PANAMA CANAL AT 15 KTS.

BT

Name: ________________________

Date: ________________________

Advisor: ________________________

Task VI – Transit, Day 2
Show all work on separate sheets of paper attached to this one.

Today is the second day of your transit to the Panama Canal. You have traveled **384** nautical miles (nm) South-Southeast of San Diego from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		40° 23.15'	23° 08' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 yesterday until 12:00 today?

$$nm\ traveled\ in\ 24\ hours \times gallons\ per\ nm\ at\ 15\ knots = fuel\ burned\ in\ 24\ hours$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

Percentage burned:

$$\frac{Fuel\ burned\ in\ 24\ hours\ at\ 15\ knots}{(Full\ load\ in\ gallons)} = Percentage\ burned$$

Percentage left onboard:

$$100\% - Percentage\ burned$$

c. At 12 noon your ship receives an order from Commander, Third Fleet (COMTHRIDFLT) to proceed to 23° 43’ N / 111° 23’ W – a position just offshore of the Mexican resort of Cabo San Lucas to pick up two important passengers at the fishing port for transport to Panama City. Your ship must be in position by 12:00 noon tomorrow (24 hours from now). The position is 402 nm from your ship’s current position.

   Is 15 knots sufficient speed to travel to this new position in 24 hours? Why not?

$$15\ knots * 24\ hours = 360\ nm < 402\ nm$$

d. Is 20 knots sufficient speed?

$$20\ knots \times 24\ hours = 480\ nm > 402\ nm$$

e. What percentage of fuel will she consume at 20 knots? What percentage will remain?

$$\frac{(Gallons\ burned\ per\ nm\ at\ 20\ knots) \times 402\ nm}{Full\ load\ in\ gallons} = Percentage\ of\ fuel\ burned\ at\ 20\ knots$$

$$Percentage\ onboard\ at\ close\ of\ Day\ 1 - Percentage\ of\ fuel\ burned\ at\ 20\ knots$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 2$$

3. **Step 3: Speed Computation.** During the afternoon watch (12 noon – 4 PM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	**Noon – 4 pm**
1 bell	280 yards
2 bells	285 yards
3 bells	278 yards
4 bells	289 yards
5 bells	288 yards
6 bells	274 yards
7 bells	270 yards
8 bells	285 yards
Average Speed	16.87 knots

a. What is your average speed in nautical miles per hour (knots)?

   16.87 knots

b. Is this speed sufficient to get to your destination on time?

   No.

4. **Task 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information.

O 160001Z NOV 13
FM USS ___________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS________________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:
A. ENR 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z TO PICK UP PAX. ETA 141200 LOCAL.
B. PAC FIRE
C. CONDITION I DRILLS.

BT

Name: ____________________________

Date:_____________________________

Advisor:___________________________

Task VI – Transit, Day 2
Show all work on separate sheets of paper attached to this one.

Today is the second day of your transit to the Panama Canal. You have traveled 384 nautical miles (nm) South-Southeast of San Diego from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		40° 23.15'	23° 08' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 yesterday until 12:00 today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

Percentage burned:

Percentage left onboard:

c. At 12 noon your ship receives an order from Commander, Third Fleet (COMTHRIDFLT) to proceed to 23° 43’ N / 111° 23’ W – a position just offshore of the Mexican resort of Cabo San Lucas to pick up two important passengers at the fishing port for transport to Panama City. Your ship must be in position by 12:00 noon tomorrow (24 hours from now). The position is 480 nm from your ship’s current position.

   Is 15 knots sufficient speed to travel to this new position in 24 hours? Why not?

d. Is 20 knots sufficient speed?

e. What percentage of fuel will she consume at 20 knots? What percentage will remain?

3. **Step 3: Speed Computation.** During the afternoon watch (12 noon – 4 PM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	**Noon – 4 pm**
1 bell	280 yards
2 bells	285 yards
3 bells	278 yards
4 bells	289 yards
5 bells	288 yards
6 bells	274 yards
7 bells	270 yards
8 bells	285 yards
Average Speed	16.87 knots

c. What is your average speed in nautical miles per hour (knots)?

d. Is this speed sufficient to get to your destination on time?

4. **Step 4:** By midnight, you need to file a report with Commander, Task Force 5 containing all the information you calculated above. Fill out the message template below with that information.

O 160001Z NOV 13
FM USS ___________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS________________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

A. ENR 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z TO PICK UP PAX. ETA 141200 LOCAL.
B. PAC FIRE
C. CONDITION I DRILLS.

BT

Name: _________________________

Date:___________________________

Advisor:___________________________

Task VII – Transit, Day 3
Show all work on separate sheets of paper attached to this one.

Today is the third day of your transit to the Panama Canal. You have traveled **360** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		43° 06'	23°11' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

$$480\ nm * Gallons\ burned\ per\ nm\ at\ 20\ knots = Gallons\ burned\ last\ 24\ hours$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Gallons\ burned\ last\ 24\ hours}{Full\ load\ in\ gallons} = Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3$$

$$Percentage\ onboard\ at\ close\ of\ Day\ 2 - Percentage\ of\ fuel\ burned\ at\ 20\ knots$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3$$

c. The distance between San Diego and the Pacific Ocean entrance to the Panama Canal is 2844 nautical miles. How much of that distance remains after two days?

$$2844\ nm - 360\ nm - 480\ nm = 2004\ nm$$

d. If your ship travels at 15 knots for the remainder of the voyage how much fuel in gallons will remain and what percentage will remain?

$$2004\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ for\ remainder\ of\ voyage$$

$$\frac{Gallons\ burned\ for\ remainder\ of\ voyage}{Full\ load\ in\ gallons} = Percentage\ burned\ for\ remainder\ of\ voyage$$

$$(Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3) - Percentage\ burned\ for\ remainder\ of\ voyage$$
$$= Percentage\ remaining\ onboard\ at\ end\ of\ voyage$$

e. Commander, Third Fleet requires that no ship drop below 50% fuel while operating in the Third Fleet area. Therefore, you will have to stop at a port to refuel. You have two options:

Puerto Quetzel, Guatemala	**1163 nm distant**
Golfito, Costa Rica	**1703 nm distant**

What percentage of fuel will you have left when you reach each port?

Puerto Quetzel:

$$1163\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ enroute\ Puerto\ Quetzel$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - \frac{Gallons\ burned\ enroute\ Puerto\ Quetzel}{Full\ load\ in\ gallons}$$
$$= Percentage\ remaining\ at\ Puerto\ Quetzel$$

Golfito:

$$1703\ nm * (Gallons\ burned\ per\ nm\ at\ 15\ knots) = Gallons\ burned\ enroute\ Golfito$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - \frac{Gallons\ burned\ enroute\ Golfito}{Full\ load\ in\ gallons} = Percentage\ remaining\ at\ Golfito$$

f. Which port do you choose and why?

3. **Step 3:** By midnight, you need to file a report with Commander, Task Force V containing all the information you calculated above. Fill out the message template below with that information.

O 170001Z NOV 13
FM USS ______________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//
1. SHIP'S 1200 POSITION: LATITUDE __________________________

LONGITUDE __________________________

2. FUEL ONBOARD AT 1200:

GALLONS _____________________________

PERCENT _____________________________

3. EVENTS COMPLETED:
A. COMPLETED PAX TRANSFER AT 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z.
B. ENR __________________________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Name: ___________________________

Date:____________________________

Advisor:_____________________________

Task VII – Transit, Day 3
Show all work on separate sheets of paper attached to this one.

Today is the third day of your transit to the Panama Canal. You have traveled **360** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		43° 06'	23°11' S		

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

c. The distance between San Diego and the Pacific Ocean entrance to the Panama Canal is 2844 nautical miles. How much of that distance remains after two days?

d. If your ship travels at 15 knots for the remainder of the voyage how much fuel in gallons will remain and what percentage will remain?

e. Commander, Third Fleet requires that no ship drop below 50% fuel while operating in the Third Fleet area. Therefore, you will have to stop at a port to refuel. You have two options:

Puerto Quetzel, Guatemala	**1163 nm distant**
Golfito, Costa Rica	**1703 nm distant**

What percentage of fuel will you have left when you reach each port?

Puerto Quetzel:

Golfito:

f. Which port do you choose and why?

3. **Step 3:** By midnight, you need to file a report with Commander, Task Force V containing all the information you calculated above. Fill out the message template below with that information.

O 170001Z NOV 13
FM USS ______________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//
1. SHIP'S 1200 POSITION: LATITUDE ___________________________

LONGITUDE __________________________

2. FUEL ONBOARD AT 1200:

GALLONS _____________________________

PERCENT _____________________________

3. EVENTS COMPLETED:
A. COMPLETED PAX TRANSFER AT 23° 43' N / 111° 23' W IAW COMTHRIDFLT 152000Z.
B. ENR __________________________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task VII – Transit, Day 4
Show all work on separate sheets of paper attached to this one.

Today is the fourth day of your transit to the Panama Canal. You have traveled **345** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		47° 28'	23° 14' S	19° 18' N	

2. **Step 2 – Logistics Requirements Message (LOGREQ).** Complete the LOGREQ message at the end of this document by filling in the blanks. All of the information you need has already been provided or determined by you.

3. **Step 3 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you burn from 12:00 noon yesterday until 12:00 noon today?

$$345\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4$$

3. **Step 4: Weather Forecast.** Your course will take you past the Gulf of Tehuantepec on the western coast of Guatemala. The northern boundary of the gulf is 750 nautical miles from your current position.

   a. How long before you arrive at the northern boundary (in days and hours)?

   $$\frac{750\ nm}{15\ knots} = 50\ hours = 2\ days, 2\ hours$$

   b. Find two online resources that describe what makes the Gulf of Tehuantepec dangerous to ships and boats passing through it. **In your own words on a separate sheet of paper**, **type** a one paragraph (3 – 4 sentences) description of the danger. **Ensure you cite both sources. Attach your paragraph to this paper.**

   c. Search online for the High Seas Forecast for the Gulf of Tehuantepec (provided by the National Hurricane Center in Miami, Florida). Find the following forecast information:

   Forecast Winds (direction and speed): ________________________________

   Forecast Seas (height in feet): ______________________________________

   d. Your ship will ride comfortably in seas up to 10 feet in height. Will your ship and her crew have a comfortable passage across the Gulf of Tehuantepec?

4. **Step 5: Daily Operations Summary.** Complete and submit the Daily OPSUM on the next page along with your LOGREQ message.

Daily Operations Summary

O 200001Z NOV 13
FM USS ________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

A. ENR _________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Logistics Requirement Message

P 200001Z NOV 13
FM USS ____________________
TO USNAO GUATEMALA CITY GU
INFO CTF FIVE
COMTHIRDFLT
BT

C O N F I D E N T I A L //N04000//

MSGID/LOGREQ/_________________________/001/NOV//
REF/NWP 1-03.1//

ALFA: ESTIMATED TIME OF ARRIVAL __________________: ______________________

BRAVO:

(1) PILOT, ONE TUG.

(2) SHIP'S LENGTH: ____

(3) SHIP'S BEAM: ____

(4) SHIP'S DRAUGHT: ____

FOXTROT: REQUEST ___________________________ GALLONS FUEL UPON ARRIVAL.

HOTEL: DRY AND REFRIGERATED FOOD STORES REQUIREMENTS PROVIDED TO HUSBANDING AGENT SEPCOR.

KILO: SEWAGE AND OILY WASTE DISPOSAL. TRASH AND GARBAGE DISPOSAL.

PAPA: REQUEST FREE PRATIQUE.

UNIFORM: __________________________________/COMMANDER/USN//

BT

Name: ____________________________

Date:_____________________________

Advisor:___________________________

Task VII – Transit, Day 4
Show all work on separate sheets of paper attached to this one.

Today is the fourth day of your transit to the Panama Canal. You have traveled **390** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
		47° 28'	23° 14' S	19° 18' N	

2. **Step 2 – Logistics Requirements Message (LOGREQ).** Complete the LOGREQ message at the end of this document by filling in the blanks. All of the information you need has already been provided or determined by you.

3. **Step 3 – Ship's Fuel Consumption.**

   a. At 15 knots on one engine, how much fuel did you burn from 12:00 noon yesterday until 12:00 noon today?

$$390\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 3 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4$$

4. **Step 4: Weather Forecast.** Your course will take you past the Gulf of Tehuantepec on the western coast of Guatemala. The northern boundary of the gulf is 750 nautical miles from your current position.

    a. How long before you arrive at the northern boundary (in days and hours)?

    $$\frac{750\ nm}{15\ knots} = 50\ hours = 2\ days, 2\ hours$$

    b. Find two online resources that describe what makes the Gulf of Tehuantepec dangerous to ships and boats passing through it. **In your own words on a separate sheet of paper**, **type** a one paragraph (3 – 4 sentences) description of the danger. **Ensure you cite both sources. Attach your paragraph to this paper.**

    c. Search online for the High Seas Forecast for the Gulf of Tehuantepec (provided by the National Hurricane Center in Miami, Florida). Find the following forecast information:

    Forecast Winds (direction and speed): ________________________________

    Forecast Seas (height in feet): ______________________________________

    d. Your ship will ride comfortably in seas up to 10 feet in height. Will your ship and her crew have a comfortable passage across the Gulf of Tehuantepec?

5. **Step 5: Daily Operations Summary.** Complete and submit the Daily OPSUM on the next page along with your LOGREQ message.

Daily Operations Summary

O 200001Z NOV 13
FM USS ________________________
TO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//
1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

B. ENR _________________ TO REFUEL. ETA AND LOGREQ TO FOLLOW.

BT

Logistics Requirement Message

P 200001Z NOV 13
FM USS ____________________
TO USNAO GUATEMALA CITY GU
INFO CTF FIVE
COMTHIRDFLT
BT

C O N F I D E N T I A L //N04000//

MSGID/LOGREQ/_________________________/001/NOV//
REF/NWP 1-03.1//

ALFA: ESTIMATED TIME OF ARRIVAL __________________: ______________________

BRAVO:

(1) PILOT, ONE TUG.

(2) SHIP'S LENGTH: ____

(3) SHIP'S BEAM: ____

(4) SHIP'S DRAUGHT: ____

FOXTROT: REQUEST ___________________________ GALLONS FUEL UPON ARRIVAL.

HOTEL: DRY AND REFRIGERATED FOOD STORES REQUIREMENTS PROVIDED TO HUSBANDING AGENT SEPCOR.

KILO: SEWAGE AND OILY WASTE DISPOSAL. TRASH AND GARBAGE DISPOSAL.

PAPA: REQUEST FREE PRATIQUE.

UNIFORM: __________________________________/COMMANDER/USN//

BT

Name: ______________________

Date: ______________________

Advisor: ______________________

Task VIII – Transit, Day 5
Show all work on separate sheets of paper attached to this one.

Today is the fifth day of your transit to the Panama Canal. You have traveled **370** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

$$370\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 4 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 5$$

3. At 1200 you receive an intelligence report that a major transfer of pure cocaine (several tons) will be conducted between a large fishing vessel and several "go-fast" high speed boats. The transfer will take place in 3 hours approximately 75 nautical miles south from your 1200 position.

   a. Plot your 1200 position on the chart and lay – out a DR track, 180° T, for three hours. Locate the estimated position of the drug transfer.

   b. At what speed will you need to travel to arrive at this position on time?

$$\frac{75\ nm}{3\ hours} = 25\ knots$$

   c. How much fuel (gallons and percent) will you burn traveling at this speed in the next three hours?

$$75\ nm \times Gallons\ burned\ per\ nm\ at\ 25\ knots = Gallons\ burned$$

$$\frac{Gallons\ burned}{Full\ load\ in\ gallons} = Percentage\ burned$$

4. At 1230 you receive the following encoded message from the Commandant of the Fifteenth Naval District headquartered at U.S. Naval Station Panama Canal.

Z 201900Z NOV 12
FM COMFIFTEEN
TO USS ___________________________
INFO CTF FIVE
COMTHIRDFLT
BT
S E C R E T (WHEN DECODED)
400 324 1 196 361 36 25 324 - 225 256 25 324 1
400 81 225 196

400 225 - 400 1 121 25 - 256 144 1 9
25 - **180** - 400

- **65** - 196 169 - 36 324 225 169 - 625 225
441 324 - **1200**
256 225 361 81 400 81 225 196
BT

a. Decode this message.

   TRANSFER OPERATION TO TAKE PLACE 180T 65 NM FROM YOUR POSITION AT 1200.

b. How does this message change (if it does change) your answers to questions 3a. and 3b.?

$$65\ nm \times Gallons\ burned\ per\ nm\ at\ 25\ knots = Gallons\ burned$$

$$\frac{Gallons\ burned}{Full\ load\ in\ gallons} = Percentage\ burned$$

Daily Operations Summary

O 210001Z NOV 13
FM USS ____________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS ________________//

1. SHIP'S 1200 POSITION: LATITUDE ___________________________

LONGITUDE __________________________

2. FUEL ONBOARD AT 1200:

GALLONS _____________________________

PERCENT _____________________________

3. EXPENDED _______________________________ GALLONS OF FUEL IN SUPPORT OF COMFIFTEEN 201900Z NOV 13 OPERATIONS.

4. EVENTS COMPLETED:

A. CONDUCTING OPERATIONS IAW COMFIFTEEN 201900Z NOV 13.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task VIII – Transit, Day 5
Show all work on separate sheets of paper attached to this one.

Today is the fifth day of your transit to the Panama Canal. You have traveled **370** nautical miles (nm) South-Southeast from 12:00 local time yesterday until 12:00 local time today.

1. **Step 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 15 knots, how much fuel did you consume from 12:00 noon yesterday until 12:00 noon today?

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

5. At 1200 you receive an intelligence report that a major transfer of pure cocaine (several tons) will be conducted between a large fishing vessel and several "go-fast" high speed boats. The transfer will take place in 3 hours approximately 75 nautical miles south from your 1200 position.

    a. Plot your 1200 position on the chart and lay – out a DR track, 180° T, for three hours. Locate the estimated position of the drug transfer.

    b. At what speed will you need to travel to arrive at this position on time?

    c. How much fuel (gallons and percent) will you burn traveling at this speed in the next three hours?

6. At 1230 you receive the following encoded message from the Commandant of the Fifteenth Naval District headquartered at U.S. Naval Station Panama Canal.

Z 201900Z NOV 13
FM COMFIFTEEN
TO USS ___________________________
INFO CTF FIVE
COMTHIRDFLT
BT
S E C R E T (WHEN DECODED)
400 324 1 196 361 36 25 324 - 225 256 25 324 1
400 81 225 196

400 225 - 400 1 121 25 - 256 144 1 9
25 - **180** - 400

- **65** - 196 169 - 36 324 225 169 - 625 225
441 324 - **1200**
256 225 361 81 400 81 225 196
BT

a. Decode this message.

   TRANSFER OPERATION TO TAKE PLACE 180T 65 NM FROM YOUR POSITION AT 1200.

b. How does this message change (if it does change) your answers to questions 3a. and 3b.?

Daily Operations Summary

O 210001Z NOV 13
FM USS ___________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//

1. SHIP'S 1200 POSITION: LATITUDE _________________________

   LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

   GALLONS ___________________________

   PERCENT ___________________________

3. EXPENDED _____________________________ GALLONS OF FUEL IN SUPPORT OF COMFIFTEEN 201900Z NOV 13 OPERATIONS.

4. EVENTS COMPLETED:

A. CONDUCTING OPERATIONS IAW COMFIFTEEN 201900Z NOV 13.

BT

Name: ________________________

Date:_________________________

Advisor:_________________________

Task XI – Transit, Day 7
Show all work on separate sheets of paper attached to this one.

Today is the seventh day of your transit to the Panama Canal. You left your refueling port two days ago and have traveled **895** nautical miles (nm) South-Southeast from 12:00 local time two days ago until 12:00 local time today.

1. **Step 1 – Ship's Position.**

   a. Find the declination of the Sun for today from and online resource.

   Sun's Declination is: ____

   The resource I used is: ____

   b. Find a government online resource for **Complete Sun and Moon for One Day** for the Panama Canal. Use the following information:

   Latitude: 9° N
   Longitude: 80° W
   Time Zone: 8 hours west of Greenwich

   Find the following information (assume the time is local apparent noon):

   Sunrise: ____

   Sunset: ____

   GMT of LAN: ____

   Sun's Altitude: ____

2. **Step 2 – Ship's Fuel Consumption.**

   a. At 20 knots, how much fuel did you consume in the last 48 hours?

$$895\ nm \times Gallons\ burned\ per\ nm\ at\ 15\ knots = Fuel\ burned\ in\ gallons$$

   b. What percentage of fuel have you burned? What percentage do you have left onboard?

$$\frac{Fuel\ burned\ in\ gallons}{Full\ load\ in\ gallons} = Percentage\ burned$$

$$Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 5 - Percentage\ burned$$
$$= Percentage\ remaining\ on\ board\ at\ close\ of\ Day\ 7$$

3. **Step 3: Speed Computation.** During mid-watch (12 midnight – 4 AM), you measure the distance the ship travels in 30 seconds every half hour. The following are your results:

Time	**Midnight – 4AM**
1 bell	290 yards
2 bells	295 yards
3 bells	288 yards
4 bells	299 yards
5 bells	298 yards
6 bells	284 yards
7 bells	280 yards
8 bells	295 yards
Average Speed	17.46 knots

O 270001Z NOV 13
FM USS ____________________________
TO CTF FIVE
INFO COMTHIRDFLT
BT
C O N F I D E N T I A L //N03120//
SUBJ: DAILY OPERATIONS SUMMARY (OPSUM), USS _______________//

1. SHIP'S 1200 POSITION: LATITUDE _________________________

LONGITUDE ________________________

2. FUEL ONBOARD AT 1200:

GALLONS ___________________________

PERCENT ___________________________

3. EVENTS COMPLETED:

a. ENROUTE PANAMA CANAL.

Name: ___________________________

Date:____________________________

Advisor:__________________________

Assessment Review – Navigation Unit

1. **Task.** In each section below, use the variables I give you to set up the formula to solve each type of problem.

   a. Latitude and Longitude:

      i. Latitude:

Variables:

Latitude (La)
Observed Altitude of Sun (O)
Sun's Declination (D)

Formula:

$$La = 90° - O + D$$
(Sun in Northern Hemisphere)

$$La = 90° - O - D$$
(Sun in Southern Hemisphere)

      ii. Longitude:

Variables:

Longitude: (Lo)
GMT of LAN (GMT)
Result of Subtraction (S)
Hours from Subtraction (H)
Minutes from Subtraction (M)

Formulas:

1. $S = 12{:}00 - GMT\ (12{:}00 > GMT)$
   $S = GMT - 12{:}00\ (12{:}00 < GMT)$

2. $Lo = H * 15° + \frac{M}{(4' per\ °)}$

   b. True Bearing:

Variables:

True Bearing (TB °T)
Magnetic Bearing (M °E or °W)
Declination (D °E or °W)

Formulas:

TB °T $= \pm$M $\pm$ D + 360° (if initial result is negative.

c. Distance Traveled:

Variables:

Distance Traveled (D nm)
Speed in Knots (S nm/hr)
Time of Journey (T hrs.)

Formulas:

$$D\ nm = \left(\frac{S\ nm}{hr}\right) * T\ hrs.$$

d. Fuel Burned:

Variables:

Fuel Burned on Journey (F gal.)
Gallons Burned per NM (G gal.)
Distance Traveled (D nm)

Formulas:

$$F\ gal. = \left(\frac{G\ gal}{nm}\right) * D\ nm$$

e. Percentage of Fuel Burned:

Variables:

Percentage of Fuel Burned (B)
Fuel Burned on Journey (F gal.)
Total Fuel Carried (T gal.)

Formulas:

$$B = \frac{F\ gal.}{T\ gal.}$$

f. Percentage of Fuel Remaining:

Variables:

Percentage of Fuel Remaining (P)
% Fuel Onboard fm Previous Day (PD)
Percentage of Fuel Burned Today (B)

Formulas:

$$P = PD - B$$

2. **Additional Information:**

Your Ship Fuel Consumption

Ship's Speed (Knots)	Gallons per Nautical Mile (GPNM) - Single Engine	Gallons per Nautical Mile (GPNM) - Duel Engine
5	95	134
6	80	112.5
7	70	97
8	61	86
9	56	78
10	52	71.5
11	48.5	67
12	46.5	63.5
13	45	60
14	44.5	58.5
15	44.5	58
16	44.9	57.5
17	45.1	58
18	47	59
19	49	60
20	50.5	62
21	54	63.5
22	56	65.5
23	60	69
24	63.5	72
25		75
26		79.5
27		84
28		88
29		94.5
30		100

Name: ___________________________

Date:____________________________

Advisor:__________________________

Assessment Review – Navigation Unit

1. **Task.** In each section below, use the variables I give you to set up the formula to solve each type of problem.

   a. Latitude and Longitude:

      i. Latitude:

      <u>Variables</u>:

      Latitude (La)
      Observed Altitude of Sun (O)
      Sun's Declination (D)

      <u>Formula</u>:

      ii. Longitude:

      <u>Variables</u>:

      Longitude: (Lo)
      GMT of LAN (GMT)
      Result of Subtraction (S)
      Hours from Subtraction (H)
      Minutes from Subtraction (M)

      <u>Formulas</u>:

   b. True Bearing:

      <u>Variables</u>:

      True Bearing (TB °T)
      Magnetic Bearing (M °E or °W)
      Declination (D °E or °W)

      <u>Formulas</u>:

c. Distance Traveled:

Variables:

Distance Traveled (D nm)
Speed in Knots (S nm/hr)
Time of Journey (T hrs.)

Formulas:

d. Fuel Burned:

Variables:

Fuel Burned on Journey (F gal.)
Gallons Burned per NM (G gal.)
Distance Traveled (D nm)

Formulas:

e. Percentage of Fuel Burned:

Variables:

Percentage of Fuel Burned (B)
Fuel Burned on Journey (F gal.)
Total Fuel Carried (T gal.)

Formulas:

f. Percentage of Fuel Remaining:

Variables:

Percentage of Fuel Remaining (P)
% Fuel Onboard fm Previous Day (PD)
Percentage of Fuel Burned Today (B)

Formulas:

2. **Additional Information:**

Your Ship Fuel Consumption

Ship's Speed (Knots)	Gallons per Nautical Mile (GPNM) - Single Engine	Gallons per Nautical Mile (GPNM) - Duel Engine
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Standard: ________________ Name: ______________________

HL: ________________ Date: ______________________

Advisor: ______________________

Navigation Unit Assessment

Show all work on separate sheets of paper stapled to this one. Put answers on this sheet

Today is the last day of your transit to the Panama Canal. You have traveled 895 nautical miles (nm) since you left Puerto Quetzel, Guatemala. You have traveled at an average speed of 15 knots since you left port.

1. **Task 1 – Ship's Position.** From the following information, find the ship's 12:00 noon latitude and longitude.

Date	GMT of LAN	Sun's Altitude	Sun's Declination	Latitude	Longitude
November 27th	17:18	60° 59'	21° 01'	8° 30' N	079° 30' W

2. **Task 2 – Ship's Fuel Consumption.**

   a. How long in hours has it taken you to reach the Panama Canal after leaving Puerto Quetzel, Guatemala? (Extra credit if you provide the time in days, hours, minutes, seconds)

$$\frac{895\ nm}{15\frac{nm}{hr}} = 59.67\ hours = 2\ days, 11.67\ hours$$

$$0.67\ hours * 60\left(\frac{min.}{hr}\right) = 40.2\ minutes$$

$$0.2\ minutes * 60\frac{sec}{min} = 12\ seconds$$

$$2\ days, 9\ hours, 40\ minutes, 12\ seconds$$

   b. On one engine, how much fuel have you burned?

$$44.5\frac{gal.}{nm} * 895\,nm = 39827.5\ gallons$$

c. What percentage of fuel have you burned?

$$\frac{39827.5\ gallons}{205{,}000\ gallons} * 100 = 19.43\%$$

d. What percentage do you have left onboard? (*your ship* can carry a total of 205,000 gallons of fuel). ***Be careful on this one.***

$$100\% - 19.43\% = 80.57\%$$

e. If you had travelled on two engines, how much fuel **in gallons and percent** would you have burned?

$$58\frac{gal.}{nm} * 895\,nm = 51910\ gallons$$

$$\frac{51910\ gallons}{205000\ gallons} = 25.32\%$$

3. As *your ship* approaches the canal, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

Magnetic Bearing	Declination	True Bearing
90° W	45° E	315° T
60° W	10° E	310° T
59° W	11° E	312° T

Standard: ________________   Name: ______________________

HL: ________________   Date: ______________________

Advisor: ______________________

Navigation Unit Assessment

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November 27th	17:18	60° 59'	21° 01'		

2. **Task 2 – Ship's Fuel Consumption.**

   a. How long in days and hours has it taken you to reach the Panama Canal after leaving Puerto Quetzel, Guatemala? (Extra credit if you provide the time in days, hours, minutes, seconds)

   b. On one engine, how much fuel have you burned?

c. What percentage of fuel have you burned?

d. What percentage do you have left onboard? (*your ship* can carry a total of 205,000 gallons of fuel). ***Be careful on this one.***

e. If you had travelled on two engines, how much fuel **in gallons and percent** would you have burned?

3. As *your ship* approaches the canal, the navigator observes the following bearings on her magnetic compass and also notes the following declinations. Find the true bearings the ship is traveling on when the navigator makes each observation.

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90° W	45° E	
60° W	10° E	
59° W	11° E	